Introduction

Reactive Power Supply of RES

Motivation

- Renewable energy sources (RESs) e.g., wind and large-scale photovoltaic plants connected at EHV/HV interfaces should have the capability to provide controllable reactive power as requested by grid codes
- Their reactive power exchange with the grid follows a voltage-reactive power droop characteristic

➢ An accurate assessment of Q(U) droop characteristic of RESs on long-term voltage stability necessitates the expansion of the conventional power flow problem

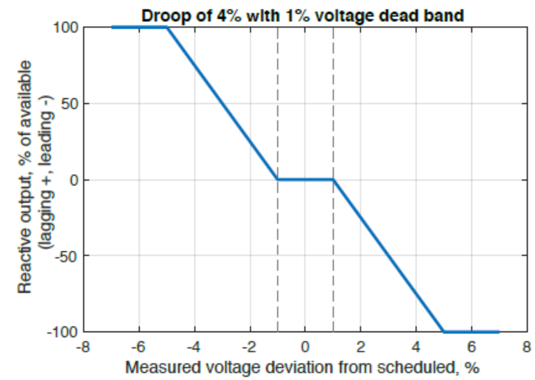
Q(U) droop characteristic

- The reactive power injection is a function of the measured voltage at PCC and the parameters of the characteristic curve

$$Q_G = Q_0 + {}^{1}/_{n_q}\,(V_0 - V)$$

- In practice, the reference voltage is changed to enable a different reactive power exchange at the connection point

Example of reactive power droop control with dead band



Theoretical Background

Power Flow and the Jacobian

- Power Flow problem
  - Used to determine bus voltages and line flows
  - Known quantities: generation, load and grid configuration
- The sum of the powers entering a bus must be zero
  - $0 = \Delta P_i = P_i^{inj} - V_i \sum_{j=1}^{N_{bus}} V_j Y_{ij} \cos(\delta_i - \delta_j - \varphi_{ij})$
  - $0 = \Delta Q_i = Q_i^{inj} - V_i \sum_{j=1}^{N_{bus}} V_j Y_{ij} \sin(\delta_i - \delta_j - \varphi_{ij})$

⇩

Nonlinear equations solved numerically using **Newton-Raphson method**

Jacobian

mismatches

corrections

$$
\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \hline \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix}
=
\left[\begin{array}{ccc:ccc}
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⇩

- Derivatives of **active power** mismatch equations with resp. to voltage angles δ_i and voltage magnitudes $|V_i|$

Theoretical Background

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Nonlinear equations solved numerically using
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- Derivatives of **active power** mismatch equations with resp. to voltage angles δ_i and voltage magnitudes $|V_i|$
- Derivatives of **reactive power** mismatch equations with resp. to voltage angles δ_i and voltage magnitudes $|V_i|$

Theoretical Background

Continuation Power Flow (CPF)

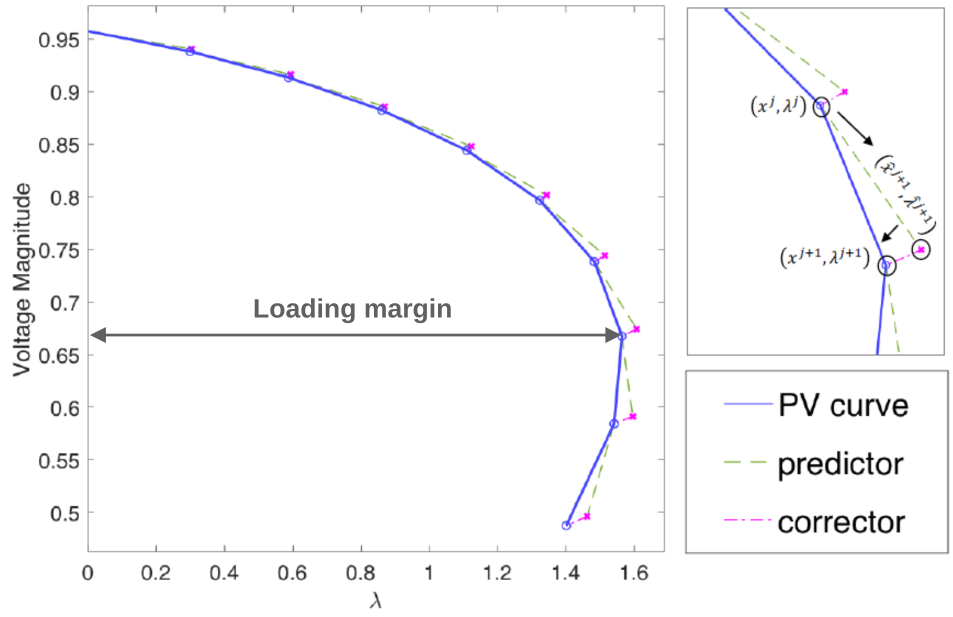
- Tracing the full course of a PV curve requires consecutive PF
  - Issue: Singular Jacobian at nose point→ PF breaks down
  - Solution: Continuation Power Flow

- **Predictor step** → Estimates the next solution using tangential approx.

predicted sol. ↓ **current sol.** ↓

- $$\begin{bmatrix}\hat{x}^{j+1}\\ \hat{\lambda}^{j+1}\end{bmatrix} = \begin{bmatrix}x^{j}\\ \lambda^{j}\end{bmatrix} + \sigma^{j}\bar{z}^{j}$$ ← **tangent vector**

↑ **step size**



- **Corrector step** → Finds the next solution by correcting the estimated solution

- Loading margin
  - Distance between initial and max load
  - Indicative of max load increase which can be taken without loss of stability

Modelling

Droop Bus

- Classification of bus types according to known quantities:

BUS TYPE	P	Q	$\|V\|$	δ
PQ bus	known	known	unknown	unknown
PV bus	known	unknown	known	unknown
Slack bus	unknown	unknown	known	known

- Number of equations reduced by removing:
  - Reactive power mismatch equation for each PV bus
  - Active and reactive power mismatch equations of slack bus

Modelling
Droop Bus

- Classification of bus types according to known quantities:

BUS TYPE	P	Q	$\|V\|$	δ
PQ bus	known	known	unknown	unknown
PV bus	known	unknown	known	unknown
Slack bus	unknown	unknown	known	known
Droop bus	known	volt. dependent	unknown	unknown

Reactive power droop control

voltage deviation

$$Q_i^{(k)} = Q_{o,i} + \frac{1}{n_q}\left(V_{o,i} - V_i^{(k)}\right)$$

$$\frac{\partial Q_i^{(k)}}{\partial V_i^{(k)}} = -\frac{1}{n_q}$$

- Number of equations reduced by removing:
  - Reactive power mismatch equation for each PV bus
  - Active and reactive power mismatch equations of slack bus

Modelling

Modified Jacobian

Jacobian

mismatches

corrections

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$$

Reactive power mismatch of the droop bus

$$
\Delta Q_i^{(k)} = \overbrace{Q_{0,i} + {}^{1}/_{n_q}\left(V_{0,i} - V_i^{(k)}\right)}^{inj.} - \overbrace{V_i^{(k)} \sum_{j=1}^{n} V_j^{(k)} Y_{ij} \, sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \varphi_{ij}\right)}^{calc.}
$$

- **Reactive power injection of droop bus is a function of its voltage magnitude → needs to be updated iteratively**

Modelling

Modified Jacobian

mismatches | Jacobian | corrections

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \frac{\partial \Delta P_2^{(k)}}{\partial \delta_2^{(k)}} & \cdots & \frac{\partial \Delta P_2^{(k)}}{\partial \delta_n^{(k)}} & \frac{\partial \Delta P_2^{(k)}}{\delta |V_2|^{(k)}} & \cdots & \frac{\partial \Delta P_2^{(k)}}{\delta |V_n|^{(k)}} \\ \vdots & J_1 & \vdots & \vdots & J_2 & \vdots \\ \frac{\partial \Delta P_n^{(k)}}{\partial \delta_2^{(k)}} & \cdots & \frac{\partial \Delta P_n^{(k)}}{\partial \delta_n^{(k)}} & \frac{\partial \Delta P_n^{(k)}}{\delta |V_2|^{(k)}} & \cdots & \frac{\partial \Delta P_n^{(k)}}{\delta |V_n|^{(k)}} \\ \hline \frac{\partial \Delta Q_2^{(k)}}{\partial \delta_2^{(k)}} & \cdots & \frac{\partial \Delta Q_2^{(k)}}{\partial \delta_n^{(k)}} & \frac{\partial \Delta Q_2^{(k)}}{\delta |V_2|^{(k)}} & \cdots & \frac{\partial \Delta Q_2^{(k)}}{\delta |V_n|^{(k)}} \\ \vdots & J_3 & \vdots & \vdots & J_4 & \vdots \\ \frac{\partial \Delta Q_n^{(k)}}{\partial \delta_2^{(k)}} & \cdots & \frac{\partial \Delta Q_n^{(k)}}{\partial \delta_n^{(k)}} & \frac{\partial \Delta Q_n^{(k)}}{\delta |V_2|^{(k)}} & \cdots & \frac{\partial \Delta Q_n^{(k)}}{\delta |V_n|^{(k)}} \end{array}\right] \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_2^{(k)} \\ \Delta |V_2|^{(k)} \\ \vdots \\ \Delta |V_n|^{(k)} \end{bmatrix}$$

Reactive power mismatch of the droop bus

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Derivative of reactive power mismatch w. r. t. voltage magnitude

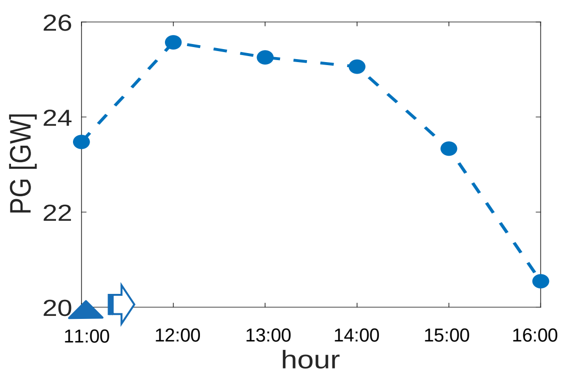
$$\frac{\partial \Delta Q_i^{(k)}}{\partial V_i^{(k)}} = -\frac{\overbrace{1}^{inj.}}{n_q} \overbrace{- \sum_{j=1}^{n} V_j^{(k)} Y_{ij} \, sin\left(\delta_i^{(k)} - \delta_j^{(k)} - \varphi_{ij}\right) + V_i^{(k)} Y_{ii} \, sin\, \varphi_{ii}}^{calc.}$$

- **Reactive power injection of droop bus is a function of its voltage magnitude → needs to be updated iteratively**
- **Only diagonal elements of submatrix J_4 are modified**

Modelling

Intra-hour Transition

- CPF is utilized to calculate the transition between discrete generation and load powers
  - Base and target cases must be defined
  - Transformers' tap ratios are adapted iteratively
- Defining base and target cases
  - Created using generation and load powers of the **current** hour
  - Created using generation and load powers of the **next** hour



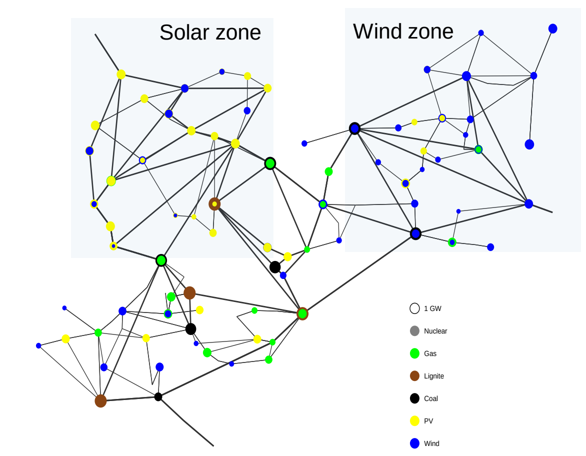
- Power plants startup and shutdown between two consecutive hours
  - These generators remain connected during each transition
  - PV bus is converted to PQ bus so that P and Q could be ramped up or down
- Adapting transformers' tap ratios
  - The variation of tap ratios between two consecutive hours are calculated
  - Total number of steps to reach the target value are determined
  - Tap ratios are adapted iteratively towards the target value

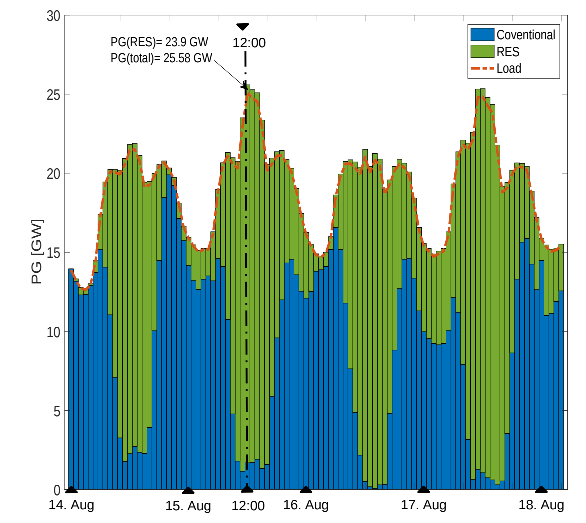
Results

Benchmark Case

- Characteristic representation of the German transmission grid
  - 120 buses at 220 kV and 380 kV voltage levels
  - Temporal component in terms annual time series (annual power flow solutions)
- RESs connected to 110 kV buses
- Initialization
  - A high share ($\approx 93\%$) of power production from RES at hour 5436
  - High load at this hour ($\approx 91\%$ of annual peak load)

120 bus grid





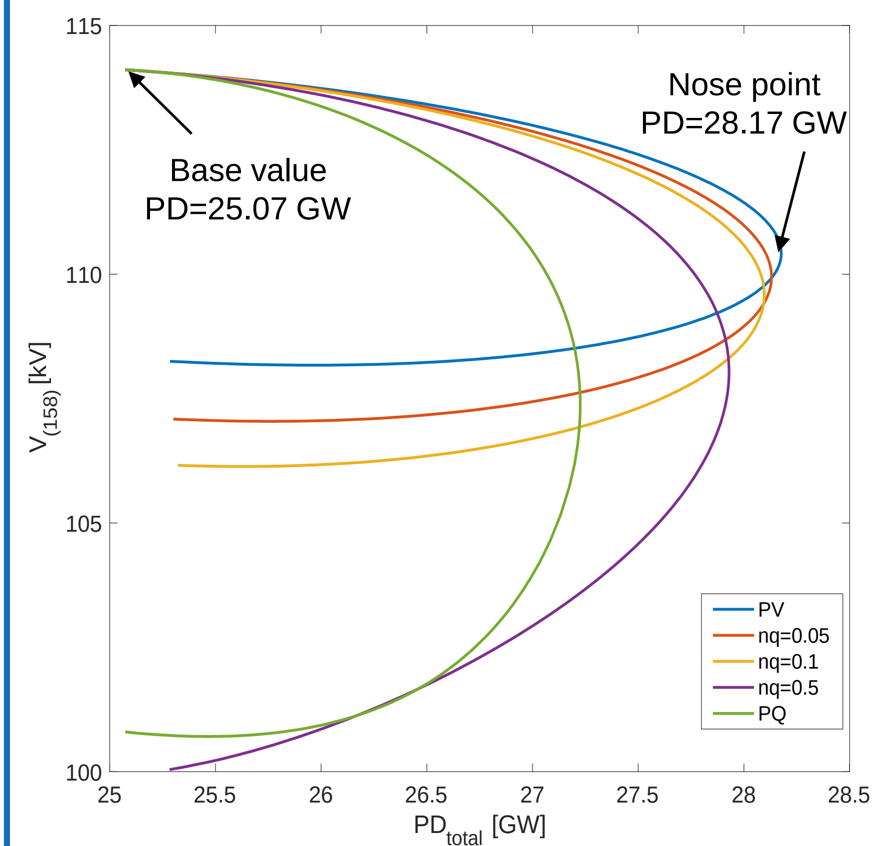
Results

Grid PV Curves

- PV curves
  - Fastest voltage decline at bus 158
- Case 1: RES modeled via PV buses
  - Highest loading margin (blue)
- Case 2: RES modeled via droop buses
  - Highest loading margin when $n_q = 0.05$ (red)
  - Loading margin decreases as n_q approaches 0.5
- Case 3: RES modeled via PQ buses
  - Lowest loading margin
  - Rapid voltage decline

Loading margin is increased when RES are modeled via droop buses

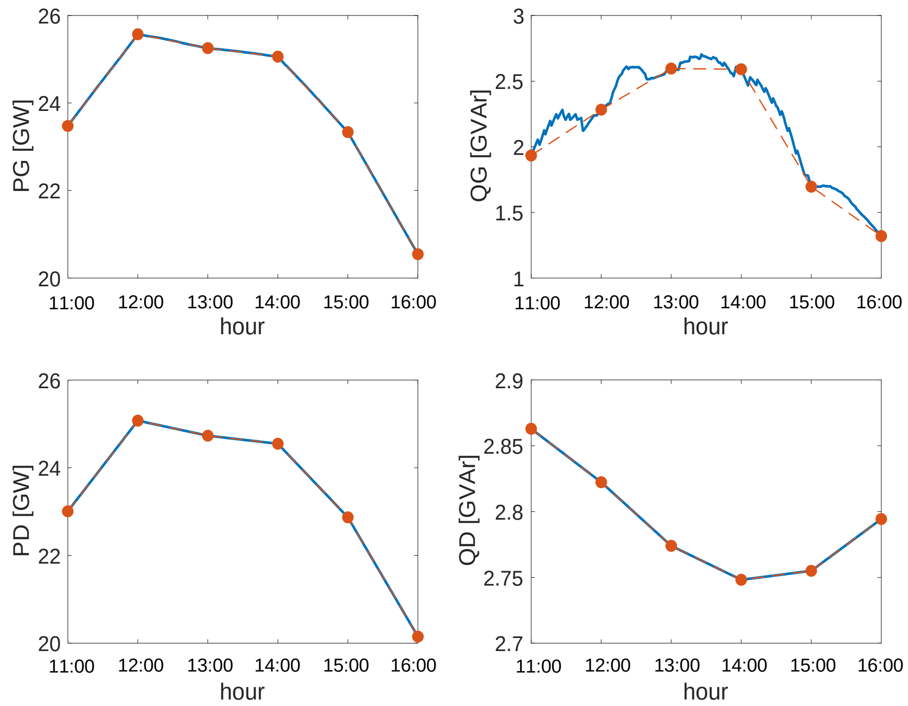
Benchmark case PV curves



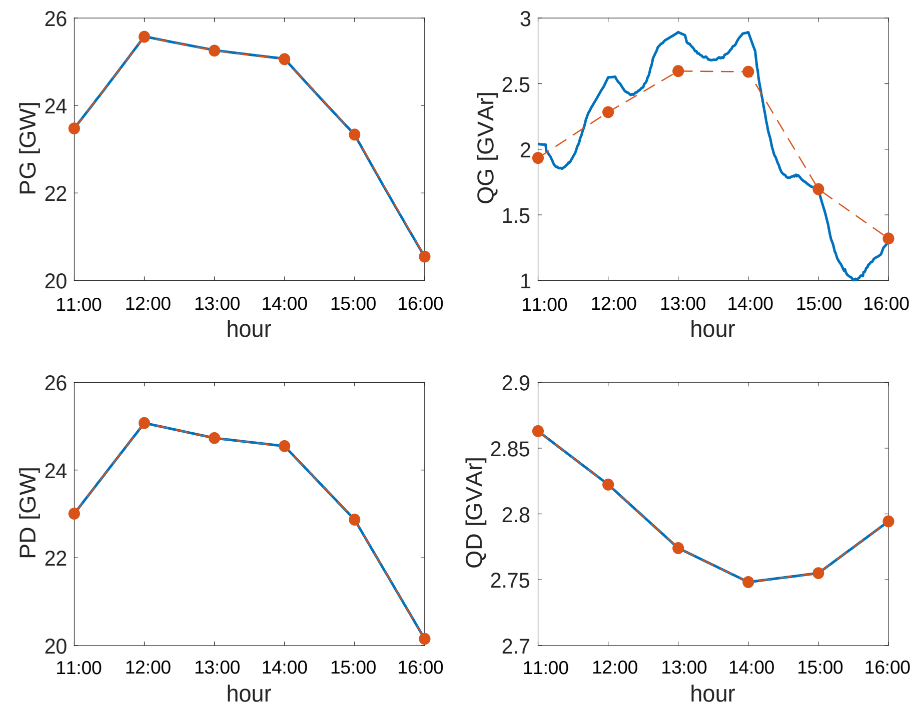
Results

Grid Transition Path

Continuous generation and load variation without droop



Continuous generation and load variation with droop



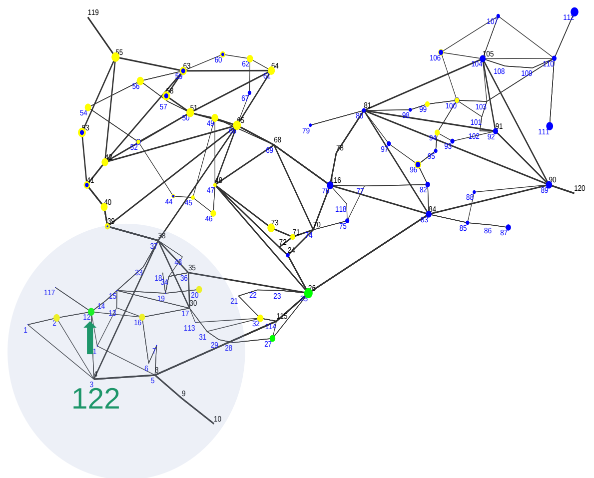
Load power and active power generation ramped up or down linearly
Discrete values are crossed

Load power and active power generation ramped up or down linearly
Discrete values of reactive power generation are not crossed

Results

Generator startup and shutdown

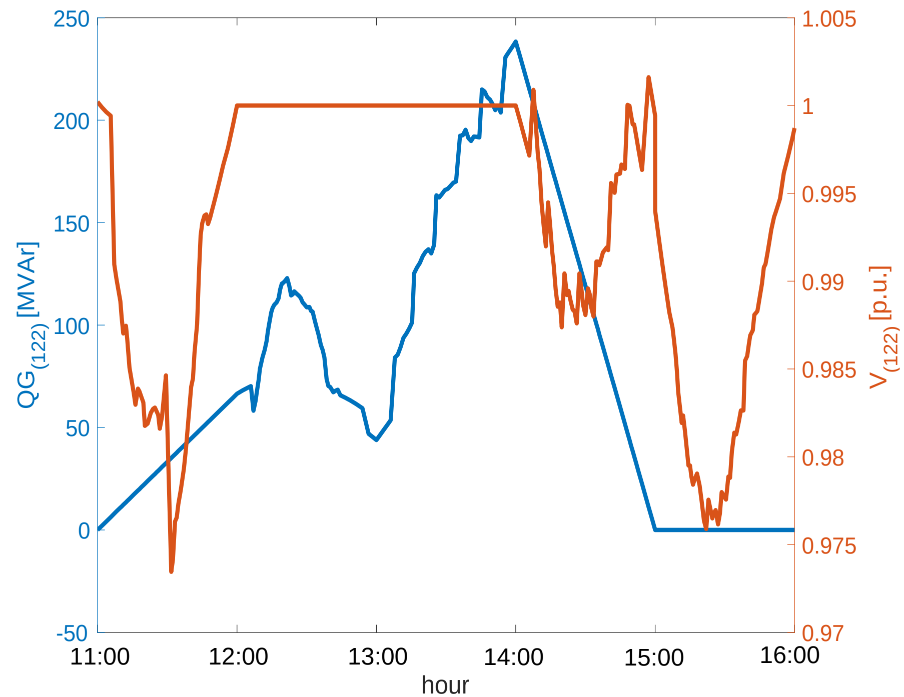
Bus 122 in Benchmark case



Generator status at bus 122

Hour	11:00	12:00	13:00	14:00	15:00	16:00
Status	off	on	on	on	off	off

Reactive power generation and voltage at gen bus 122

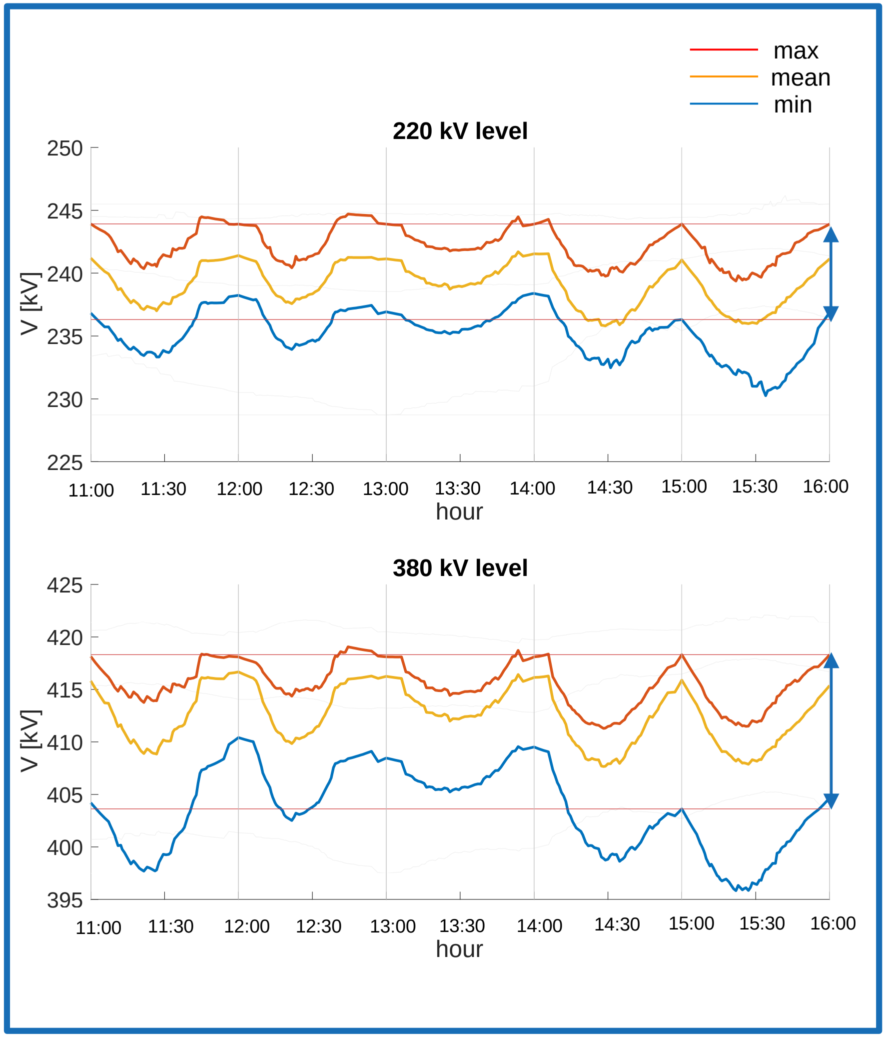


11:00 to 12:00: generator startup → converted from PV to PQ bus
12:00 to 14:00: generator modeled via PV bus

14:00 to 15:00: generator shutdown → converted from PV to PQ bus
15:00 to 16:00: generator not connected → PQ bus

Results

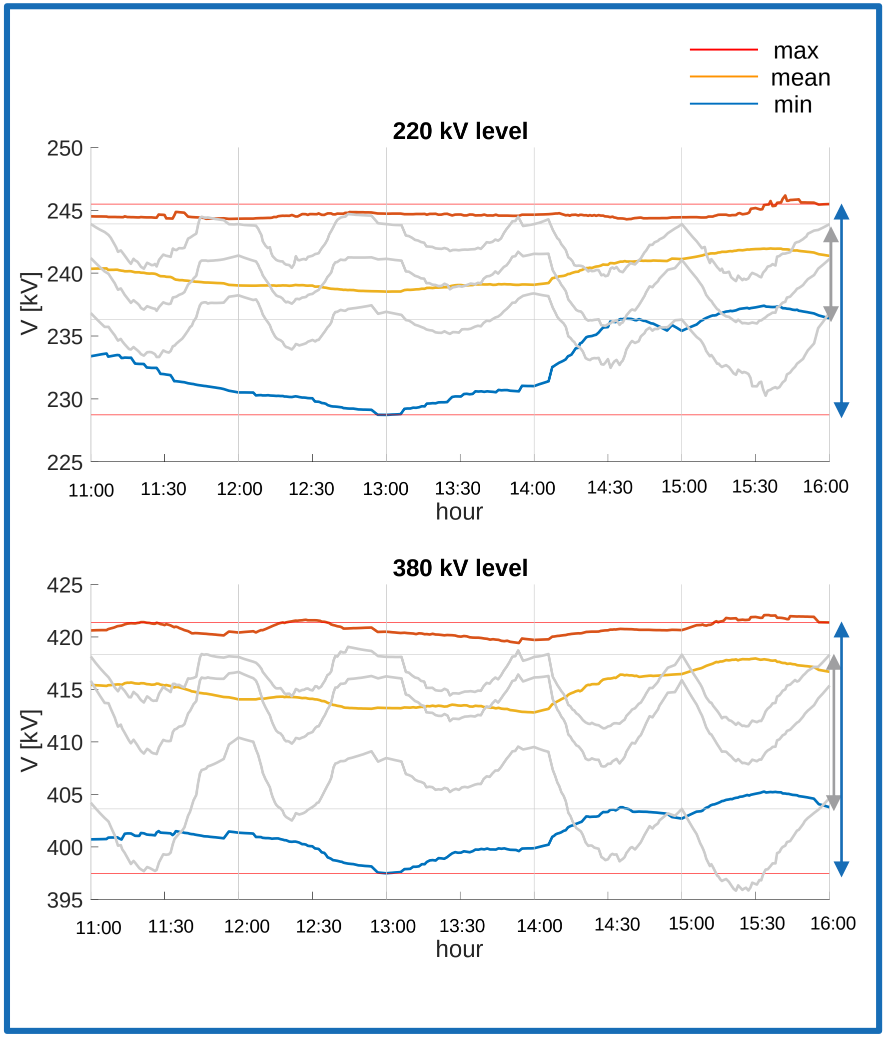
Voltage Profile Without Considering Droop



- Minimum and maximum voltages across 220 kV and 380 kV level buses shown
- Lower voltage bound is the minimum of discrete minimum voltages
- Upper voltage bound is the maximum of discrete maximum voltages

Results

Voltage Profile Considering Droop



- Without considering droop:
  - Maximum and minimum voltages repeatedly violate discrete voltage bounds
- Considering droop:
  - Maximum and minimum voltages remain constrained between discrete voltage bounds

Discussion

- Realizing droop characteristic necessitates modifying reactive power mismatch equations and the Jacobian matrix
- Considering the droop characteristic, a higher loading margin can be obtained
- The max and min voltages remain constrained between discrete values when considering the droop characteristic