

Hans Walser

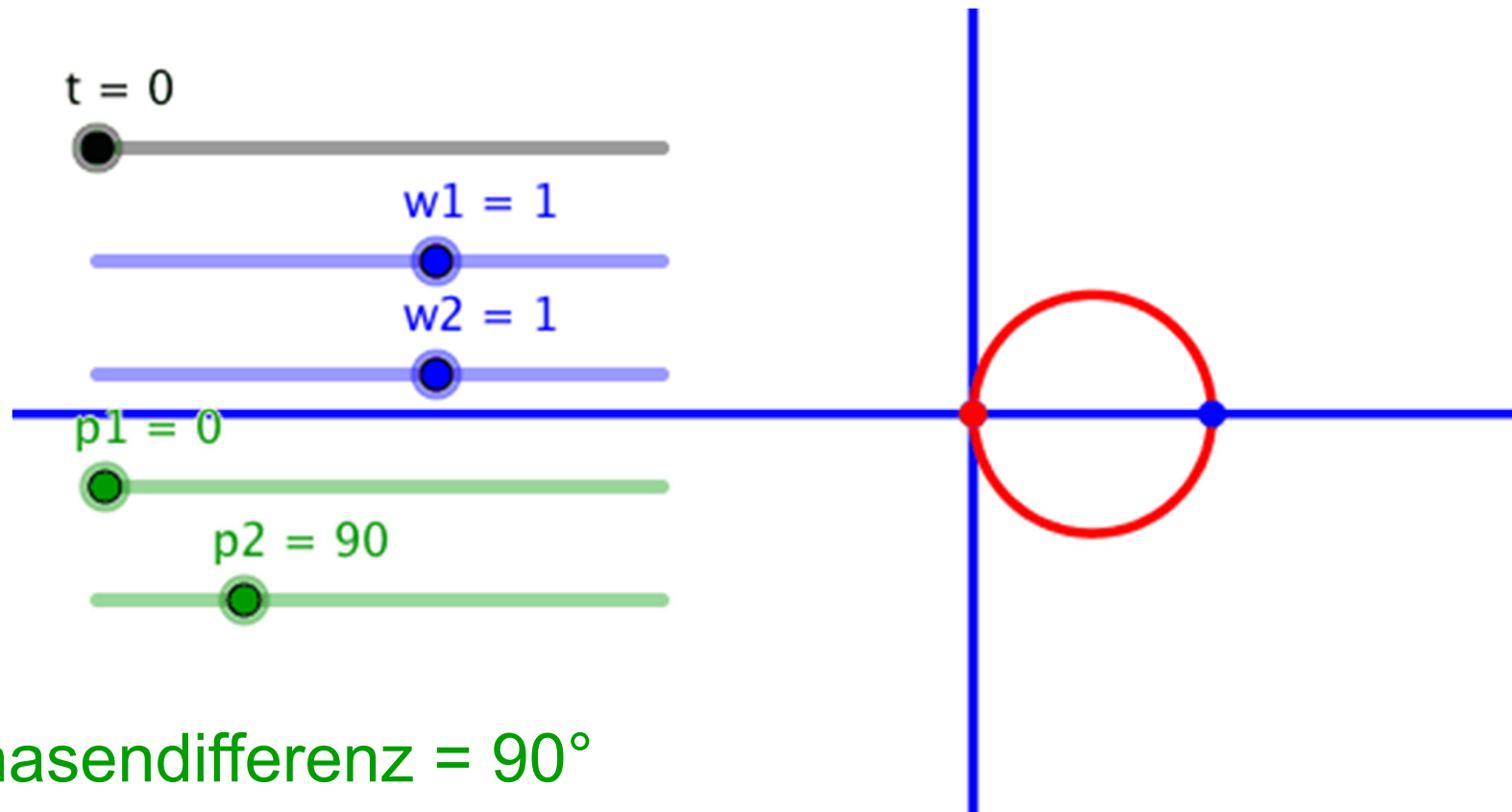
# Bewegte Figuren



Tag der Mathematik, Graz, 9. Februar 2023

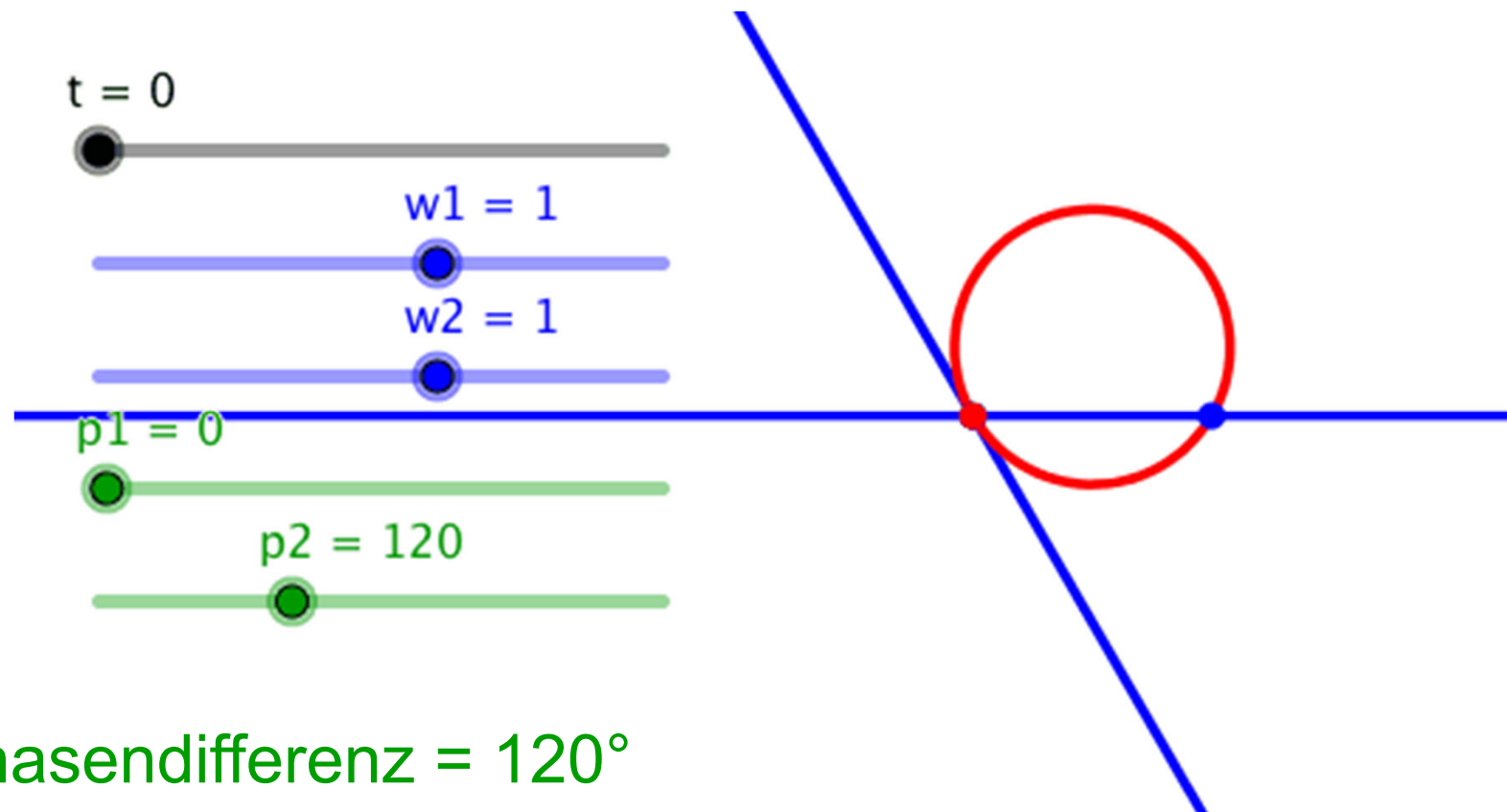
[www.walser-h-m.ch/hans/Vortraege/20230209](http://www.walser-h-m.ch/hans/Vortraege/20230209)

Was war zuerst: **der Thaleskreis** oder **der rechte Winkel**?



Phasendifferenz =  $90^\circ$

# Ortsbogen



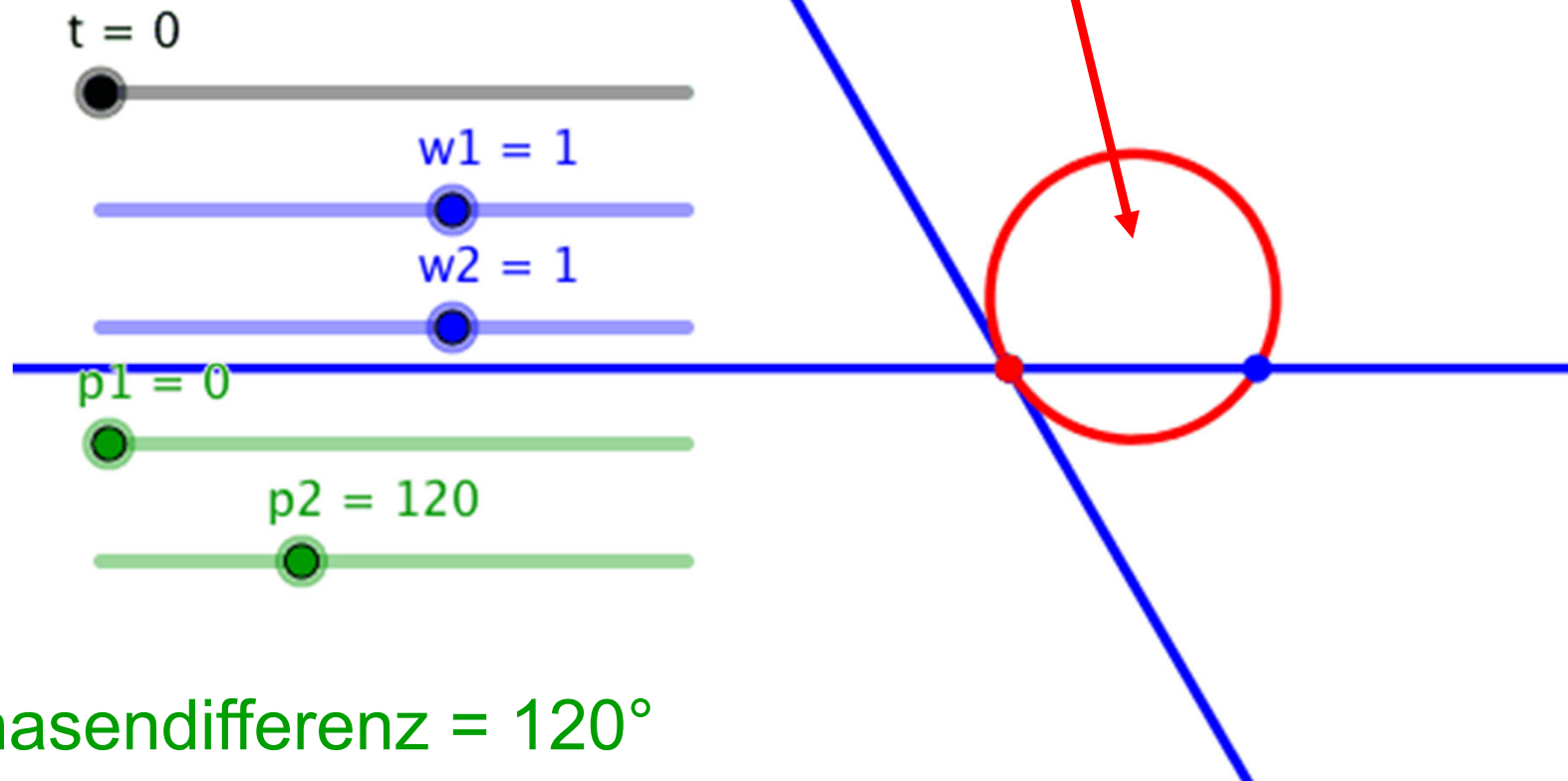
Phasendifferenz =  $120^\circ$



## Ortsbogen

Lehrer sind wie Leuchttürme.  
Sie weisen den Weg,  
ohne selber hinzugehen.

Gefahrenzone Ortsbogen

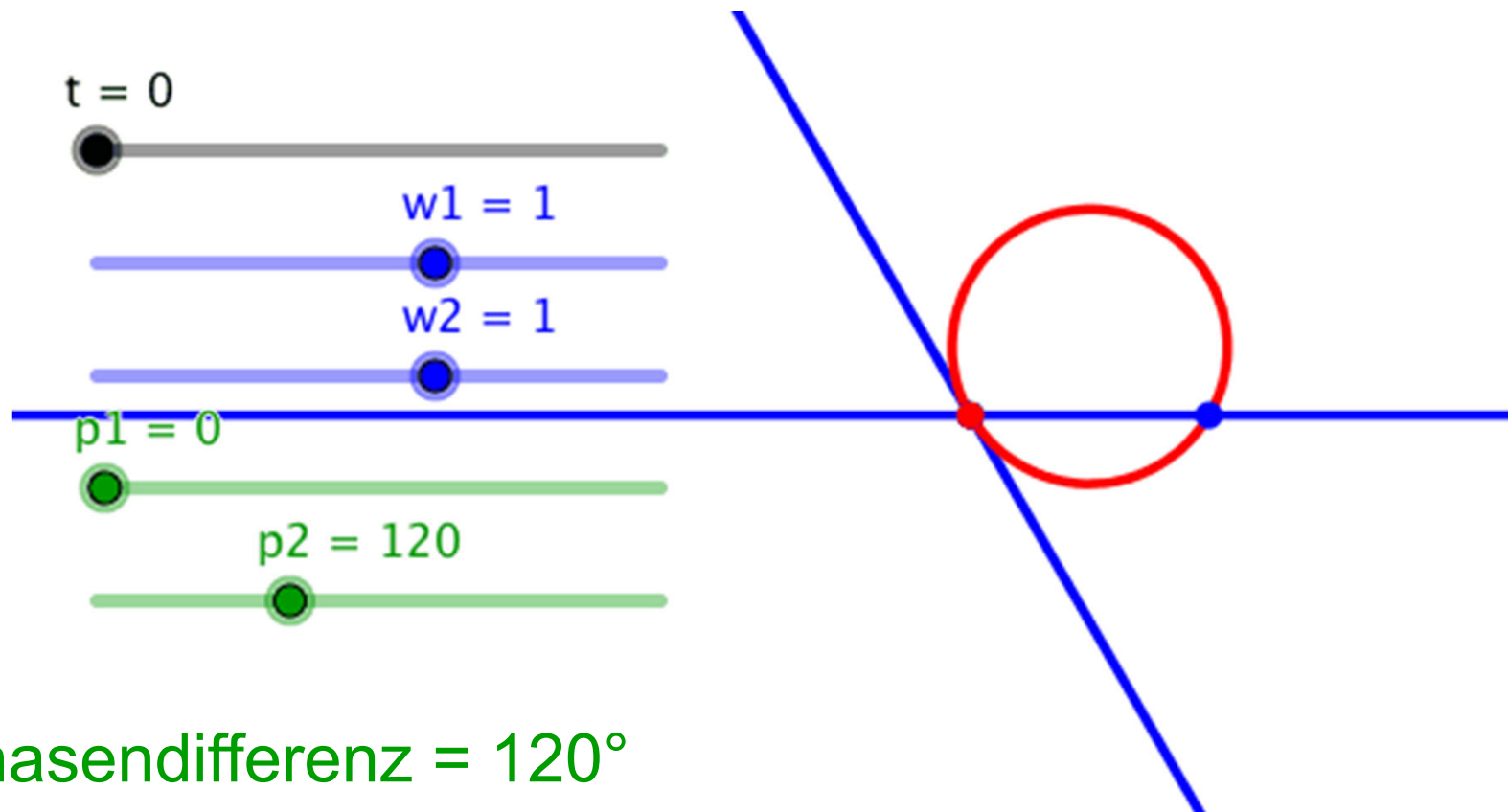


Phasendifferenz =  $120^\circ$

Sicht vom Schiff aus?

Ortsbogen?

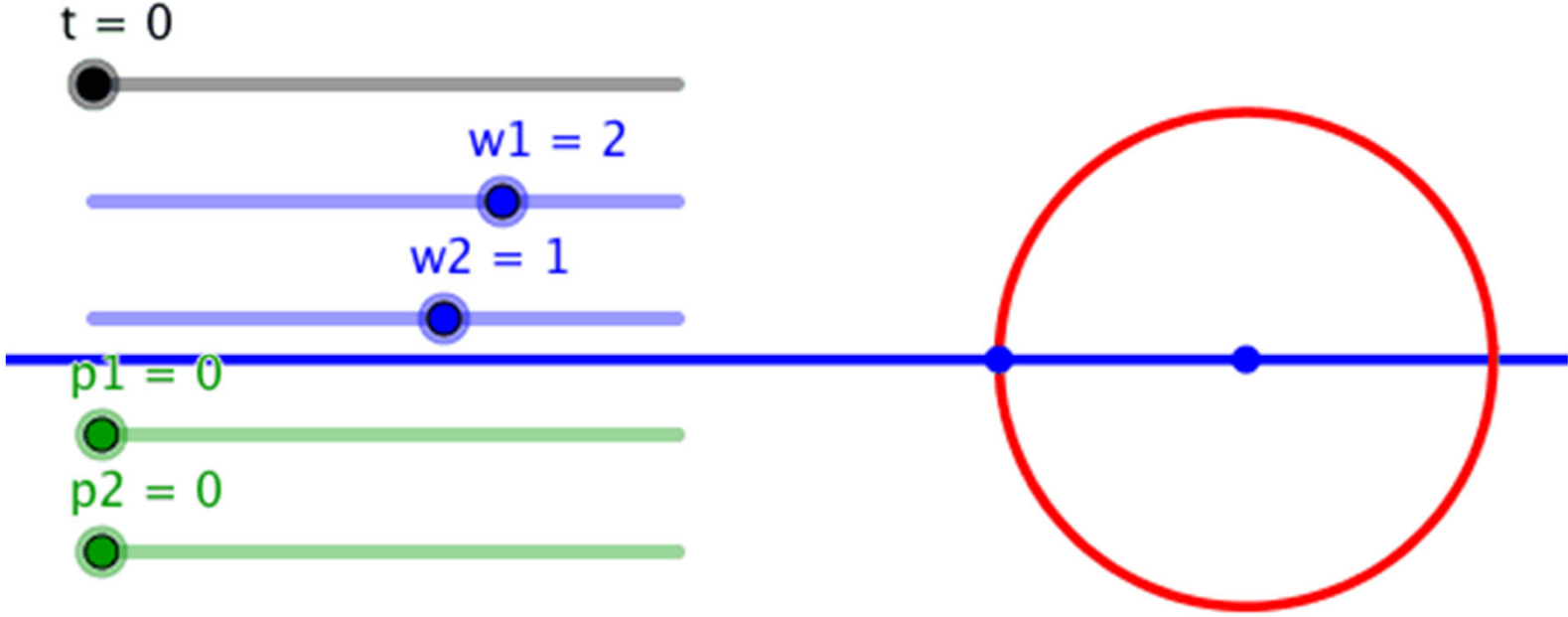
# Ortsbogen



Phasendifferenz =  $120^\circ$

Ungleiche Frequenzen

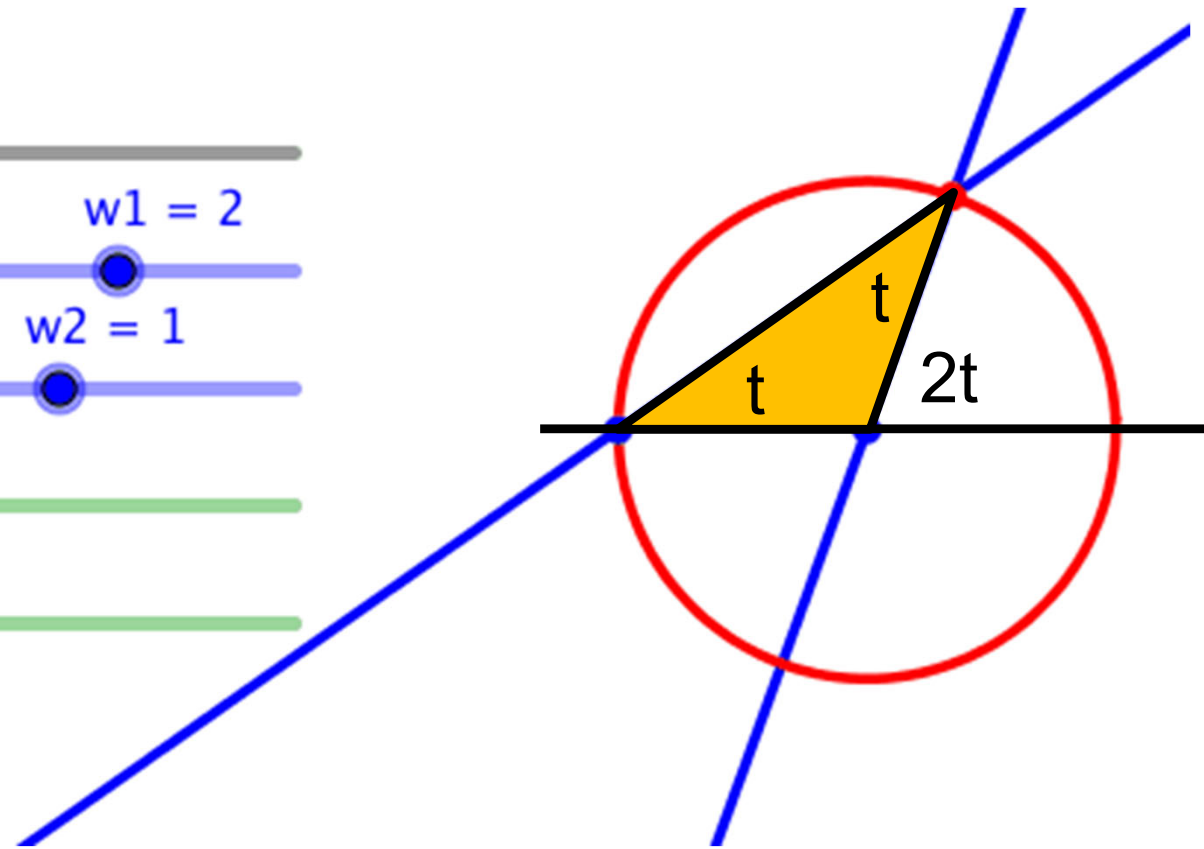
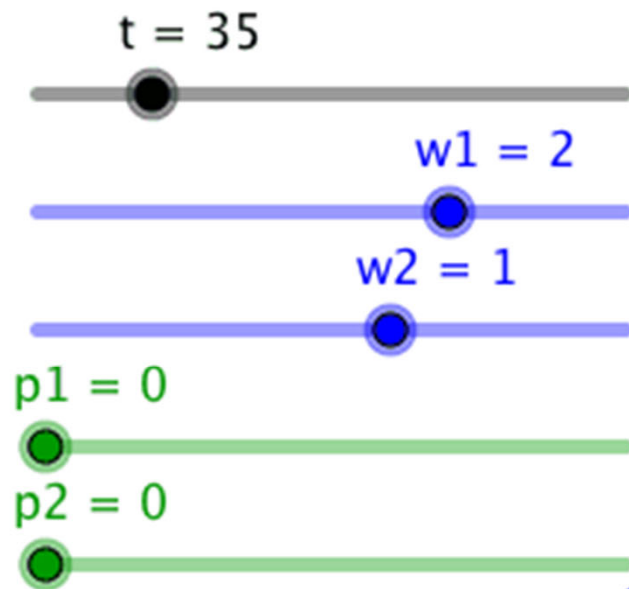
Kreis



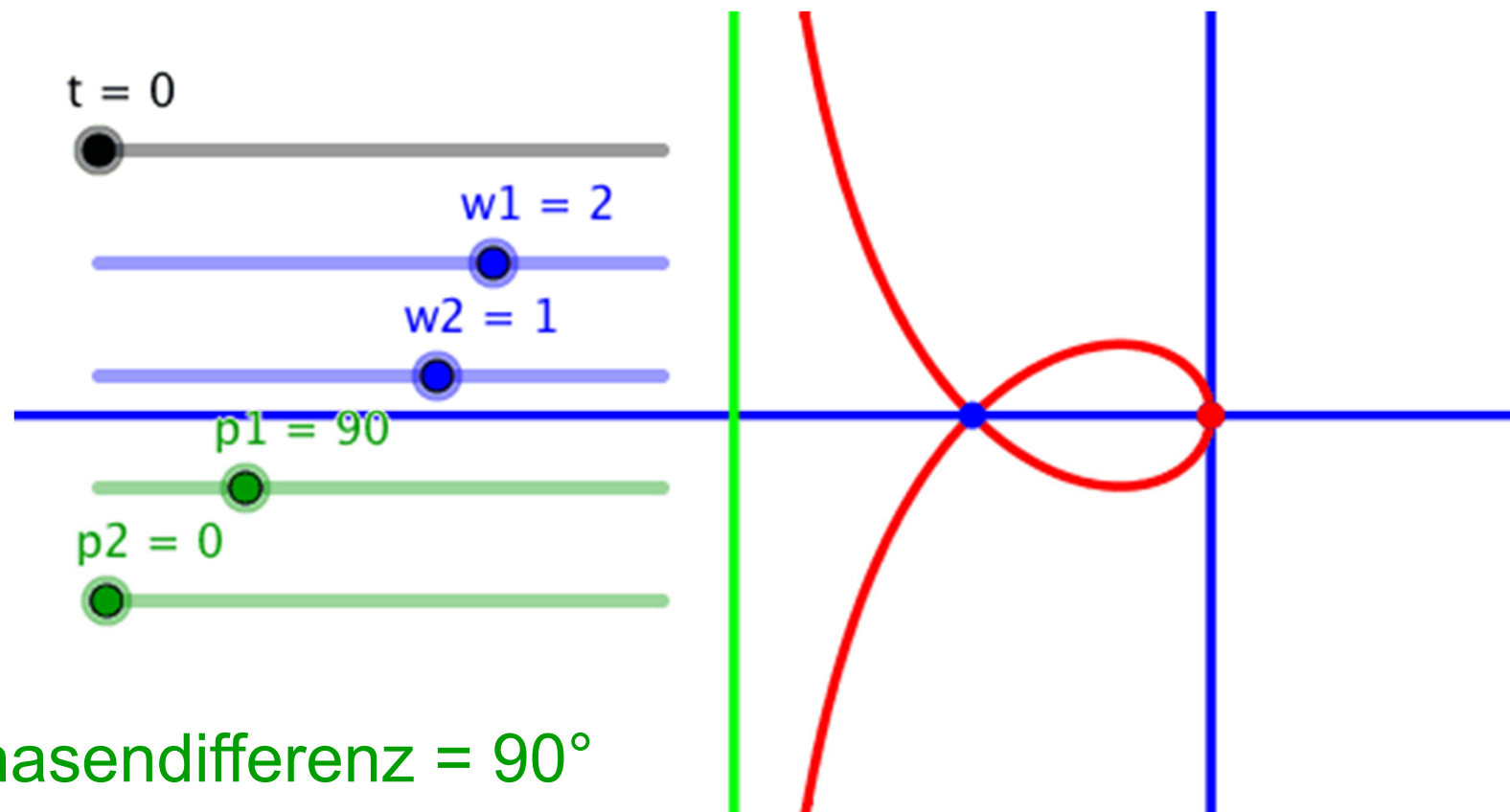


# Ungleiche Frequenzen

Kreis

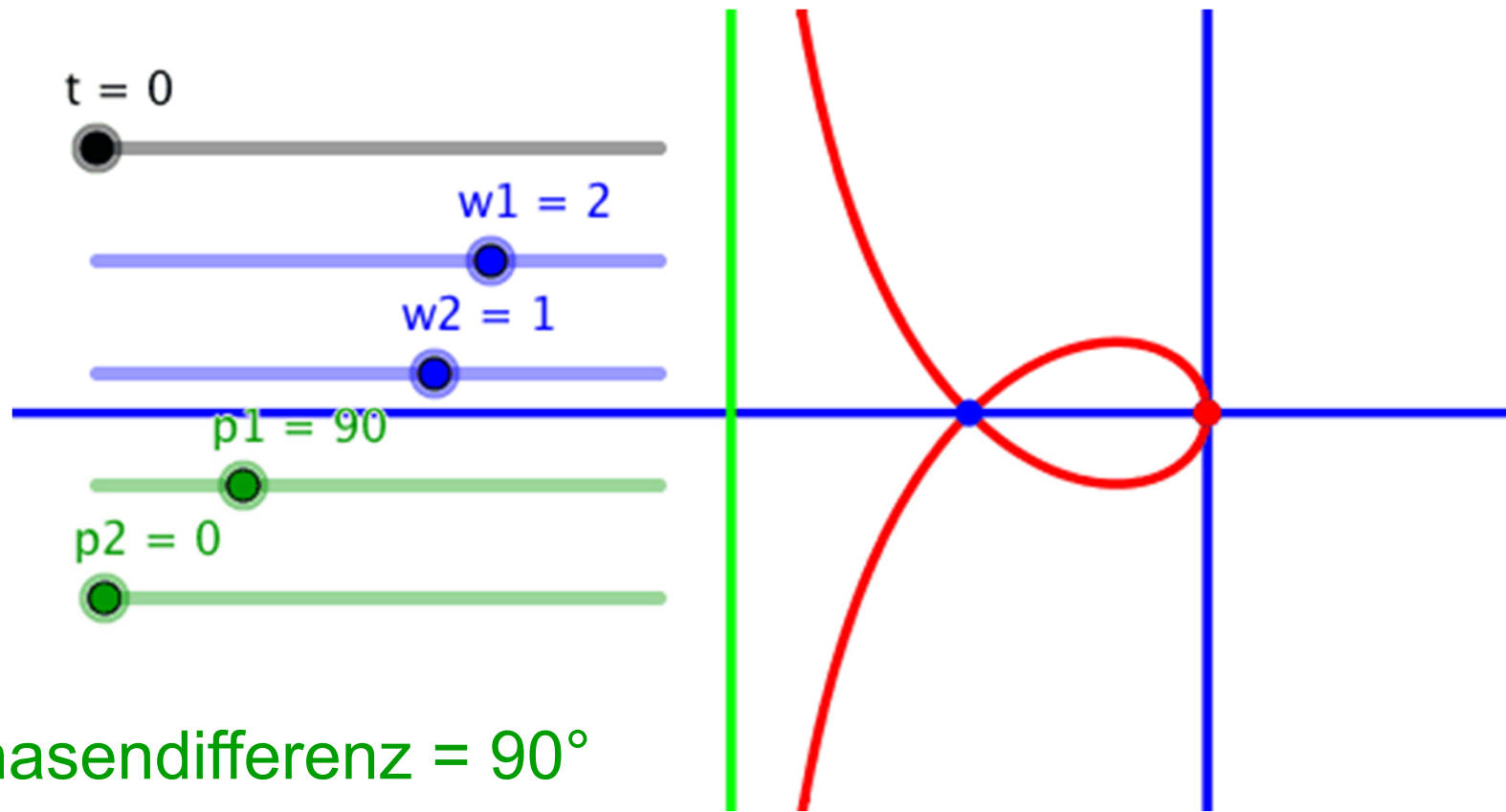


# Ungleiche Frequenzen



# Ungleiche Frequenzen

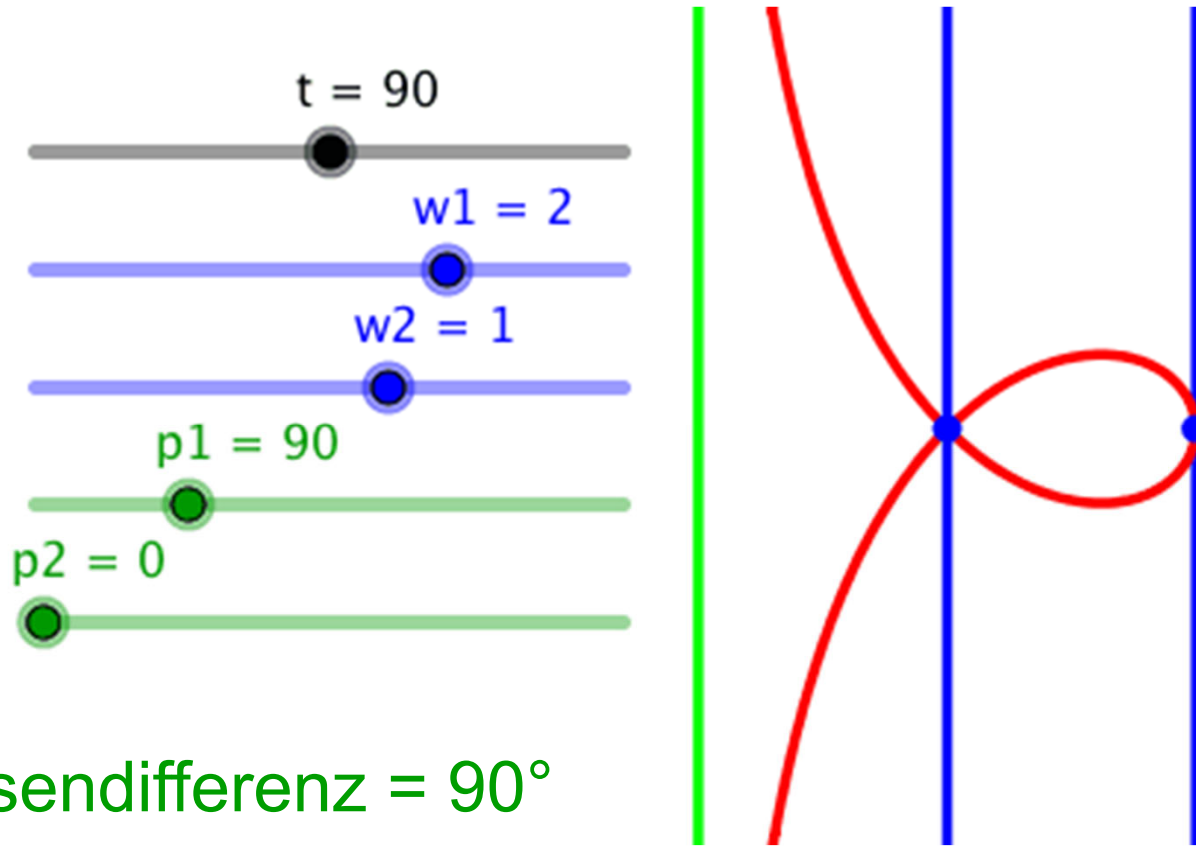
# Strophoide



Phasendifferenz =  $90^\circ$

# Ungleiche Frequenzen

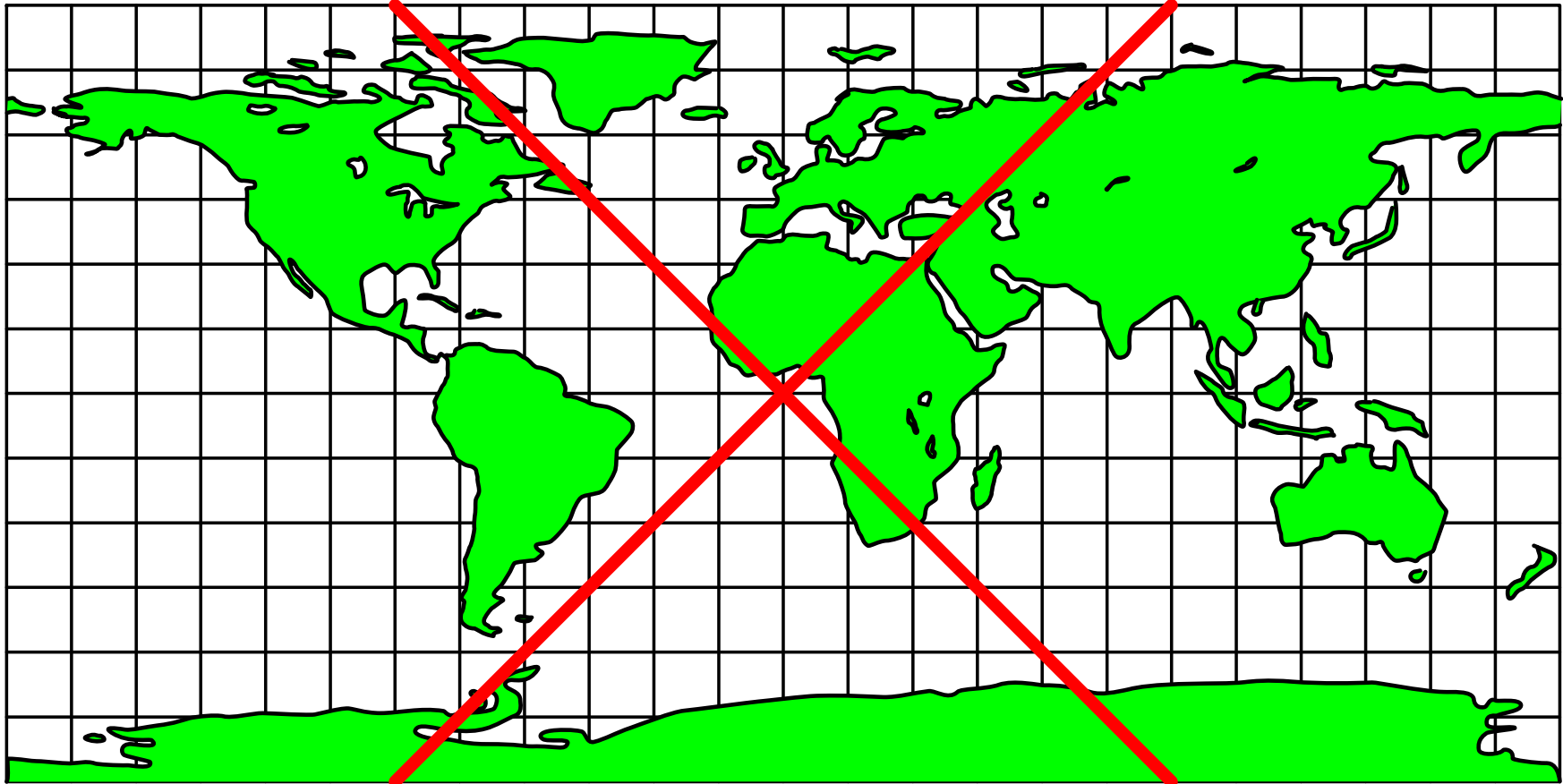
# Strophoide



Phasendifferenz =  $90^\circ$

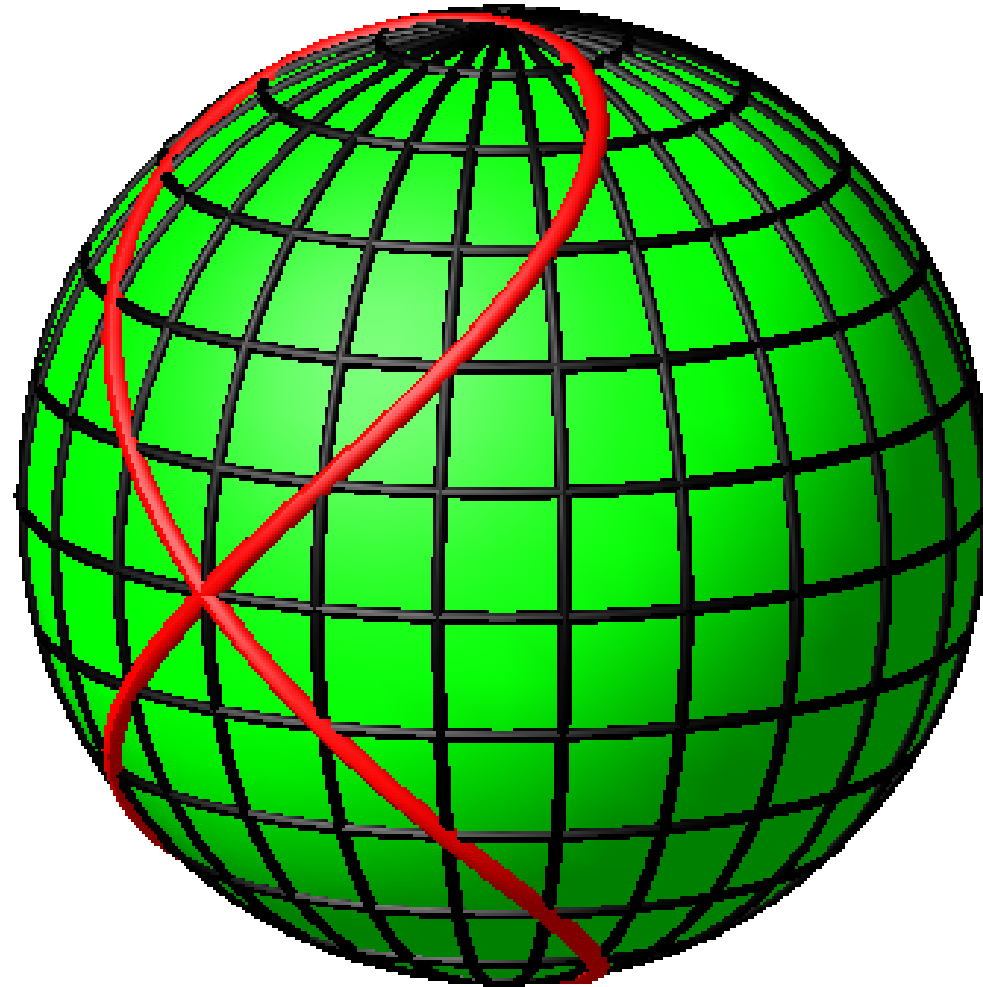
Asymptote

~~X~~ auf der Plattkarte



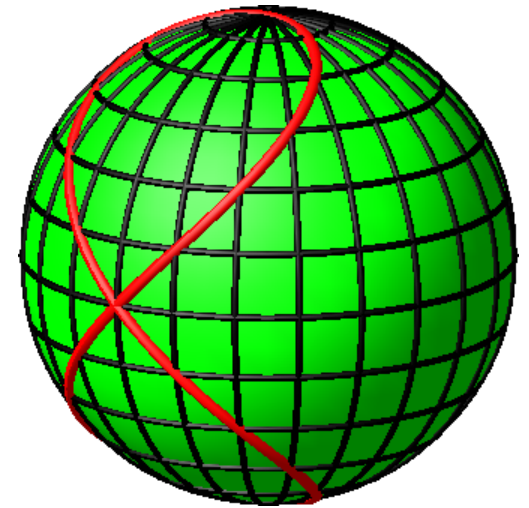
Rechter Winkel im Doppelpunkt

8 auf der Kugel

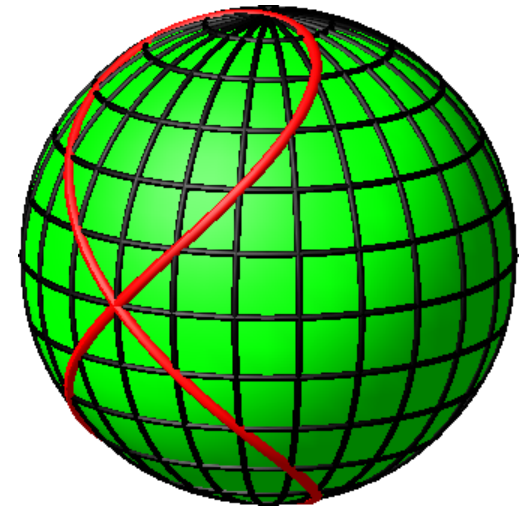
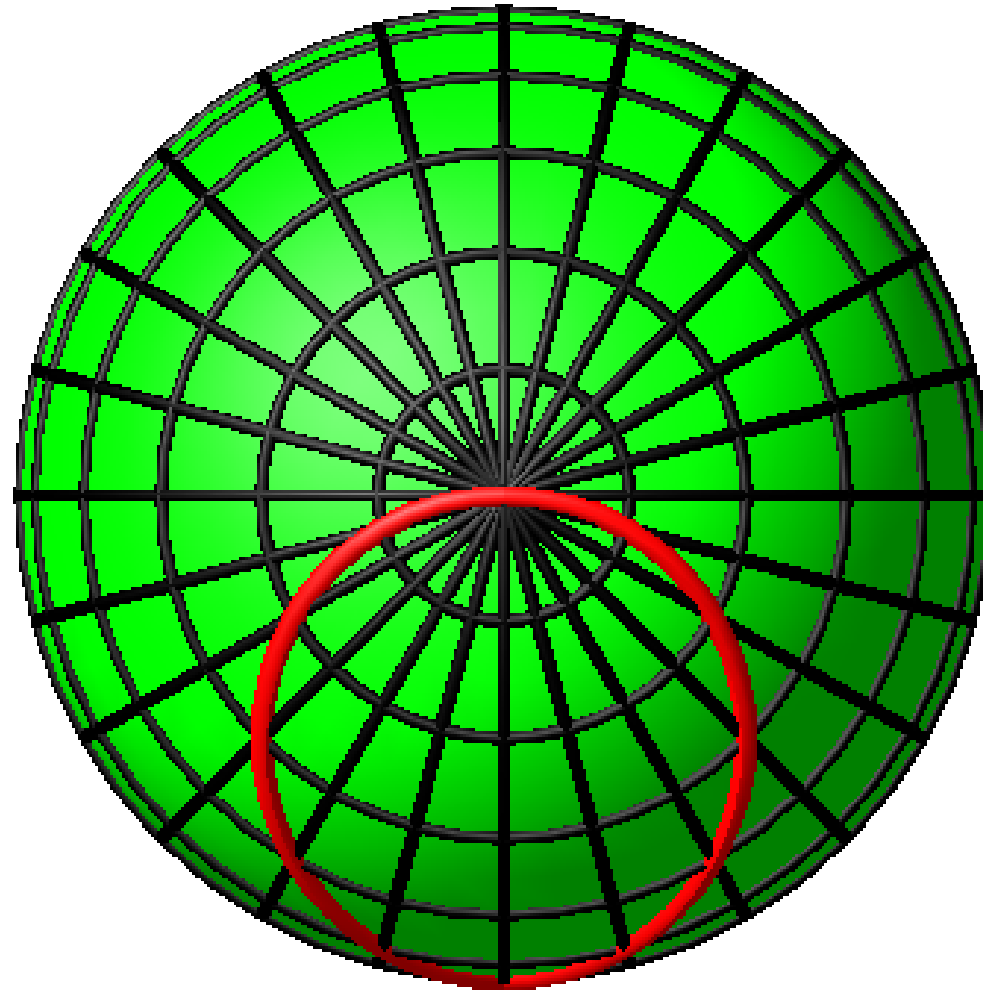


Rechter Winkel im Doppelpunkt

8 auf der Kugel

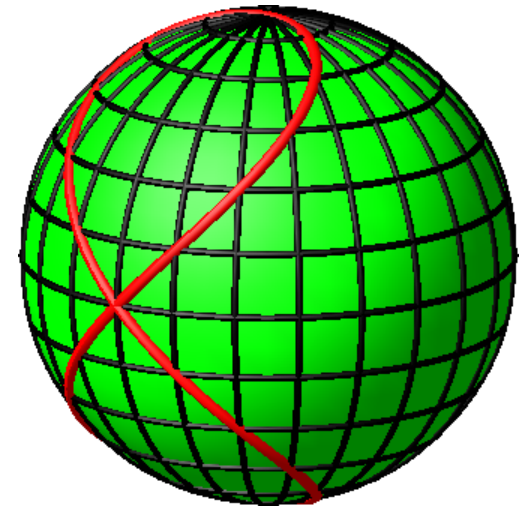
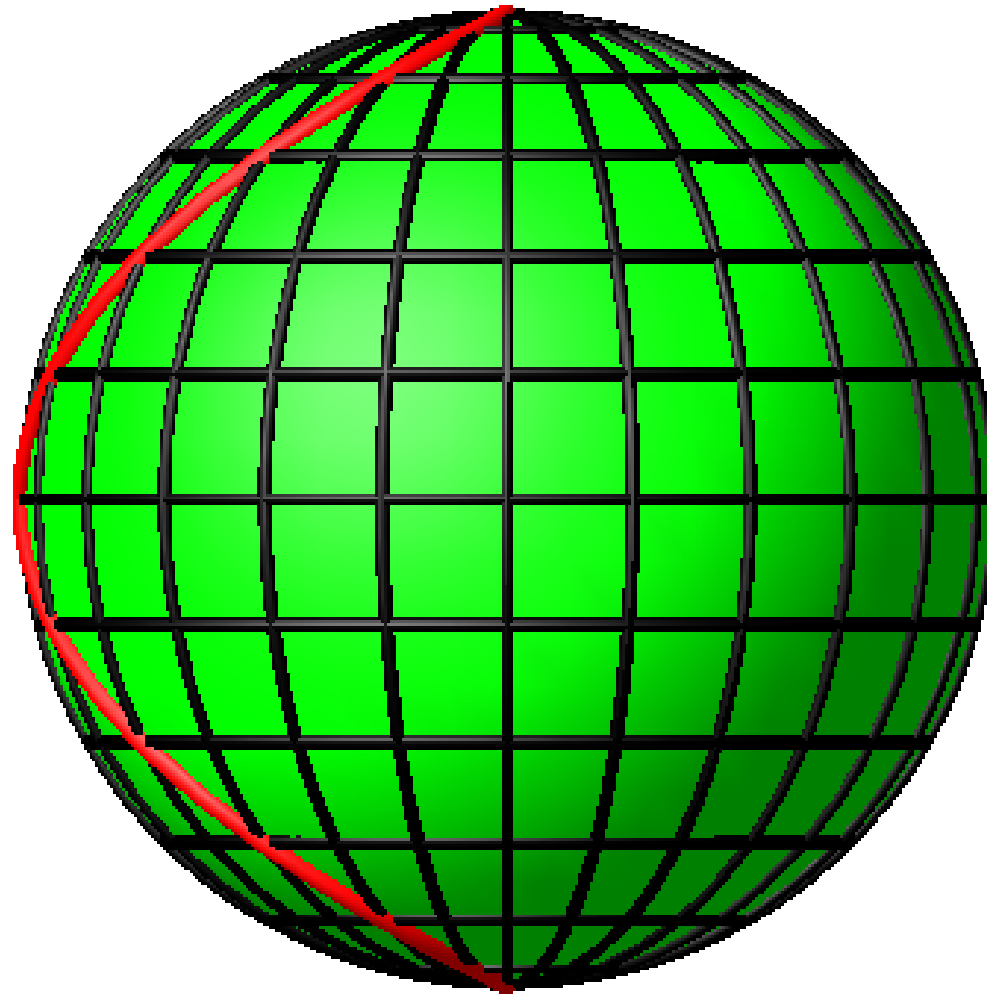


Von oben ein Kreis



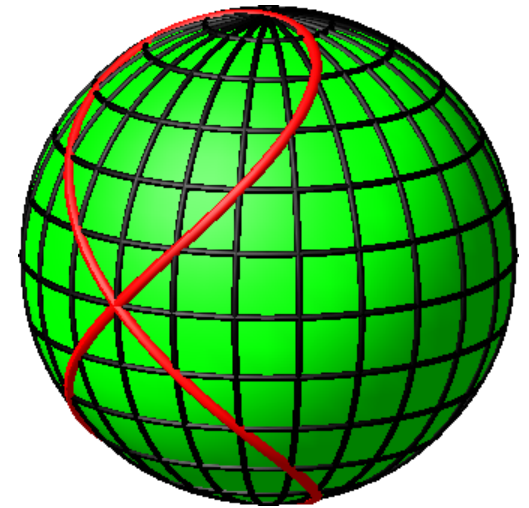
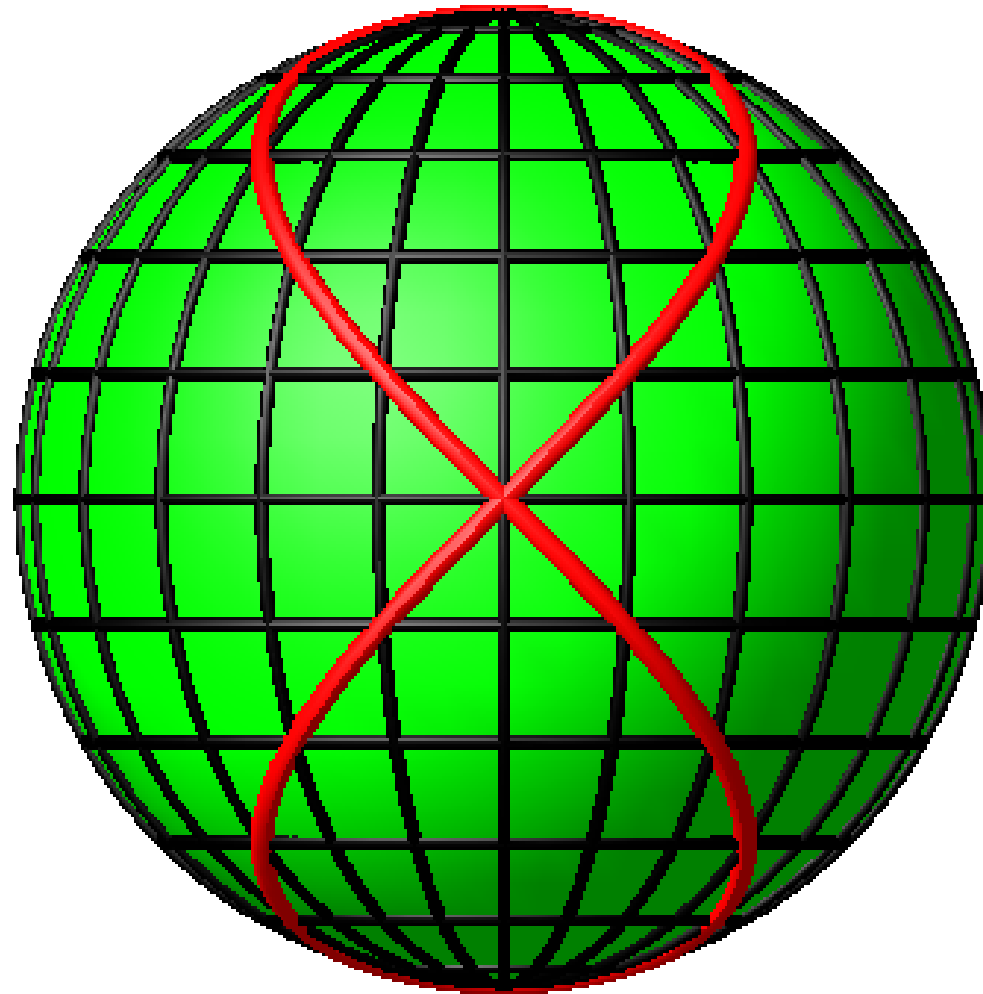


Von der Seite eine liegende Parabel



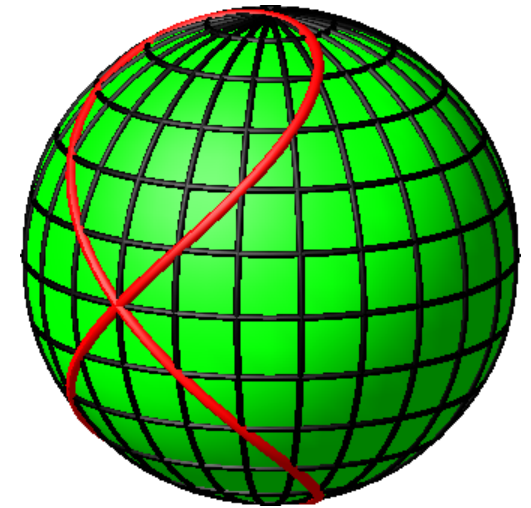
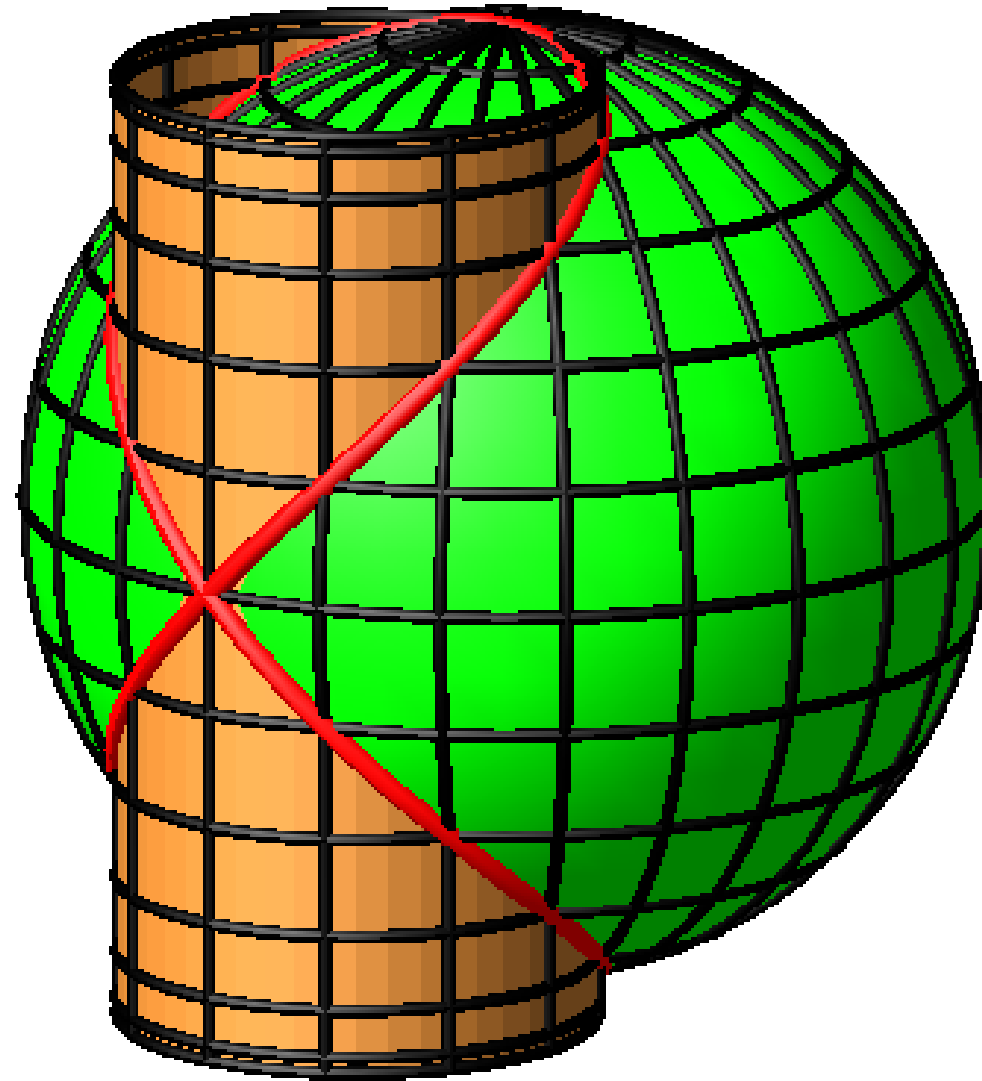
Von vorne

$$x^2 = z^2 - z^4$$



Rechter Winkel im Doppelpunkt

Schnitt mit Zylinder

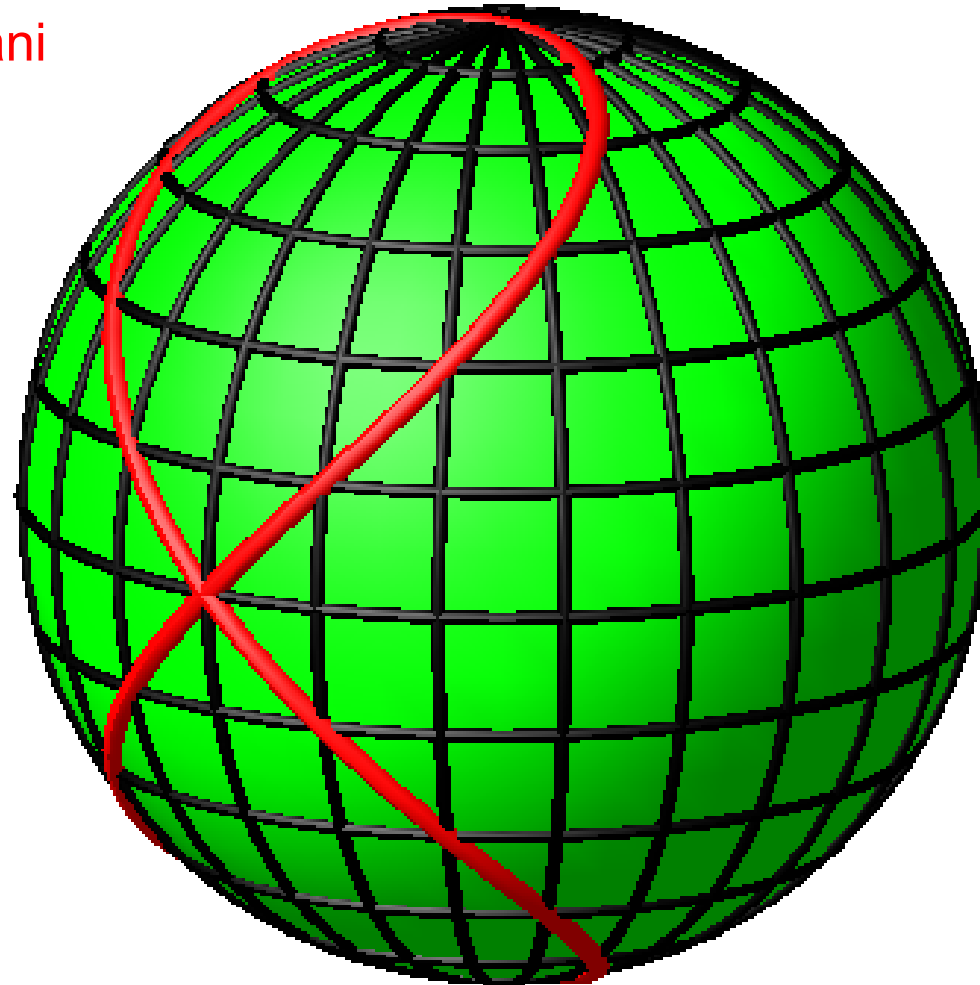


Rechter Winkel im Doppelpunkt

# Vivianische Kurve

Vincenzo Viviani

1622-1703



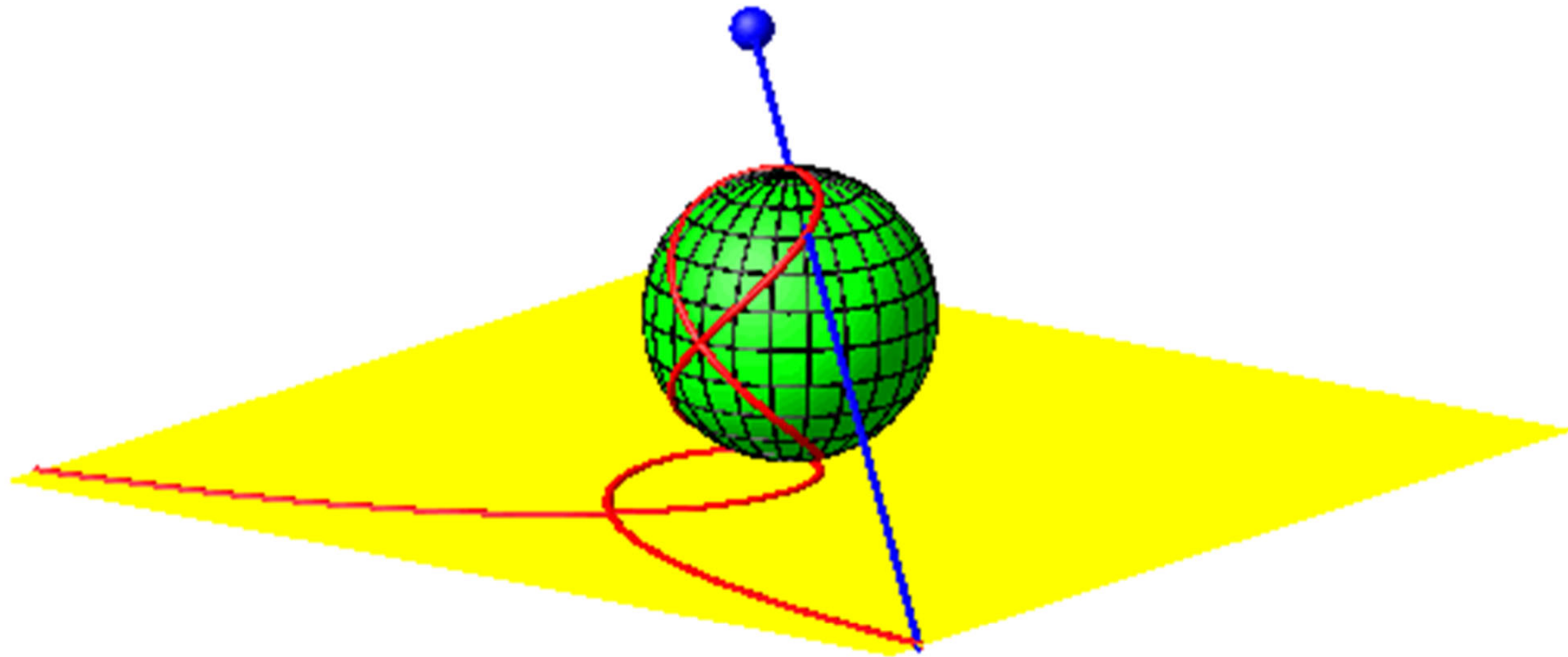
Rechter Winkel im Doppelpunkt

# Vivianische Kurve

Vincenzo Viviani

1622-1703

# Strophoide

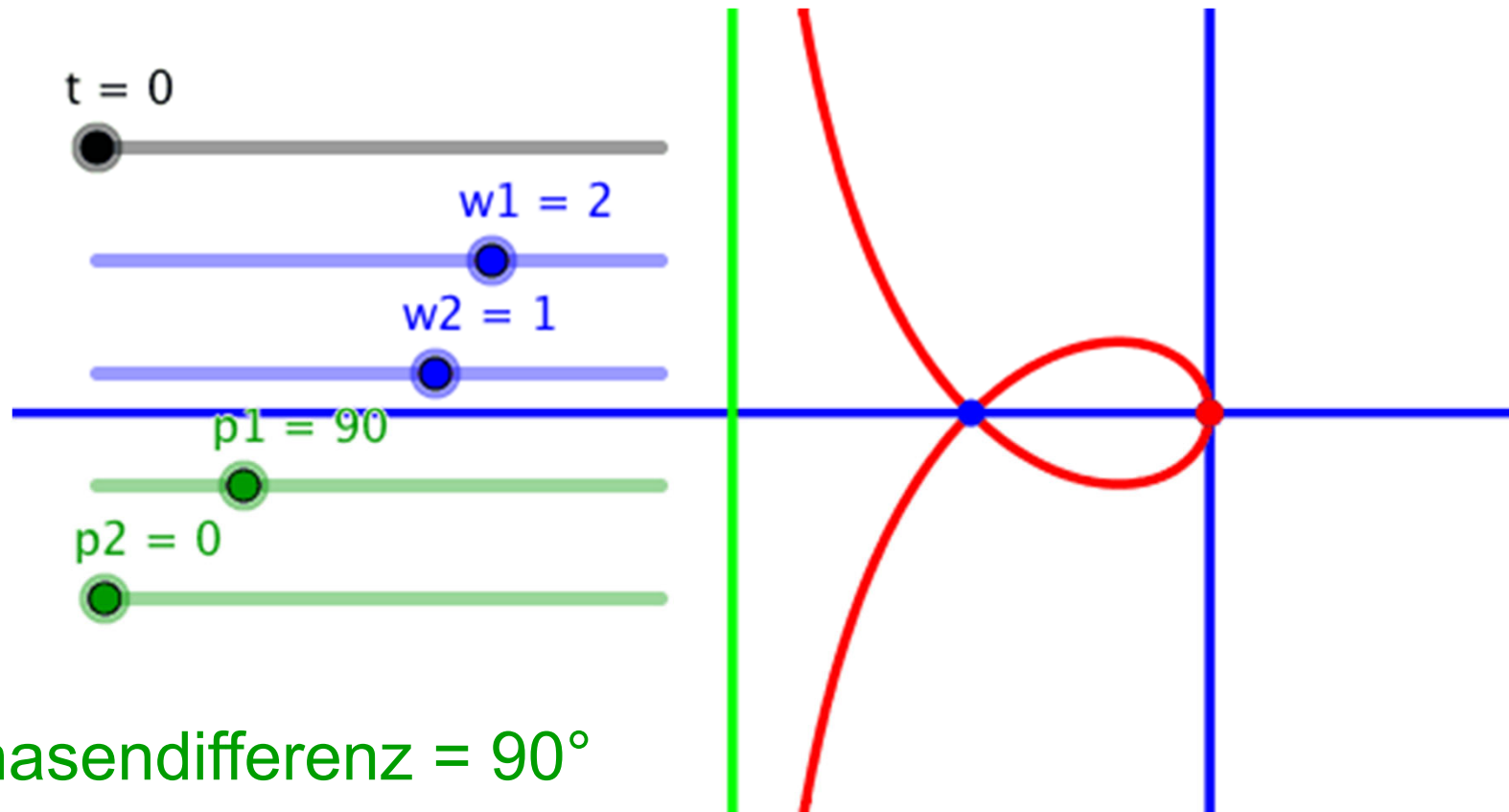


Stereografische Projektion: winkeltreu (conformal)

Rechter Winkel im Doppelpunkt

# Ungleiche Frequenzen

# Strophoide Doppelpunkt

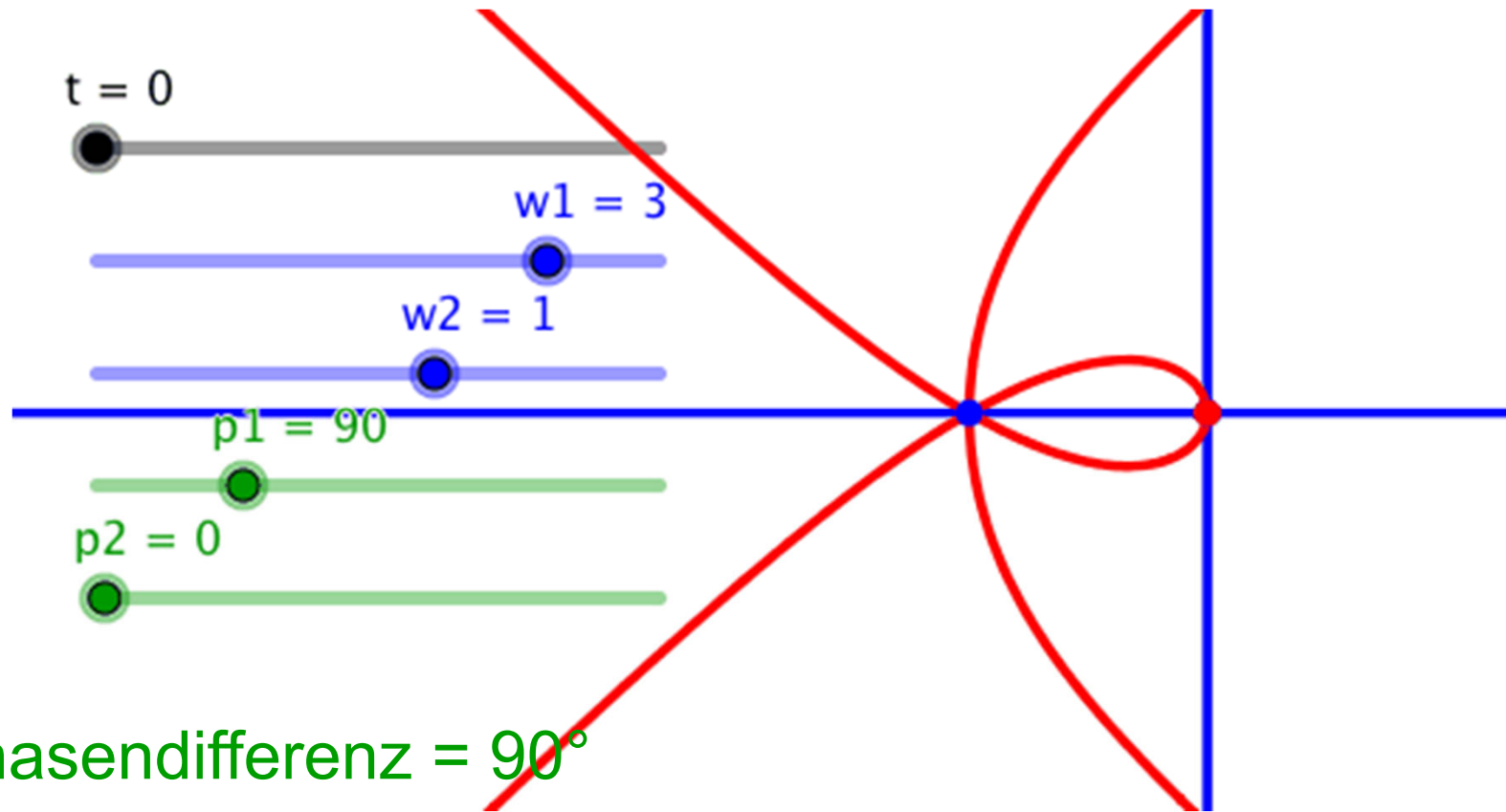


Phasendifferenz =  $90^\circ$

Rechter Winkel im Doppelpunkt

# Ungleiche Frequenzen

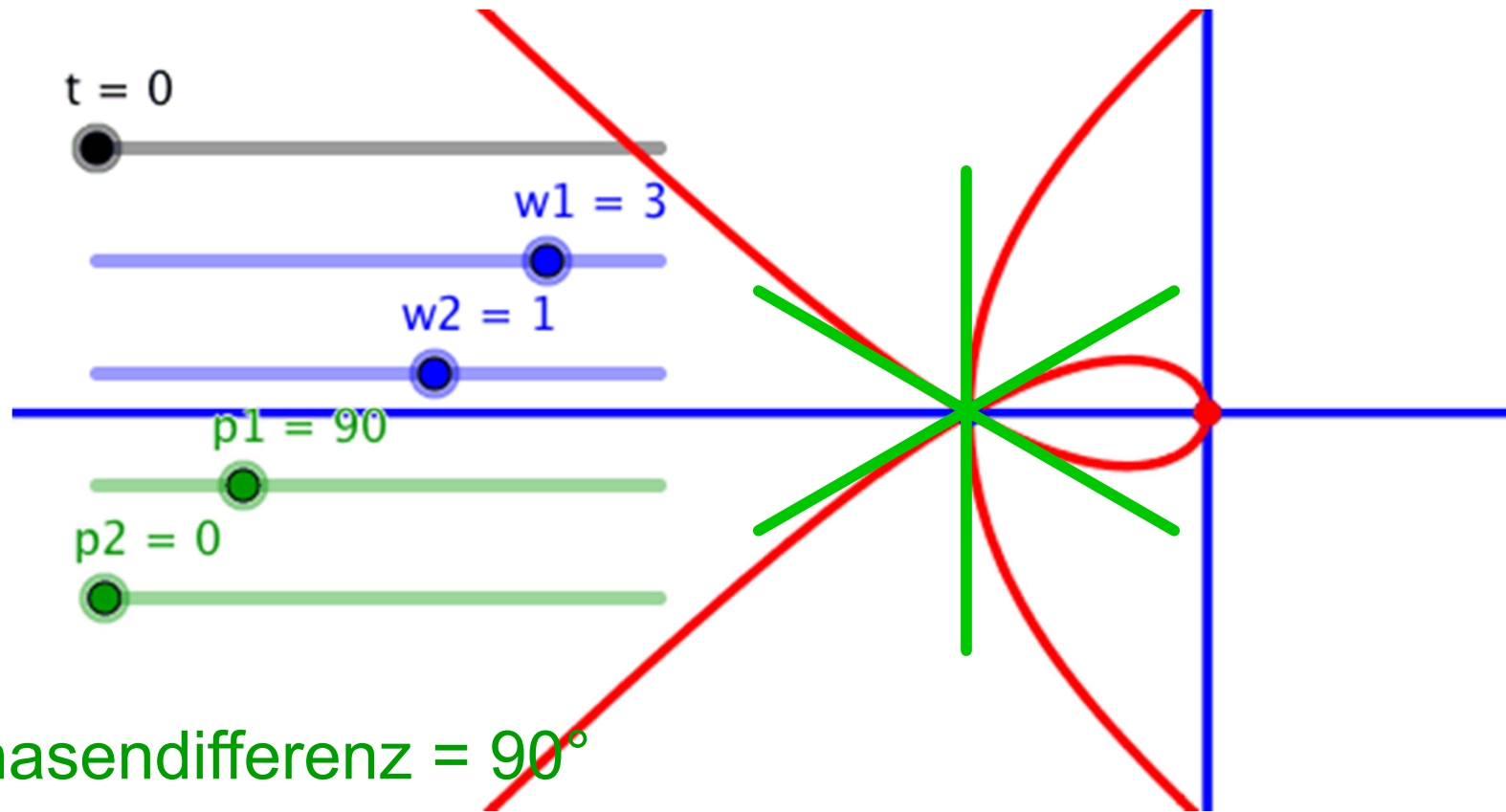
# Verallgemeinerung Tripelpunkt



Phasendifferenz =  $90^\circ$

# Ungleiche Frequenzen

# Verallgemeinerung Tripelpunkt



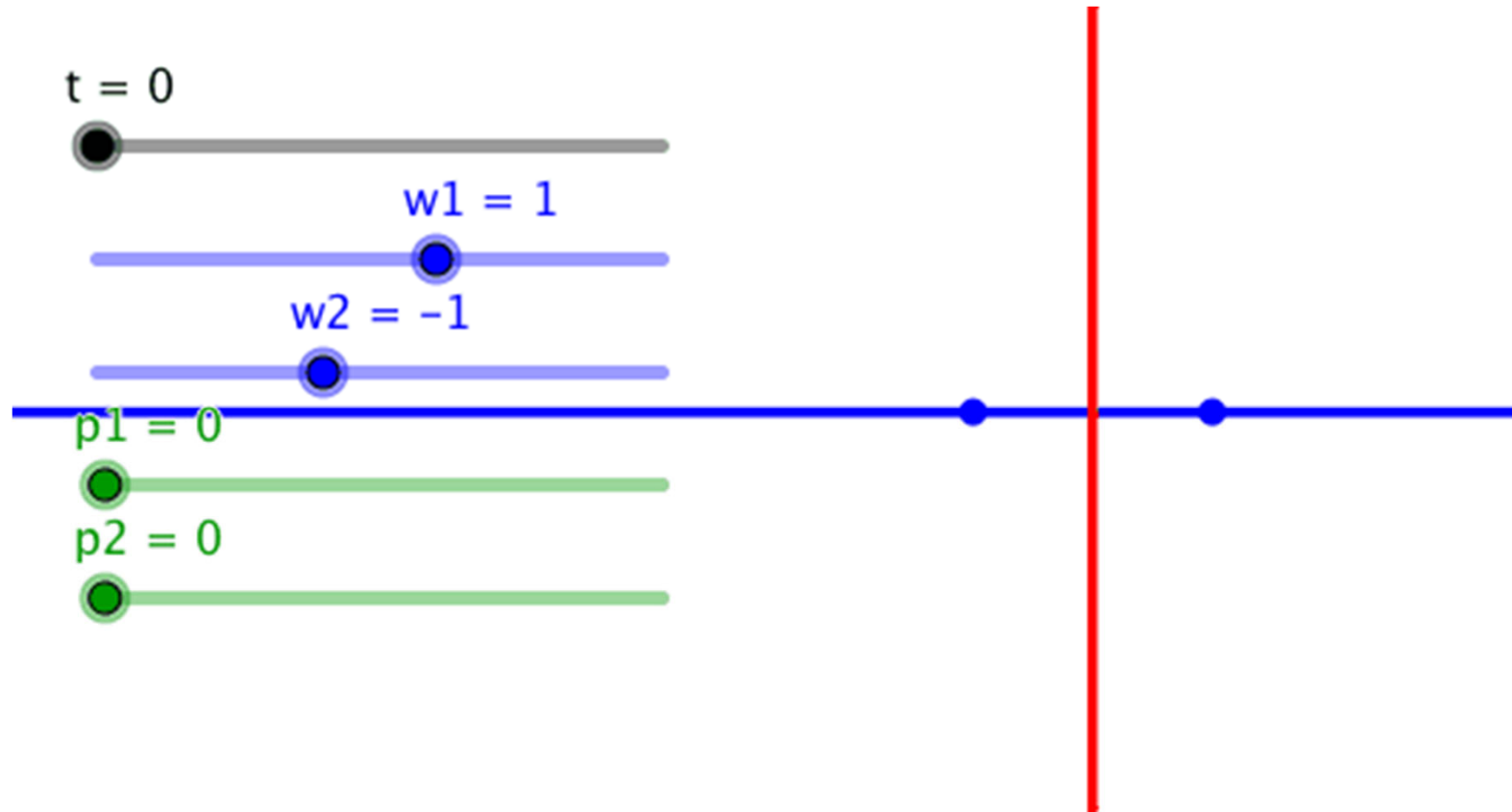
Phasendifferenz =  $90^\circ$

Gleichmaige Winkeldrittung



Entgegengesetzte Frequenzen

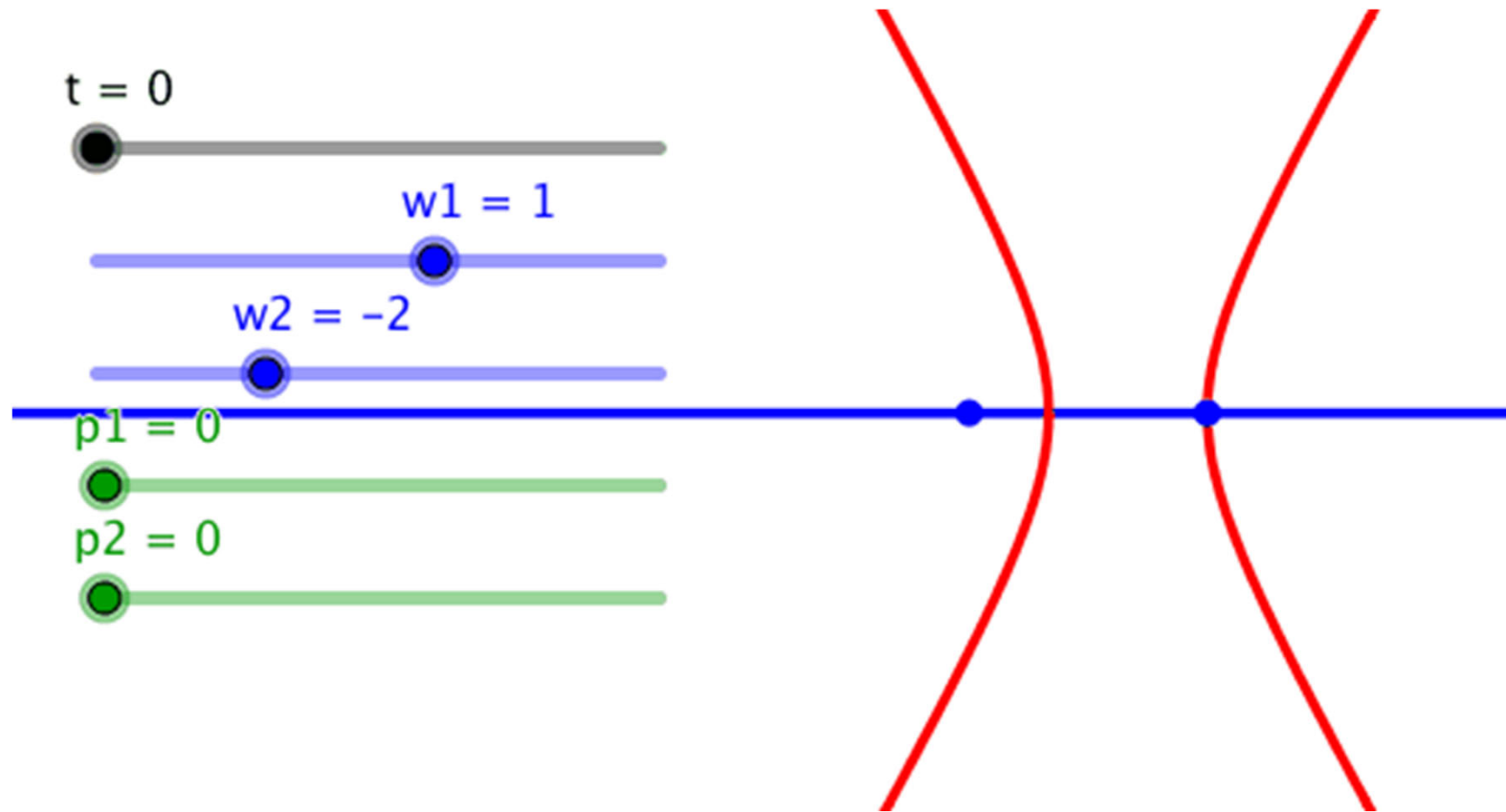
Gegengleich



Streckensymmetrale

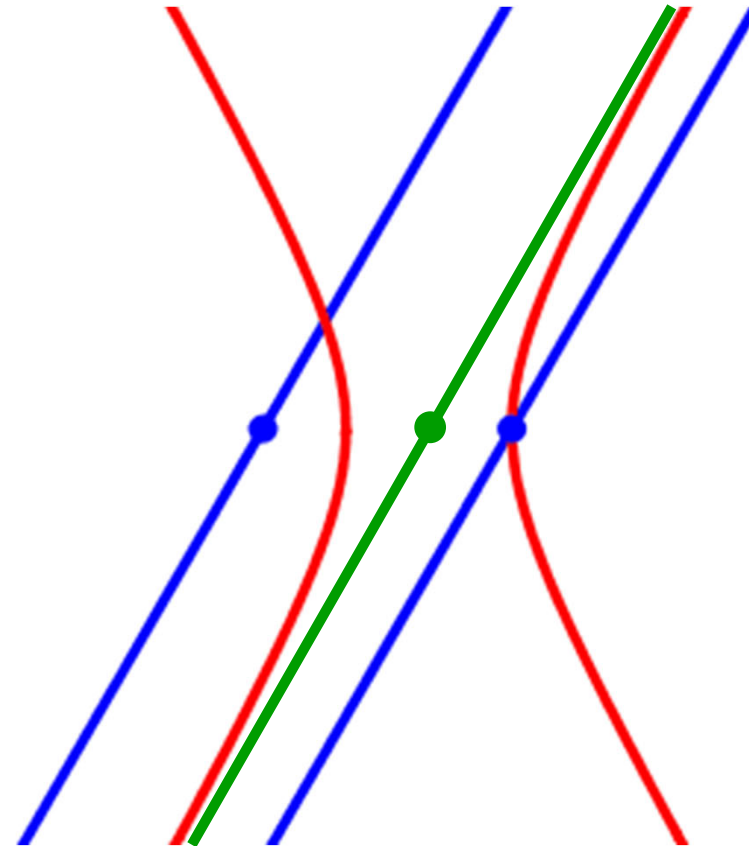
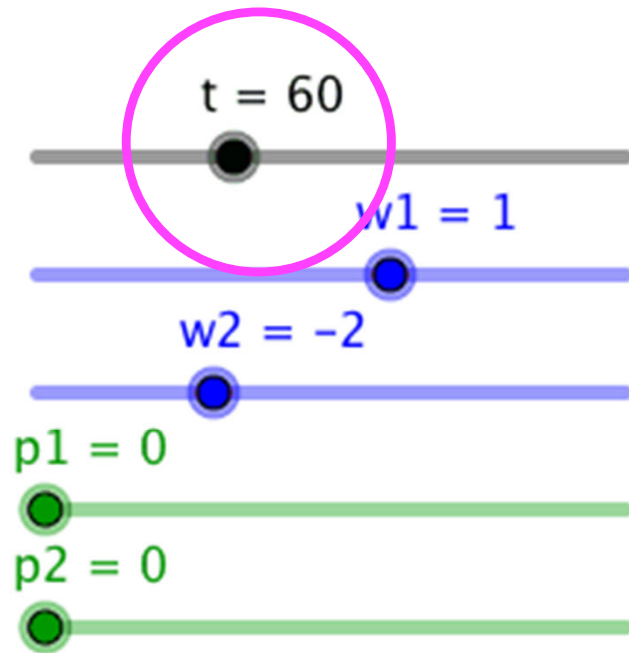
Frequenzen 1 und  $-2$

Hyperbel



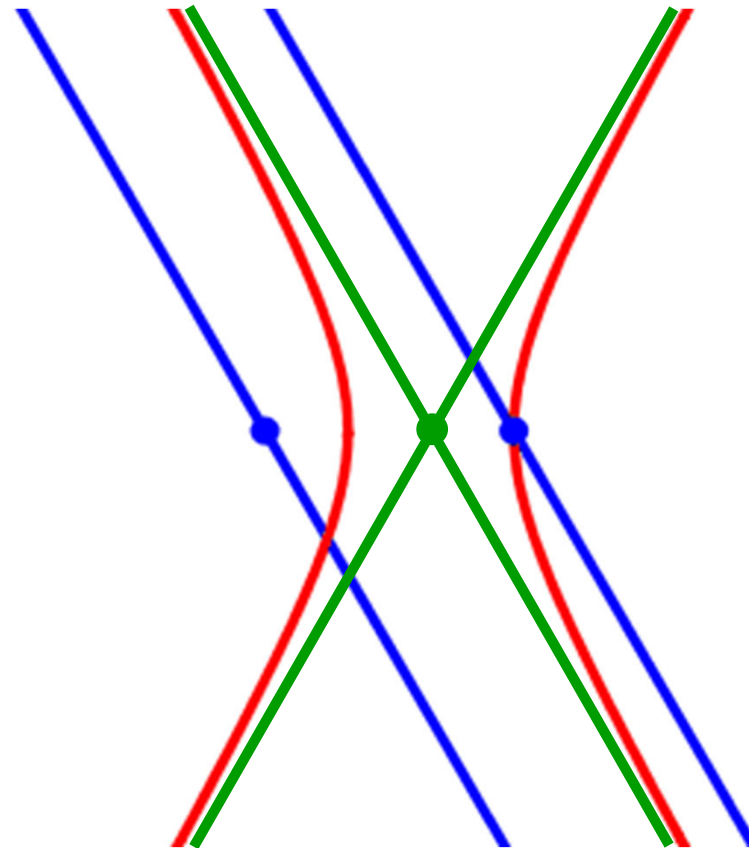
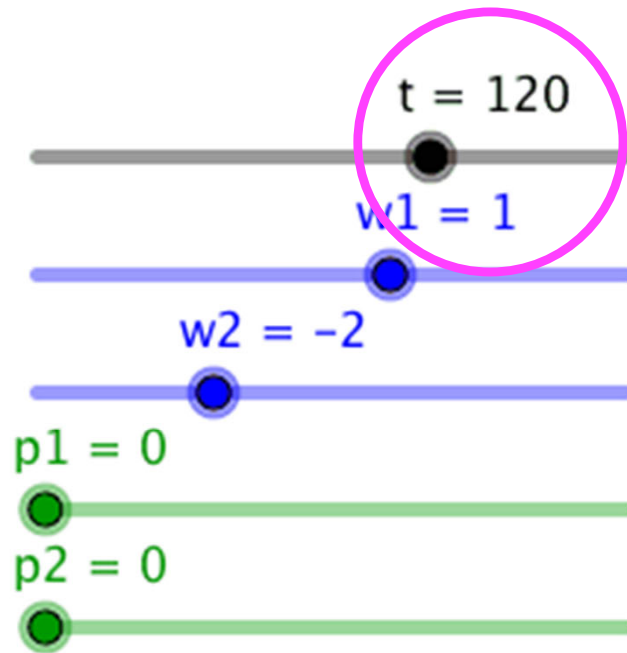
# Frequenzen 1 und -2

# Hyperbel



Frequenzen 1 und -2

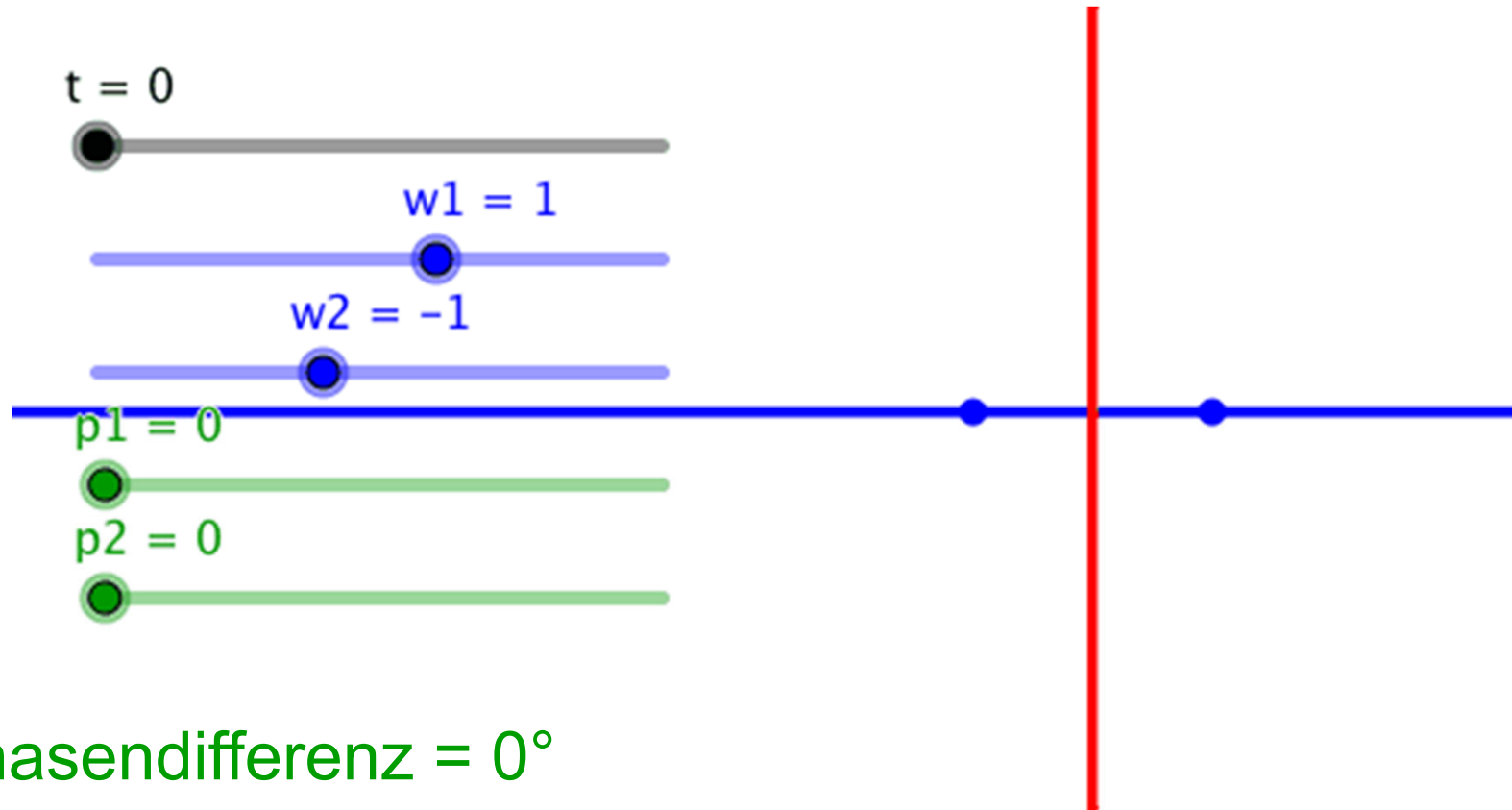
Hyperbel



Teilverhältnis 2:1

Entgegengesetzte Frequenzen

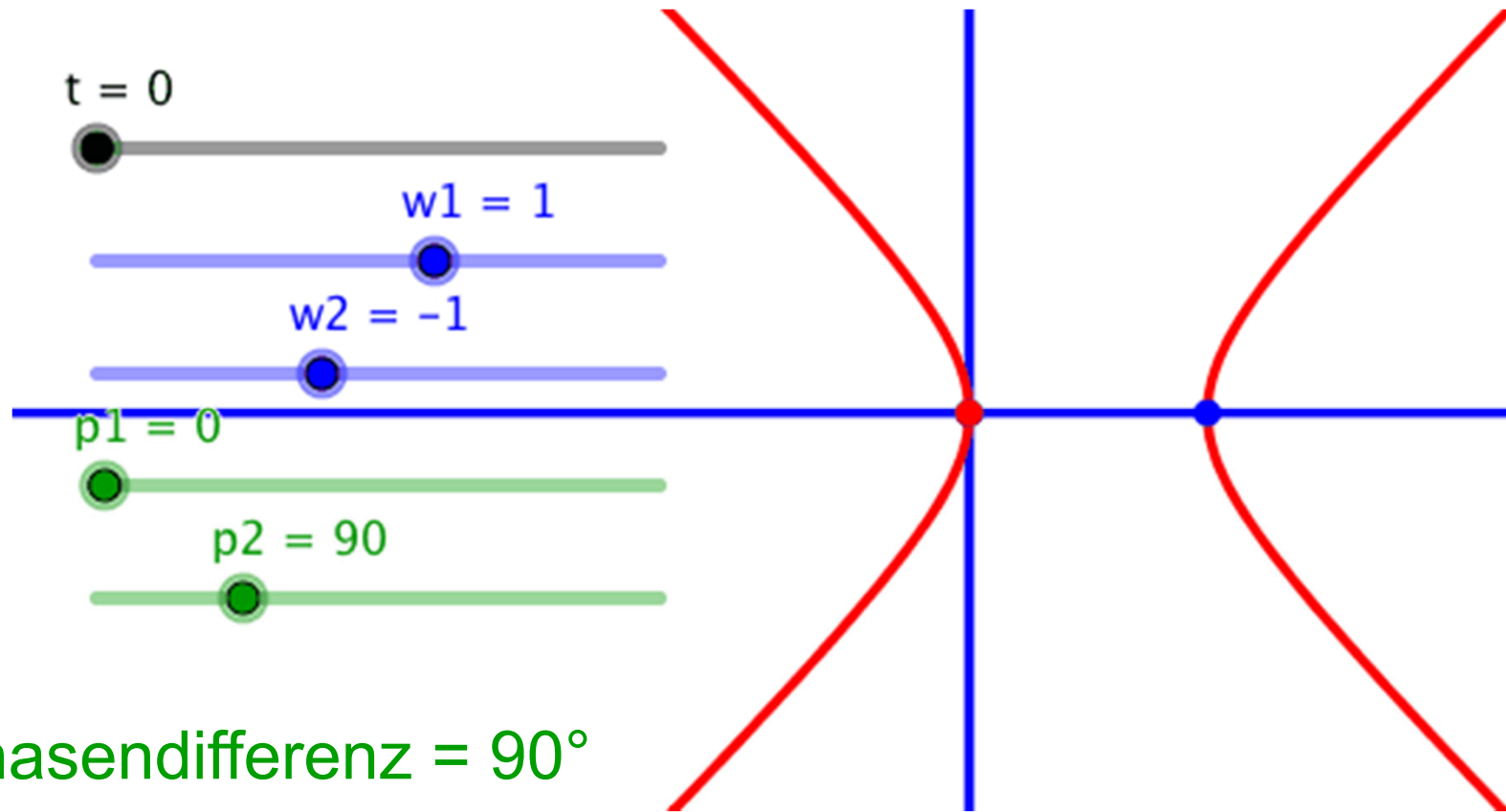
Gegengleich



Phasendifferenz =  $0^\circ$

Streckensymmetrale

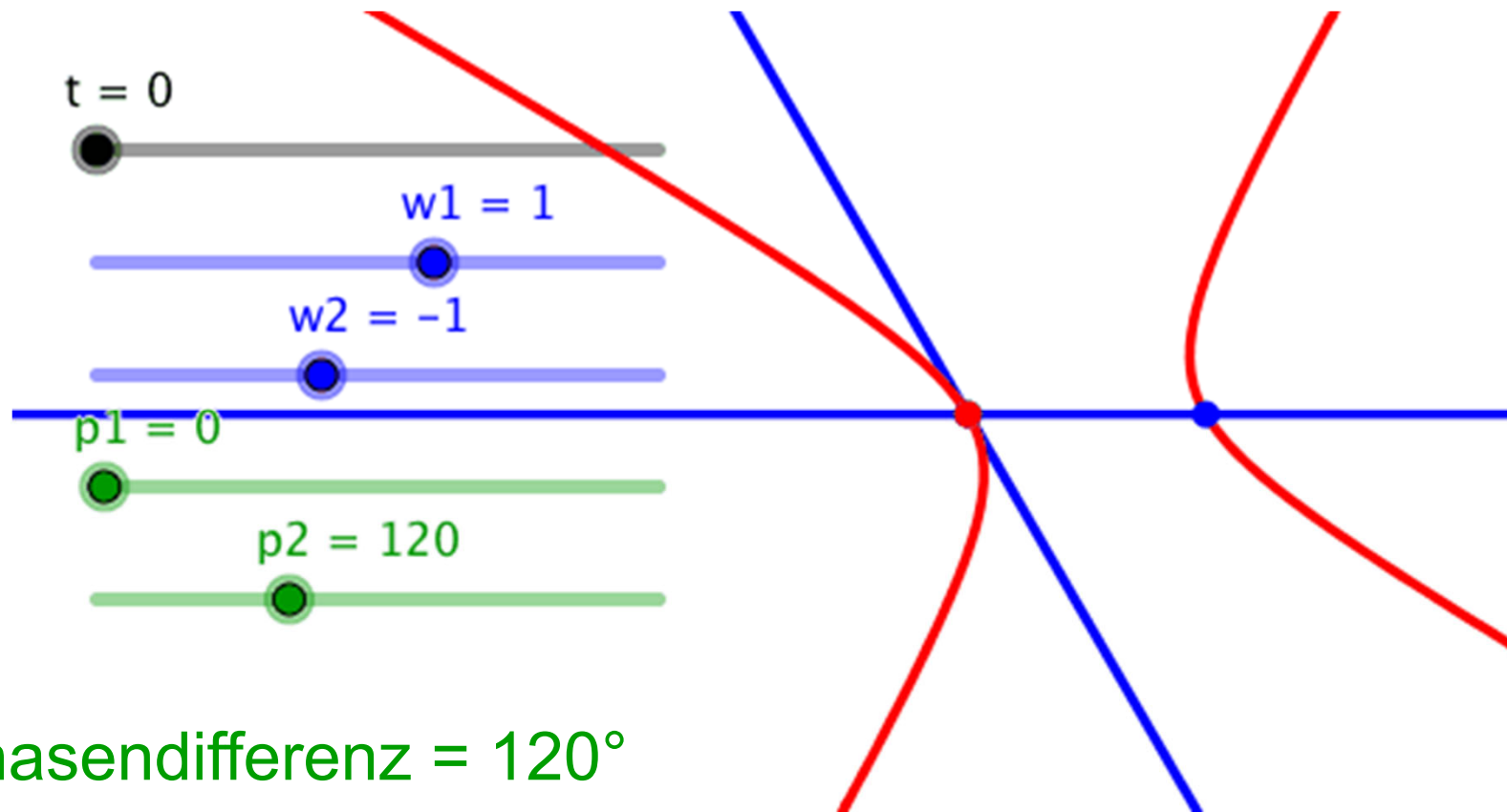
Entgegengesetzte Frequenzen Gleichseitige Hyperbel  
Pendant zum Thaleskreis



Phasendifferenz = 90°

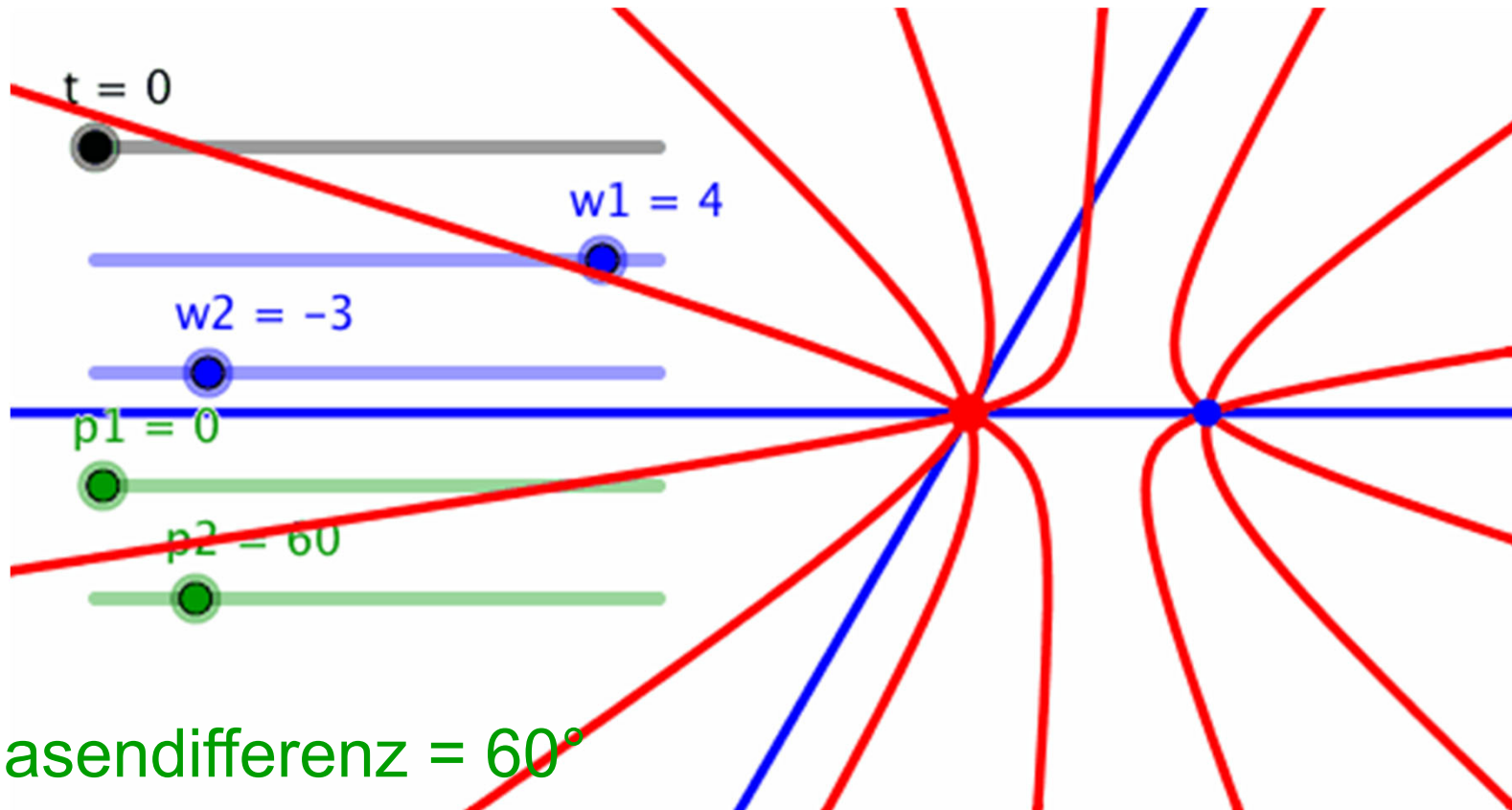
Entgegengesetzte Frequenzen

Gleichseitige Hyperbel  
Pendant zum Ortsbogen



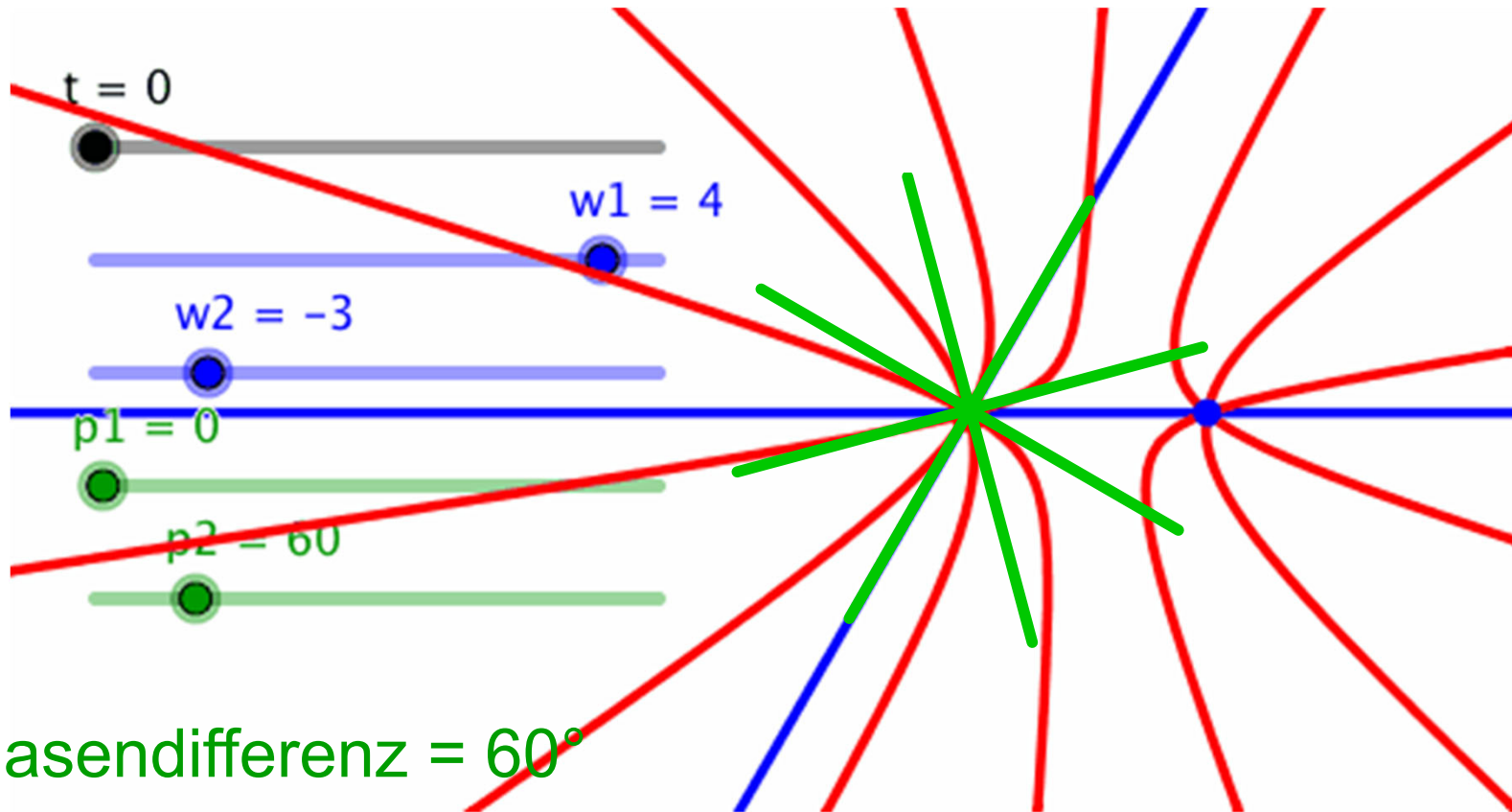
Phasendifferenz =  $120^\circ$

# Frequenzen 4 und -3





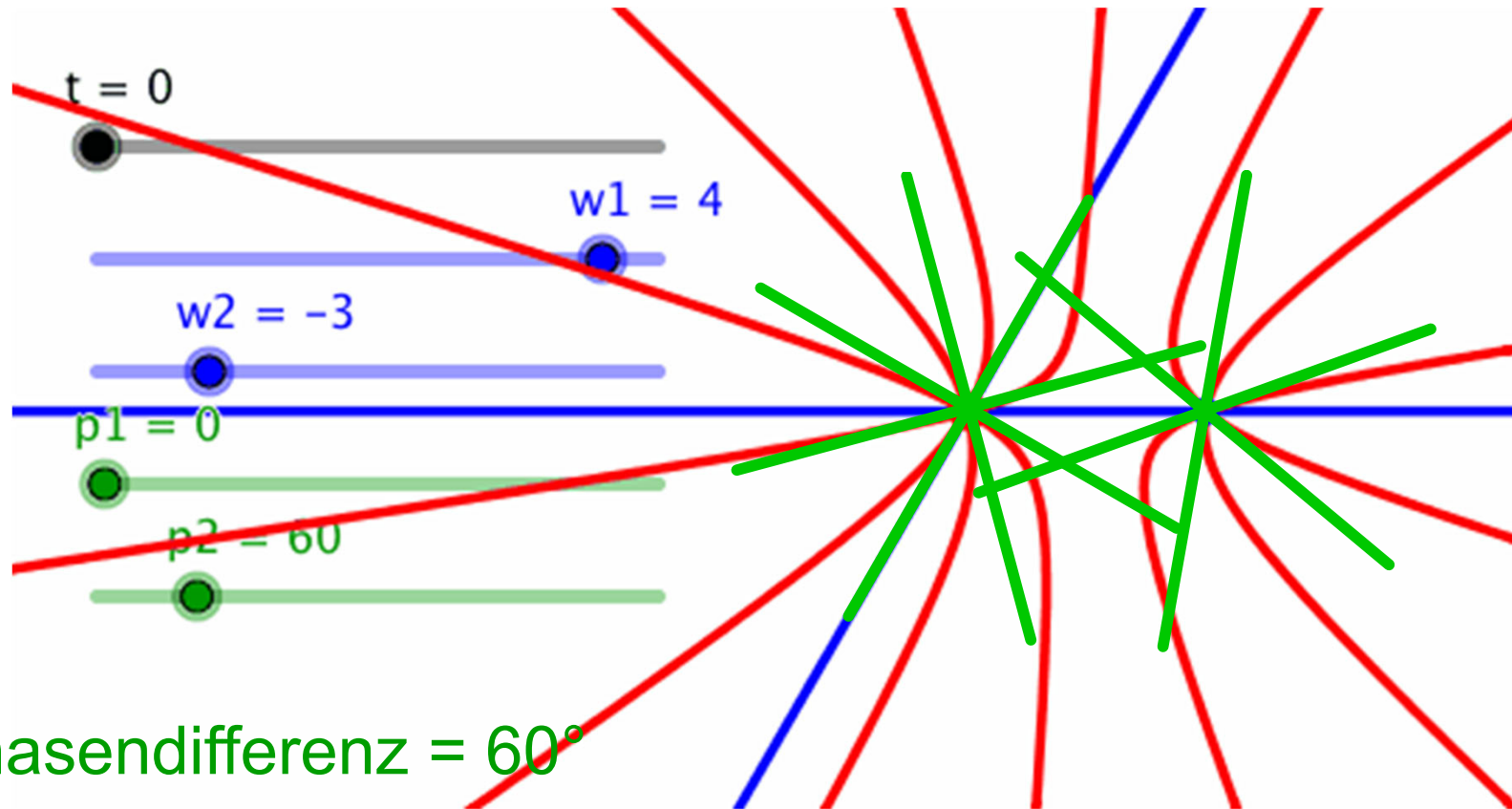
# Frequenzen 4 und -3



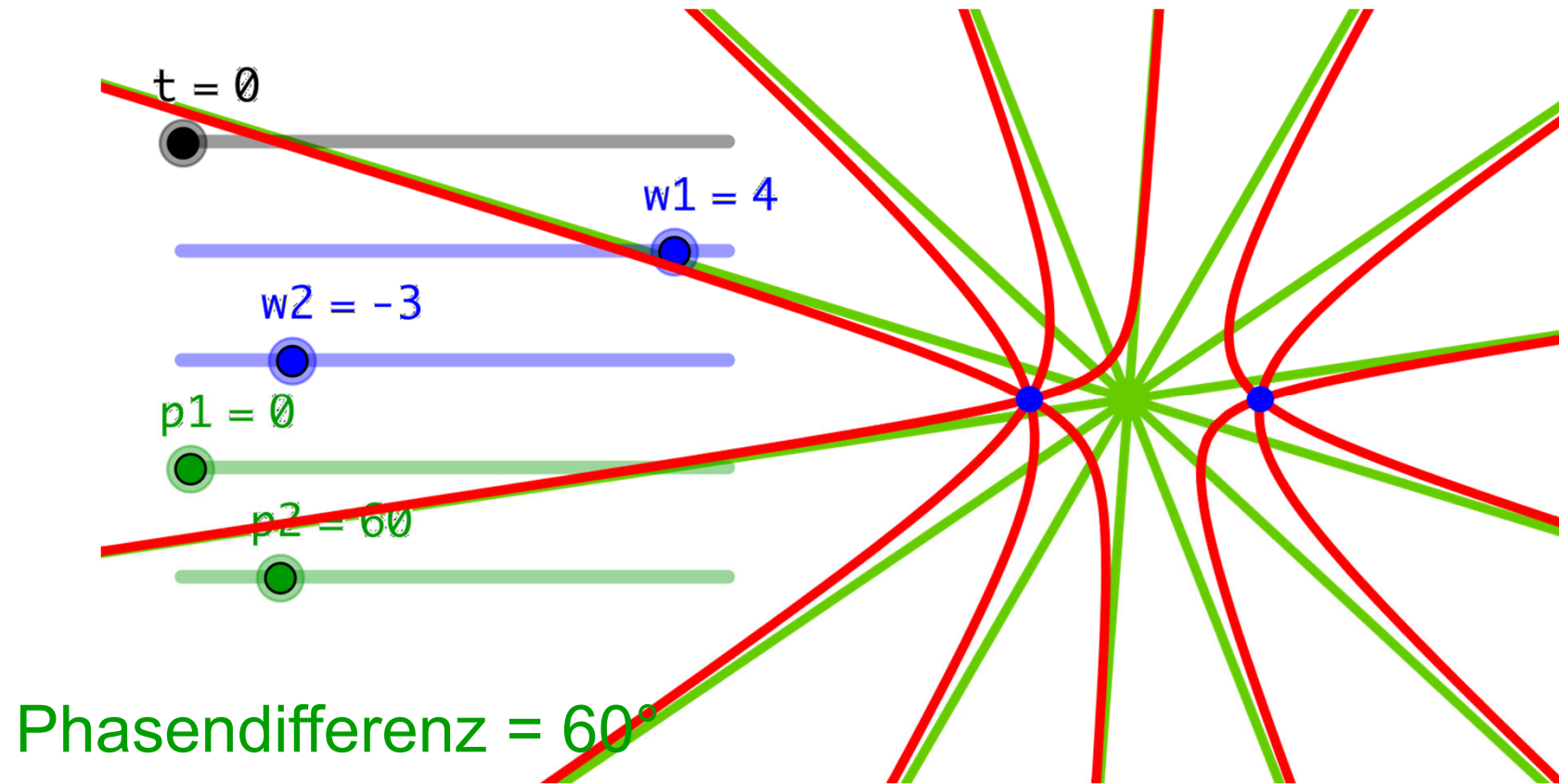
Phasendifferenz =  $60^\circ$

$45^\circ$

# Frequenzen 4 und -3



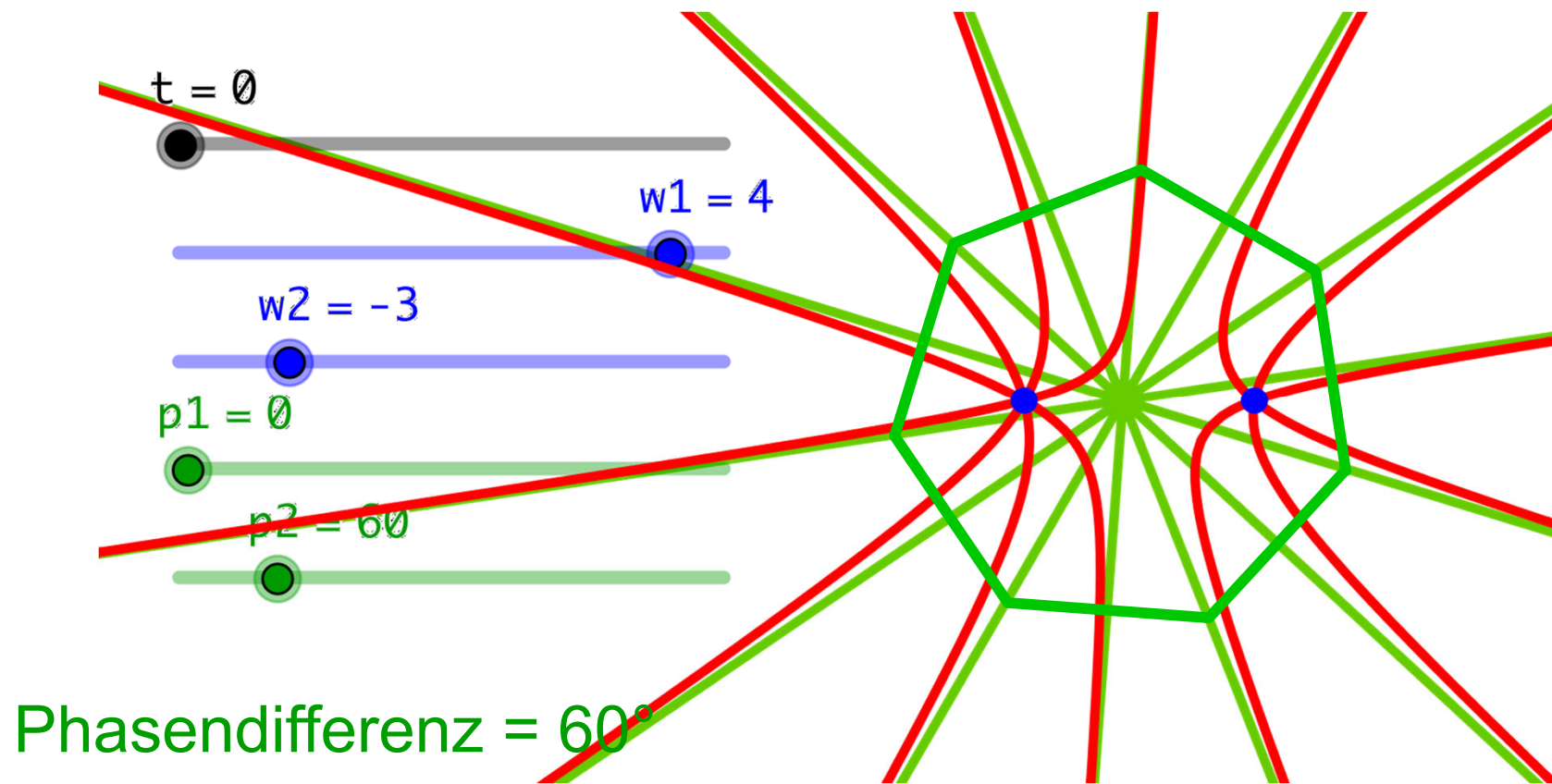
# Frequenzen 4 und -3



Phasendifferenz =  $60^\circ$

Teilverhältnis 3:4

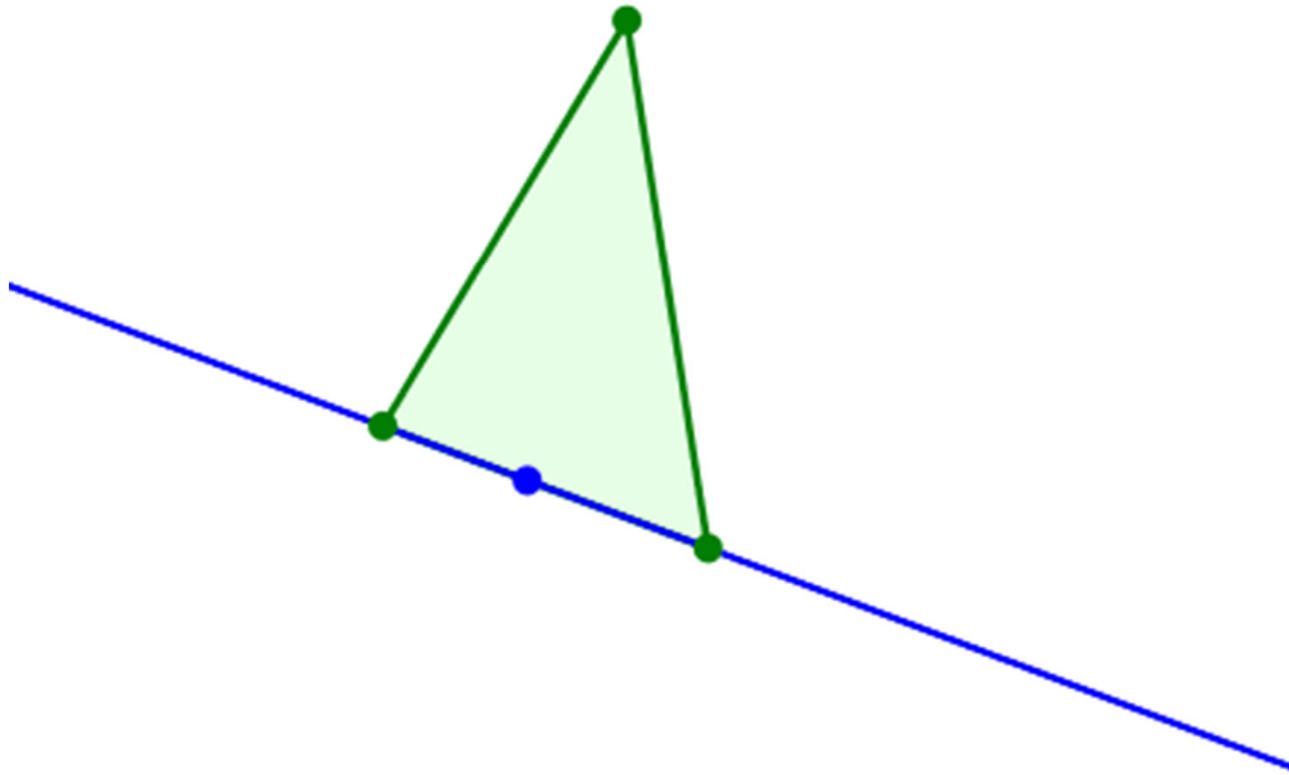
# Frequenzen 4 und -3



Phasendifferenz =  $60^\circ$

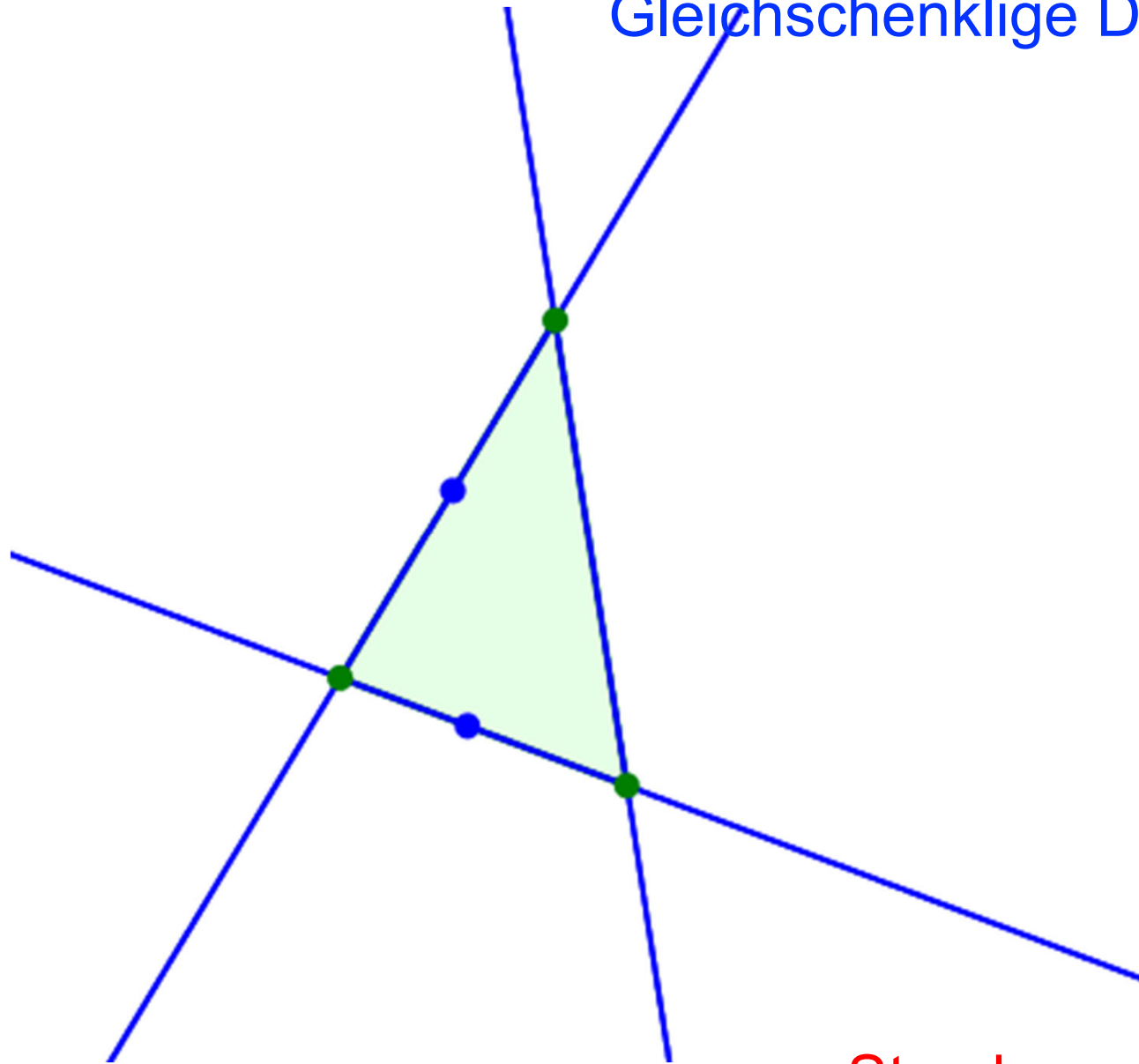
Teilverhältnis 3:4  
Siebeneck

Gegengleich



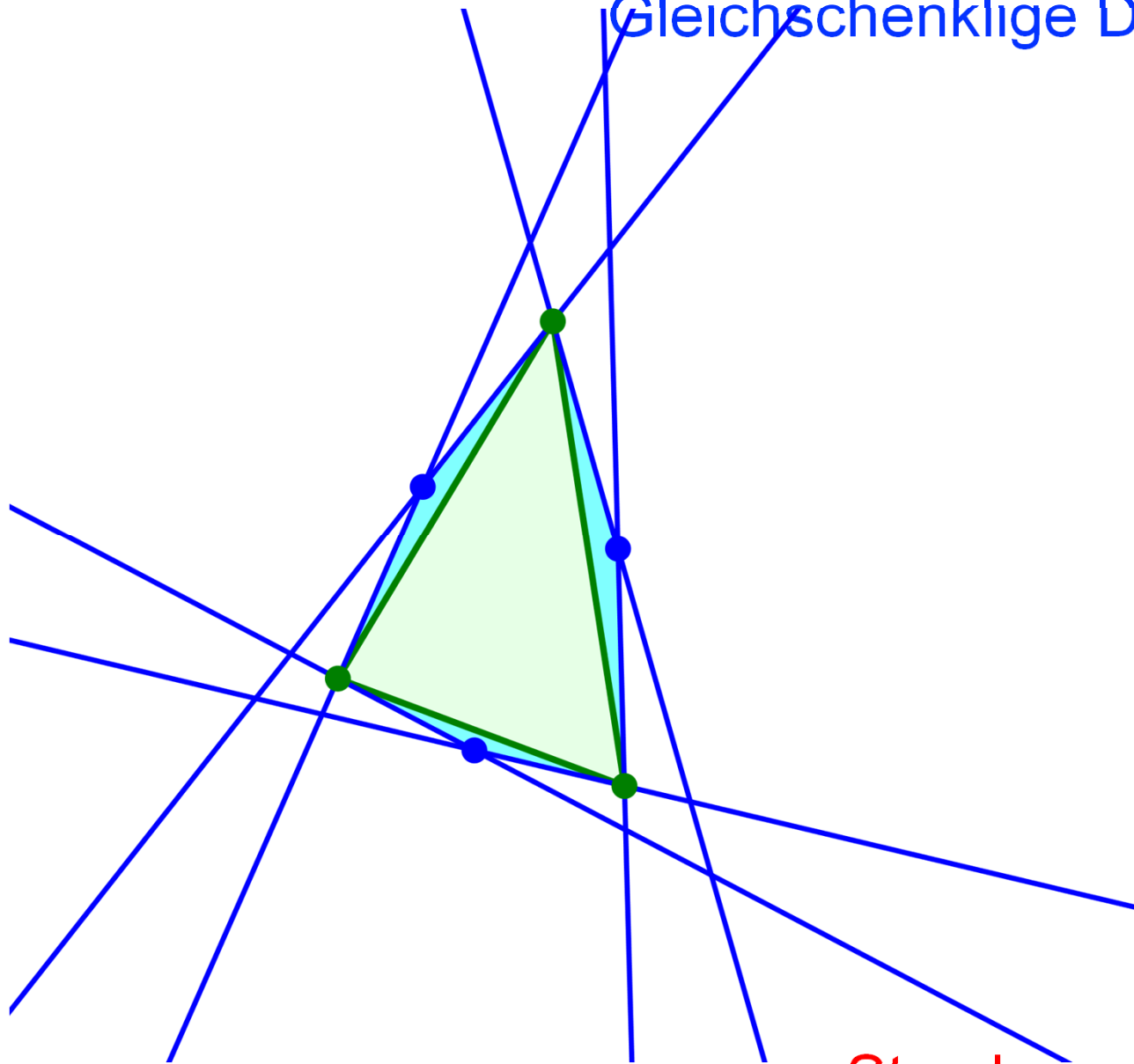
Streckensymmetrale

Gleichschenklige Dreiecke

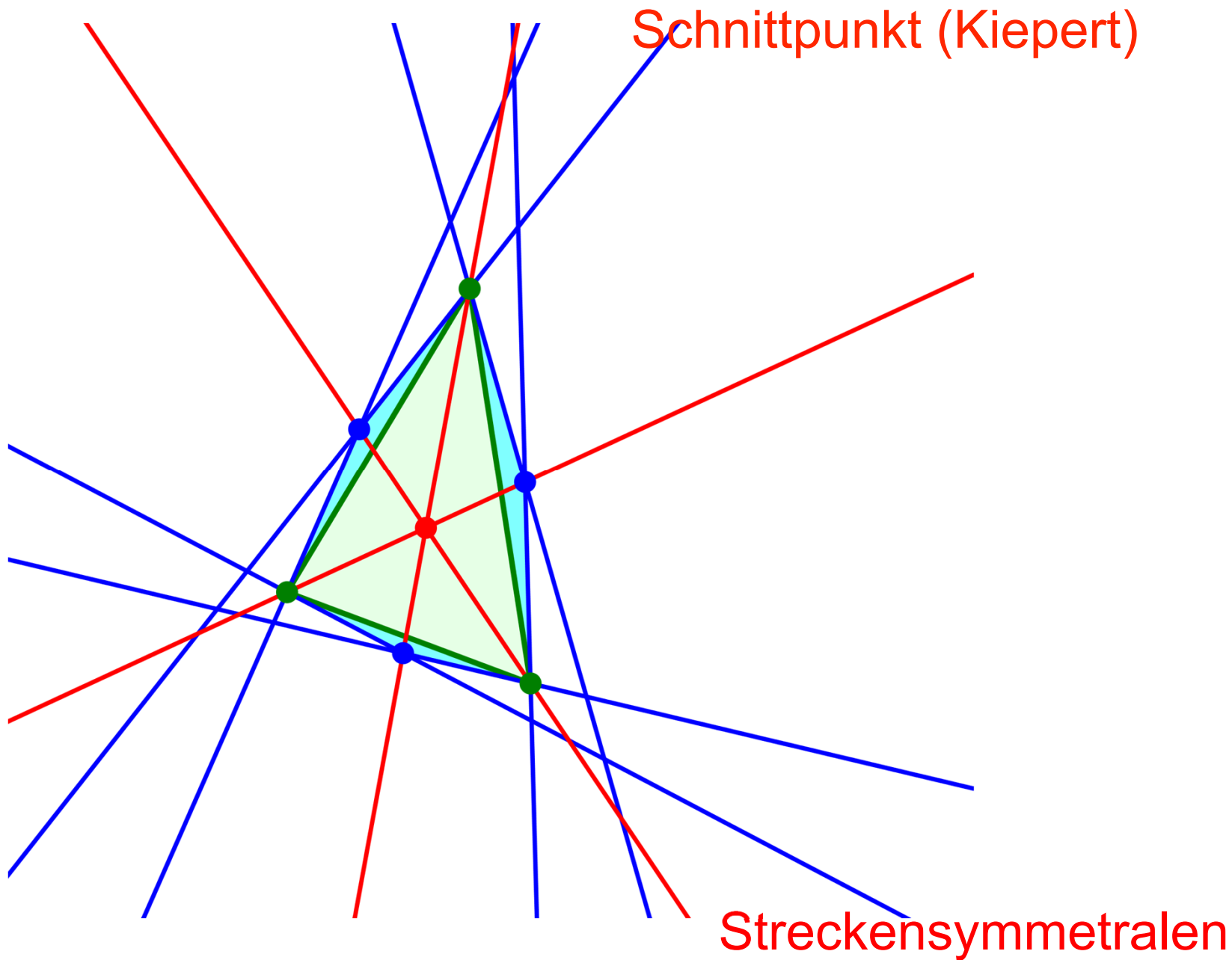


Streckensymmetralen

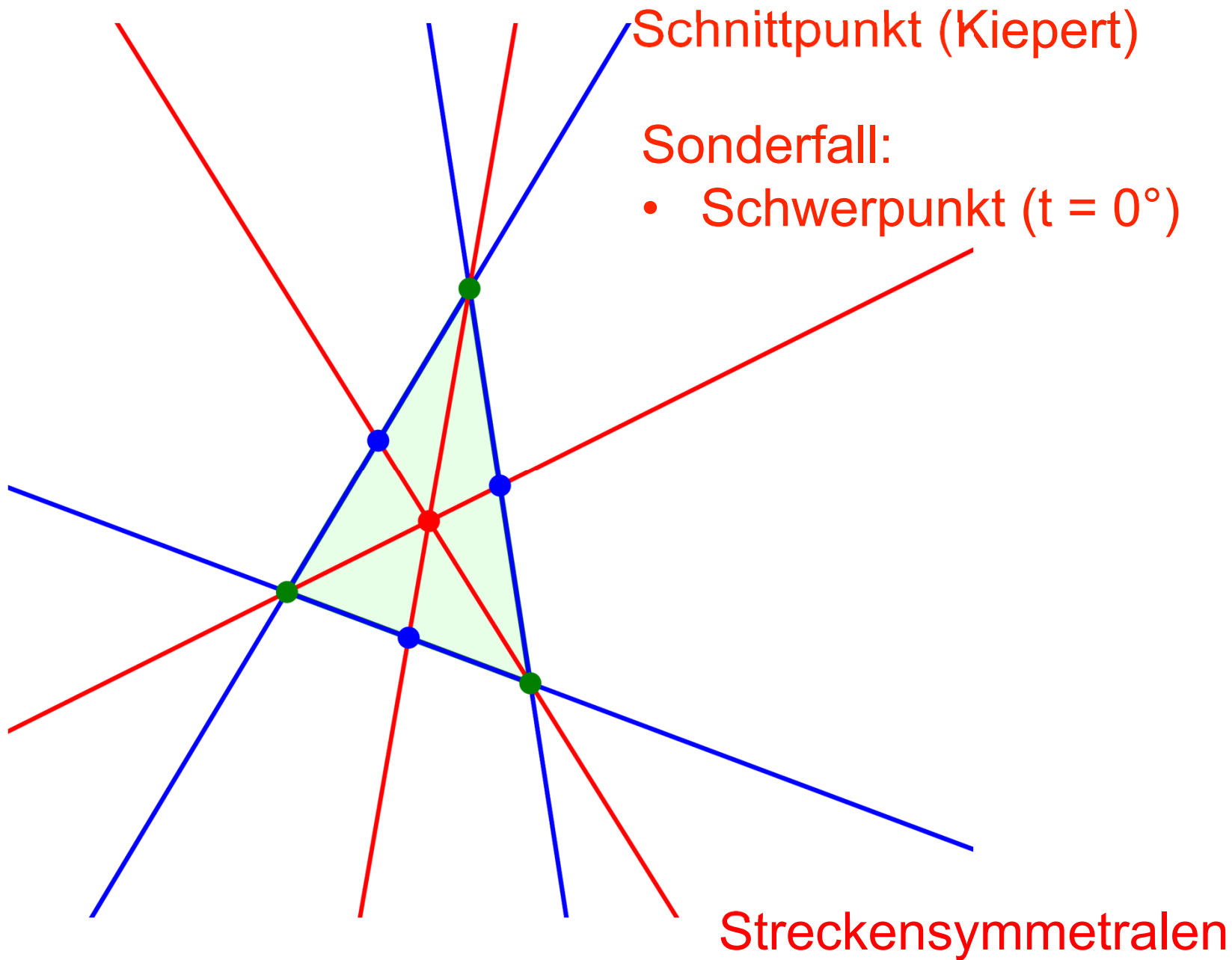
Gleichschenklige Dreiecke

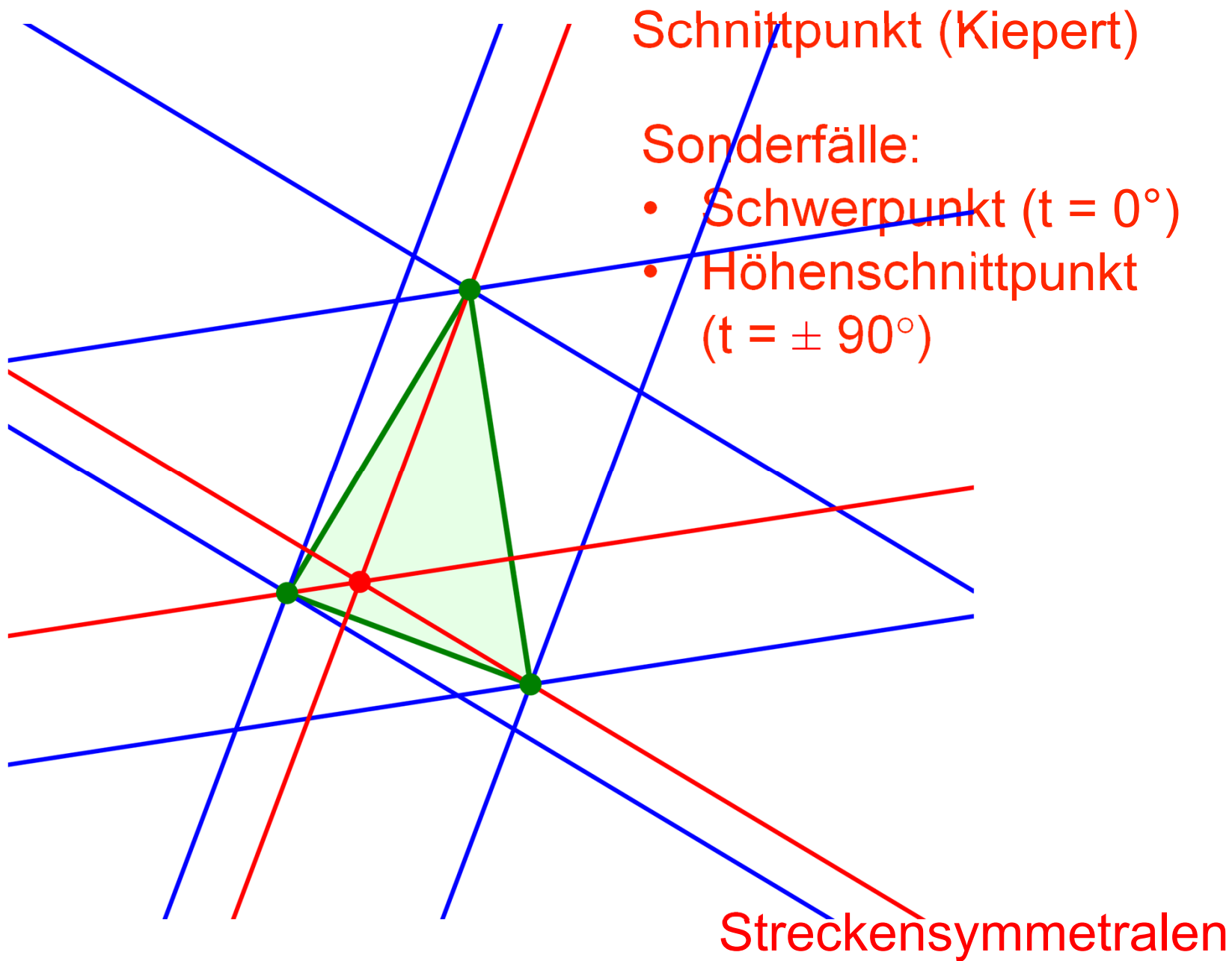


Streckensymmetralen

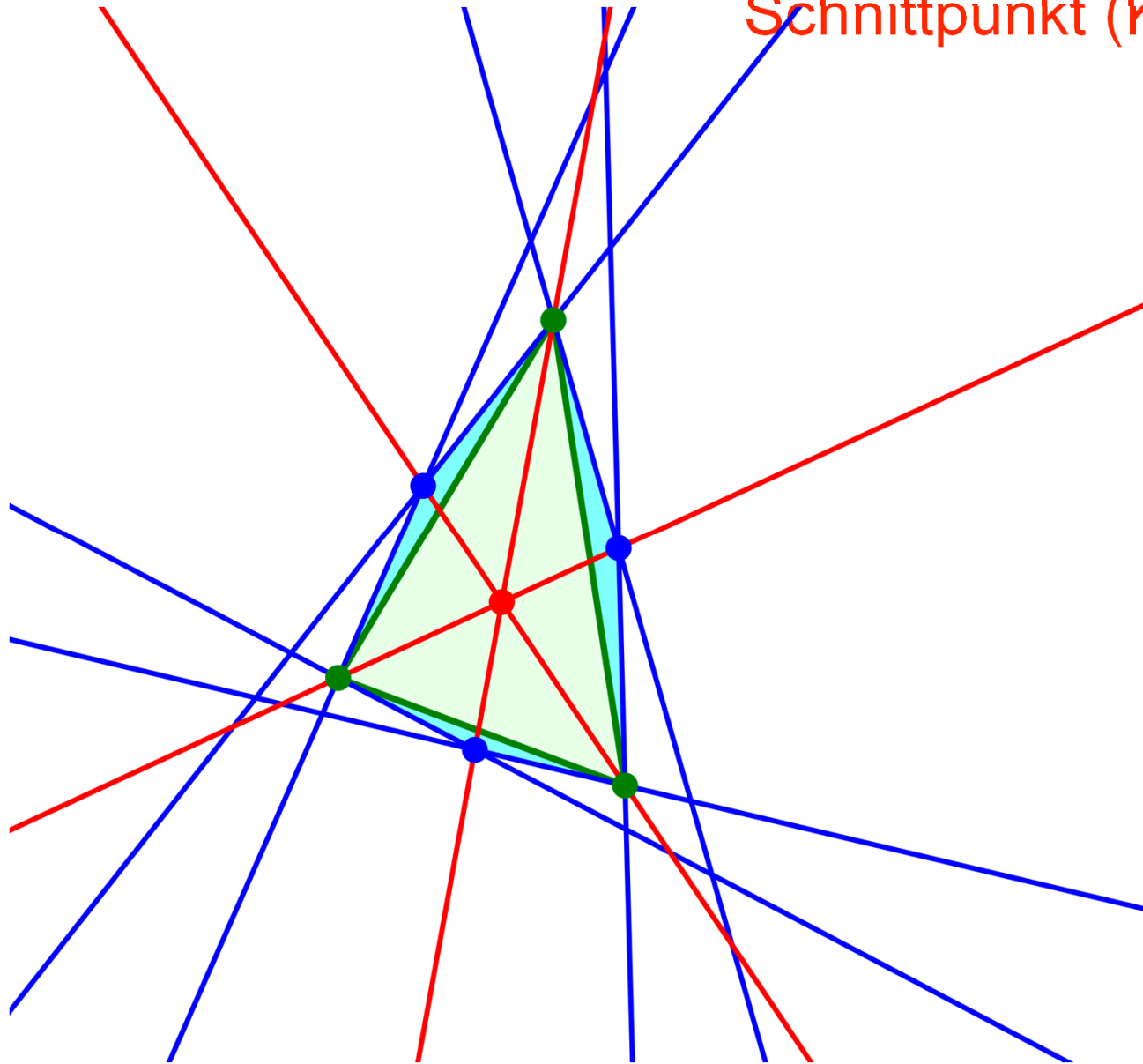




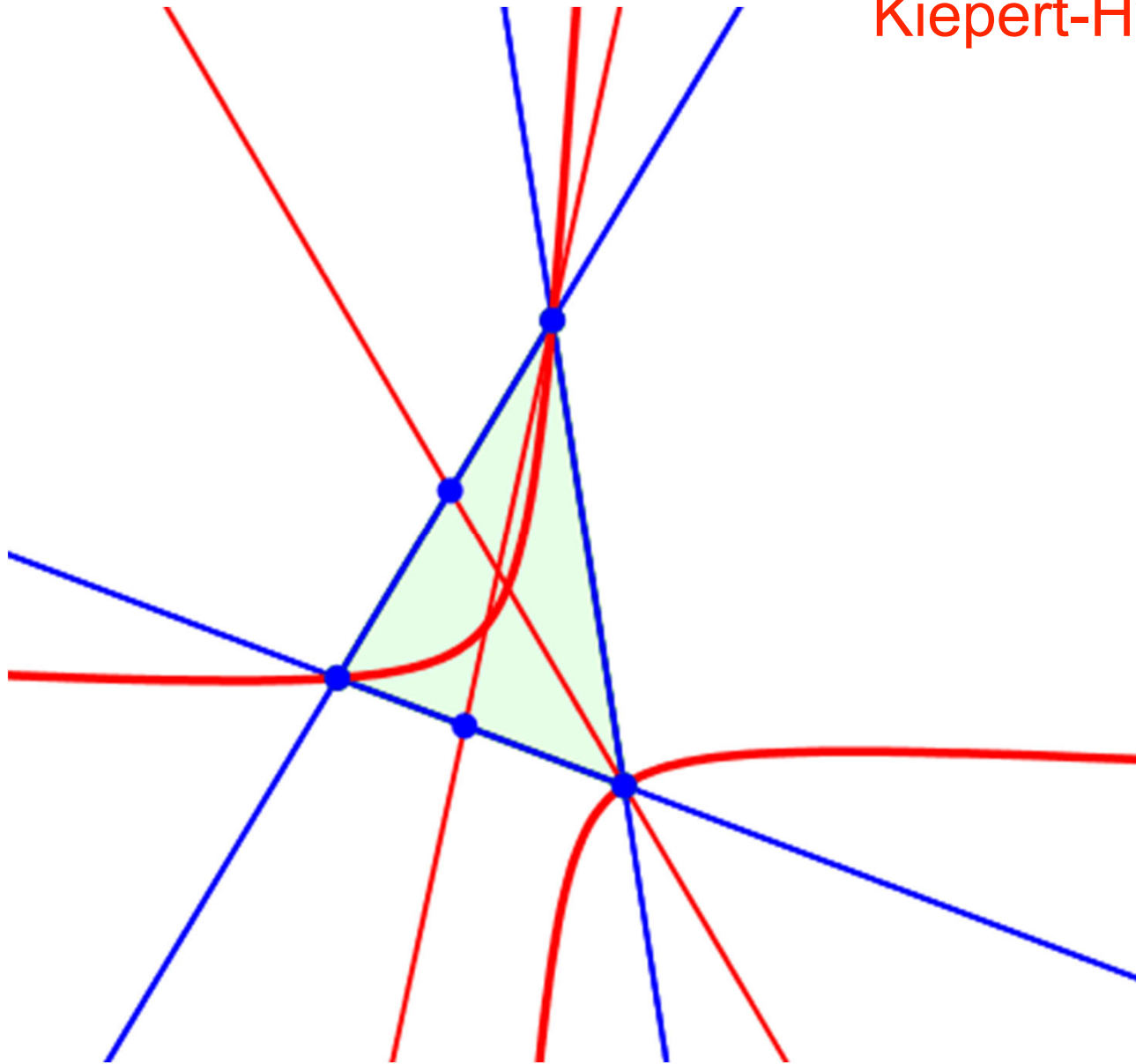




Schnittpunkt (Kiepert)

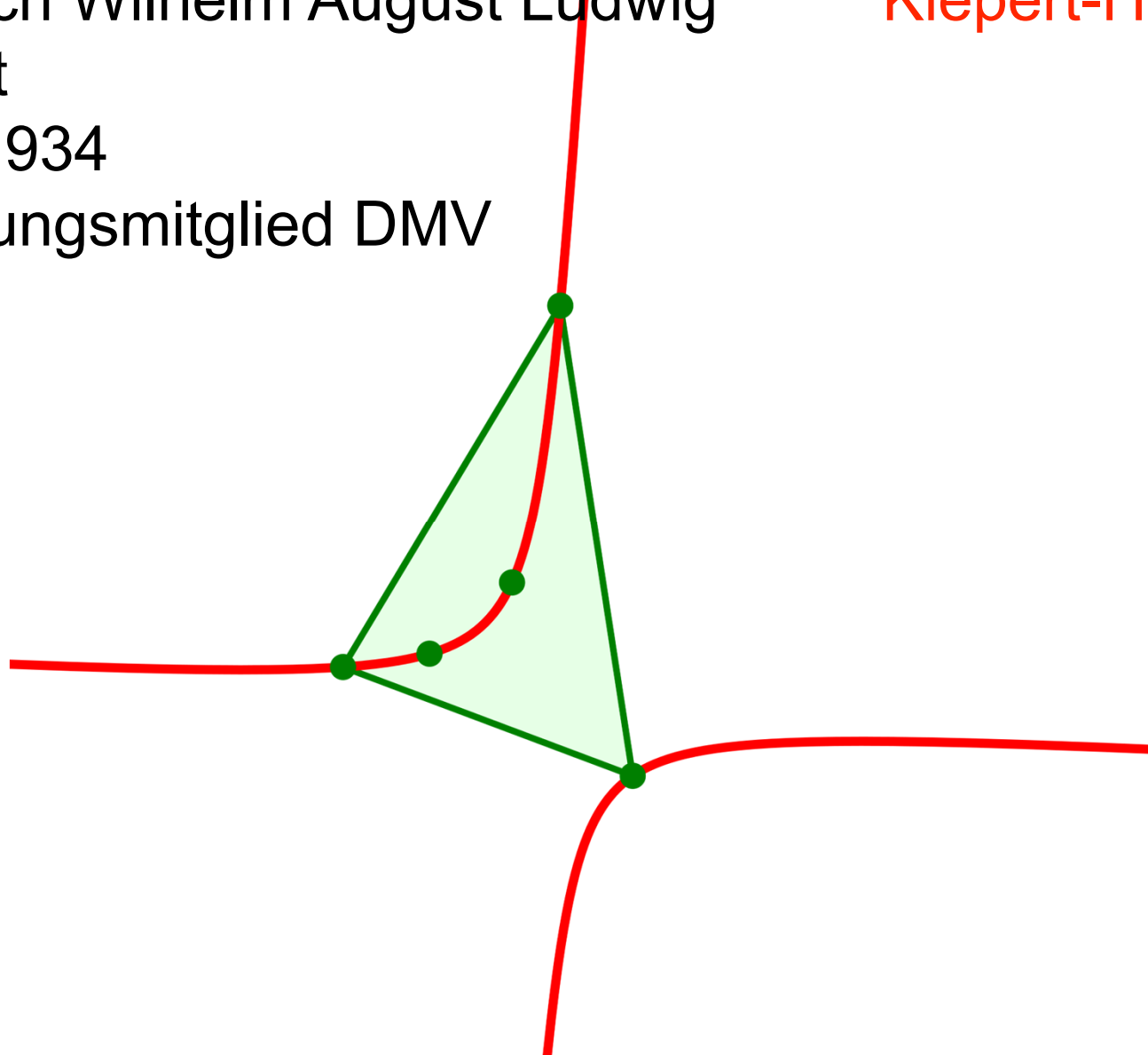


# Kiepert-Hyperbel



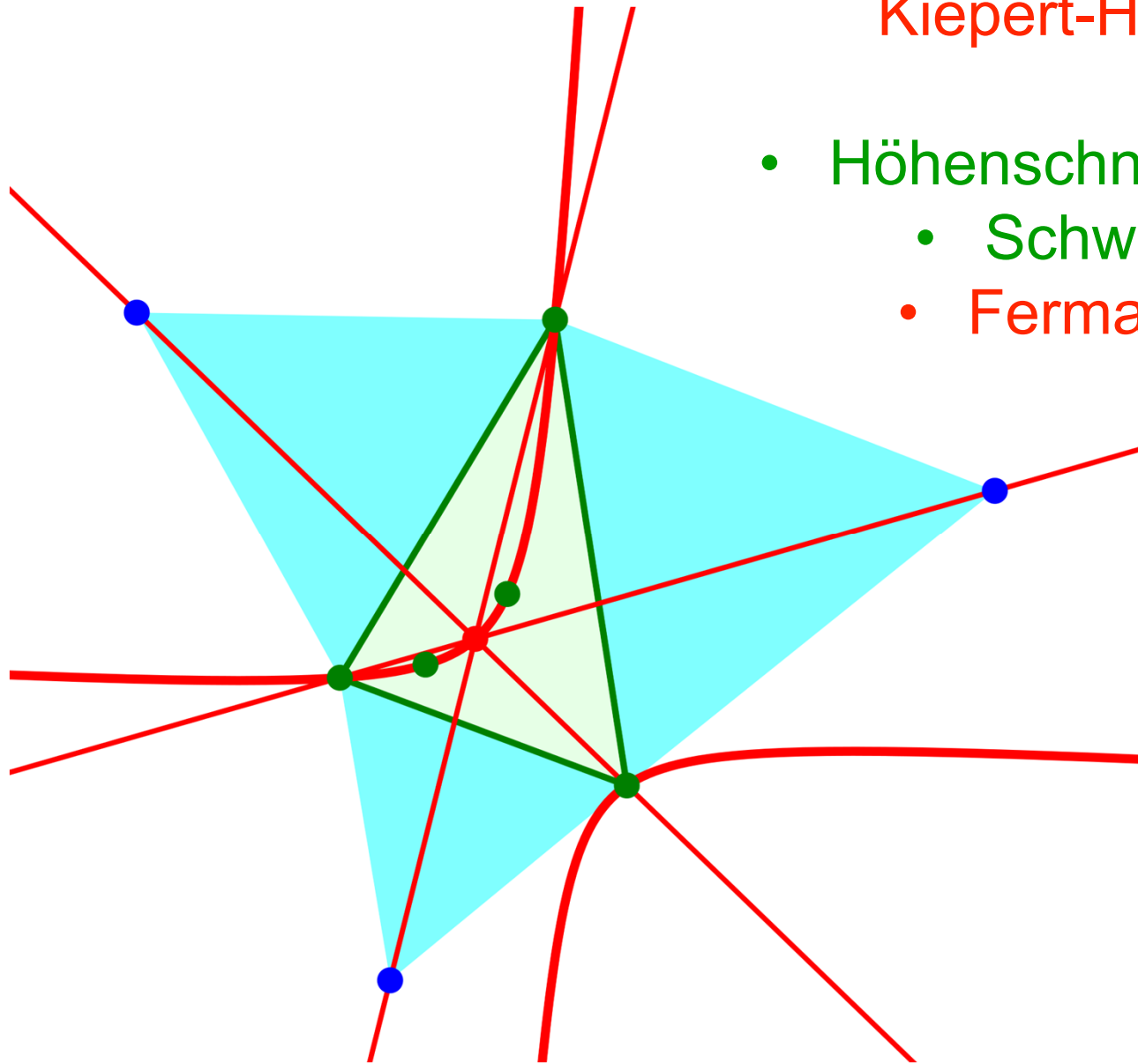
Friedrich Wilhelm August Ludwig  
Kiepert  
1846-1934  
Gründungsmitglied DMV

Kiepert-Hyperbel



# Kiepert-Hyperbel

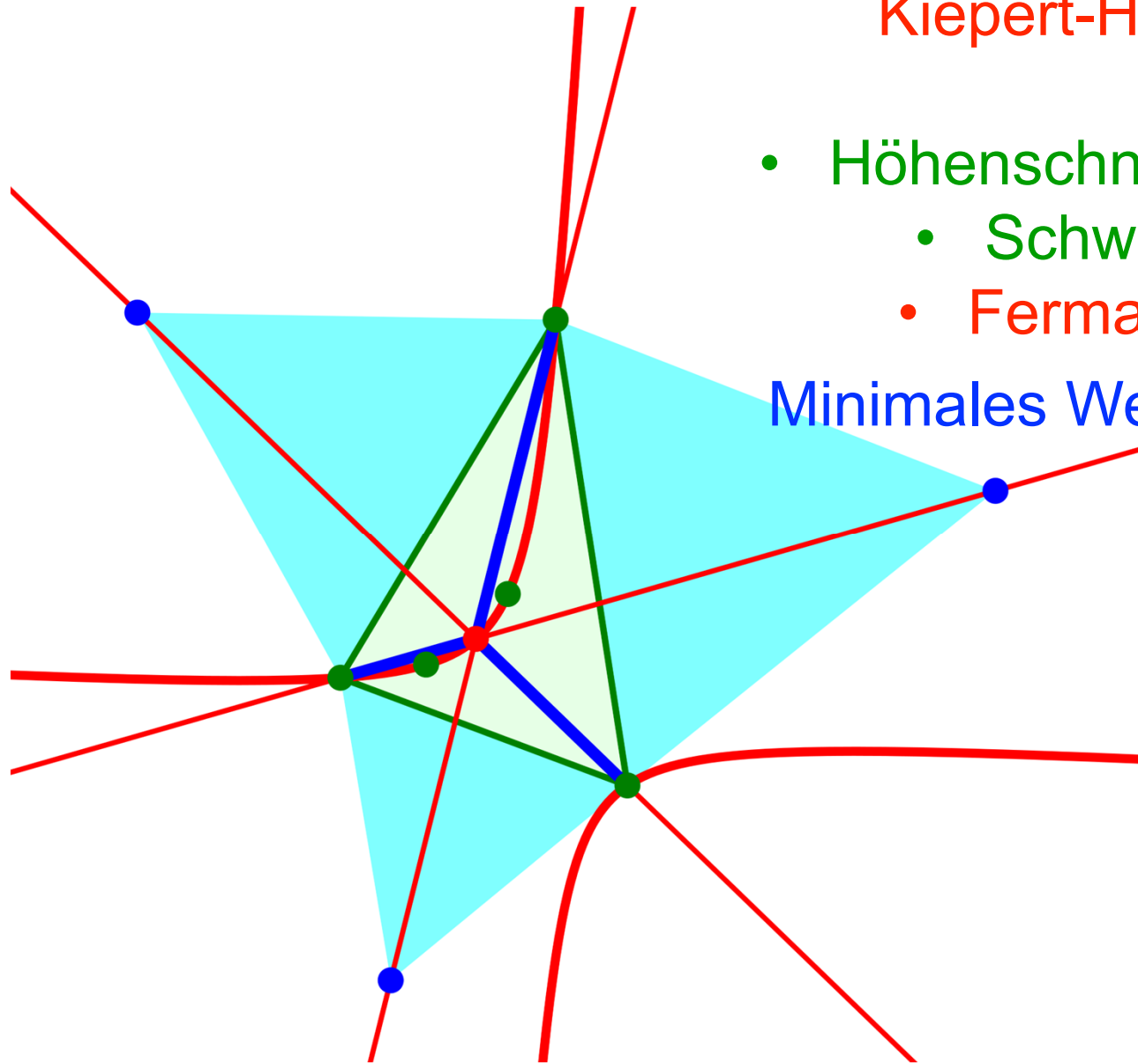
- Höhenschnittpunkt
- Schwerpunkt
- Fermat-Punkt



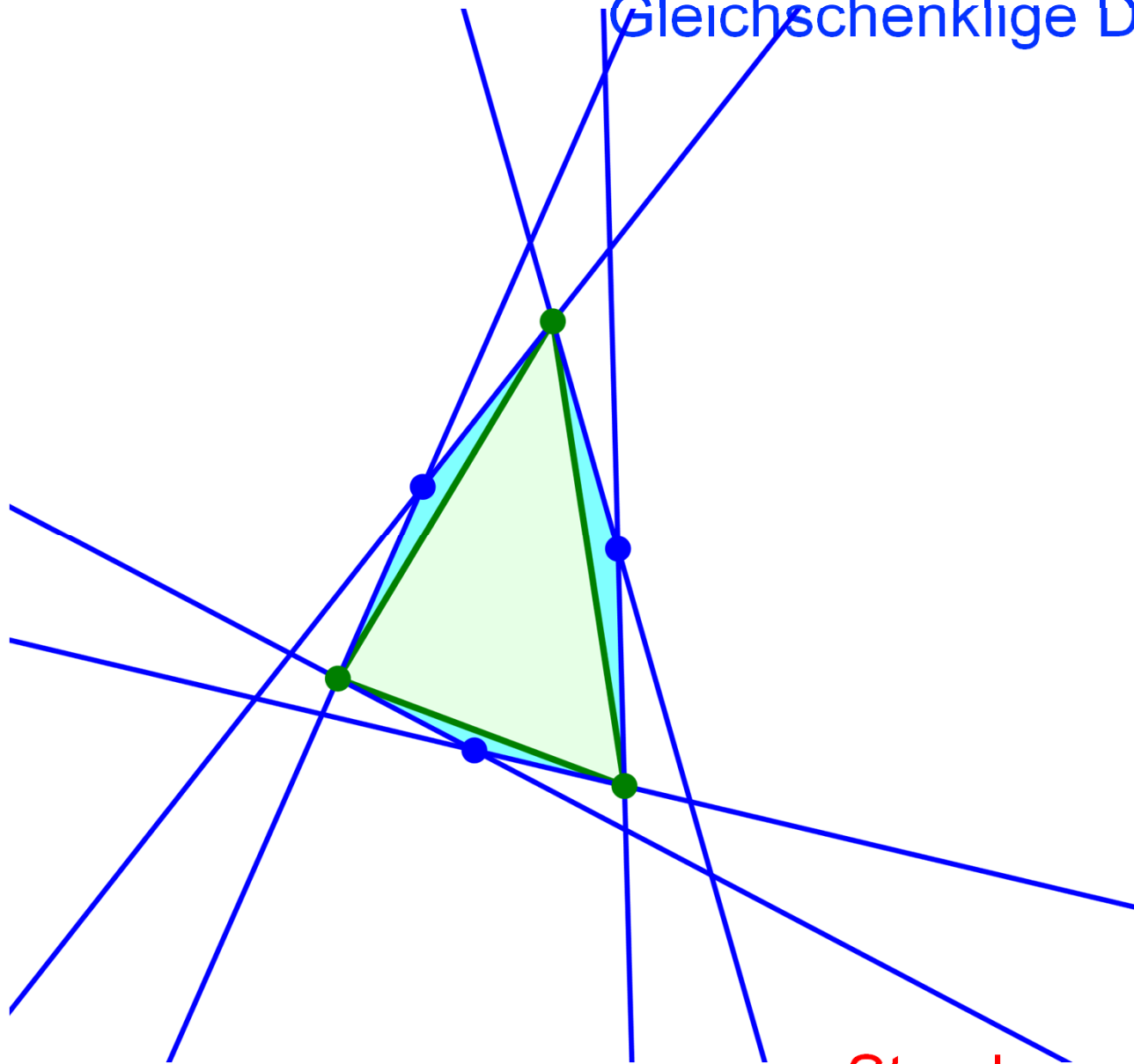
## Kiepert-Hyperbel

- Höhenschnittpunkt
  - Schwerpunkt
  - Fermat-Punkt

Minimales Wegenetz



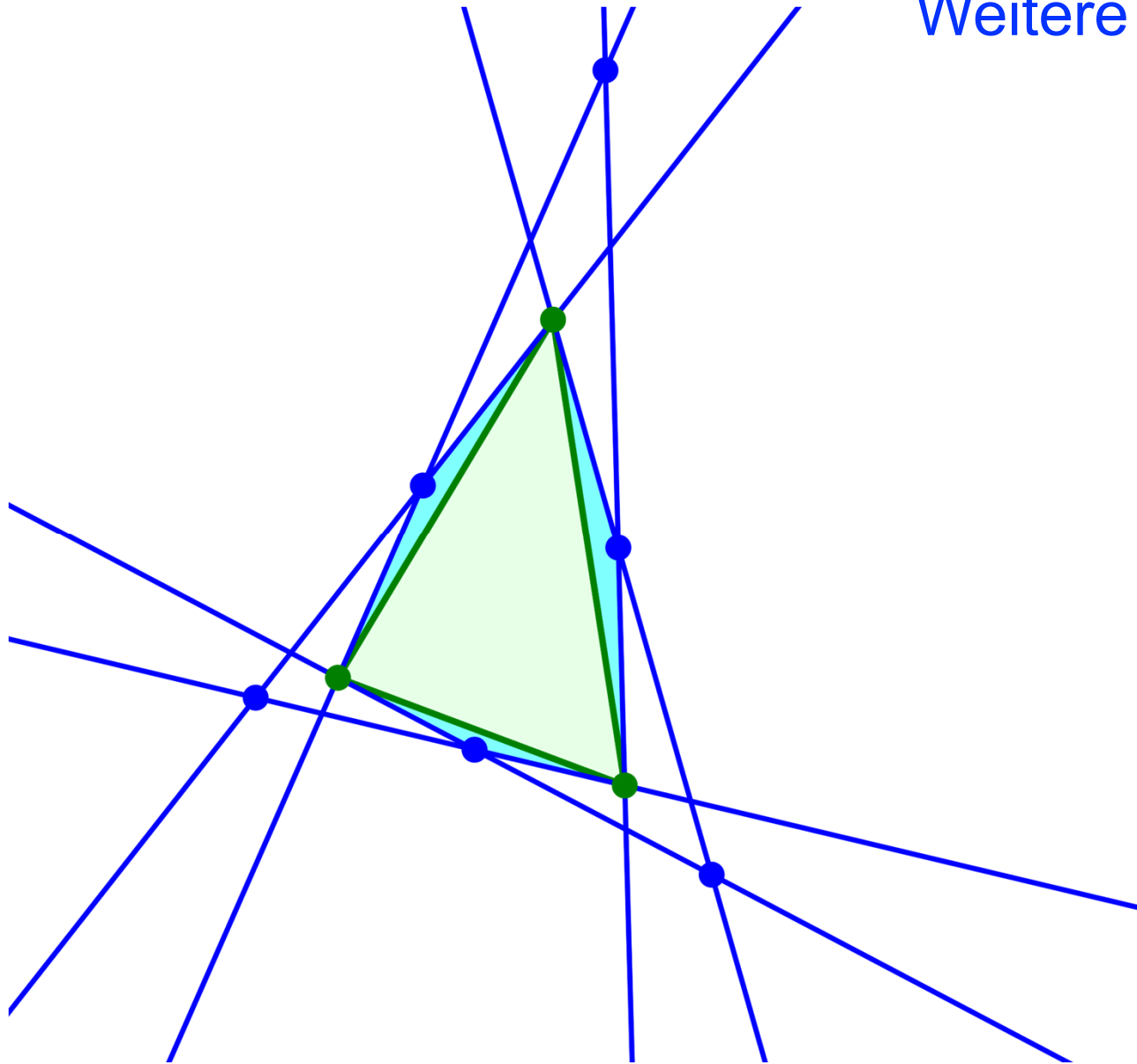
Gleichschenklige Dreiecke



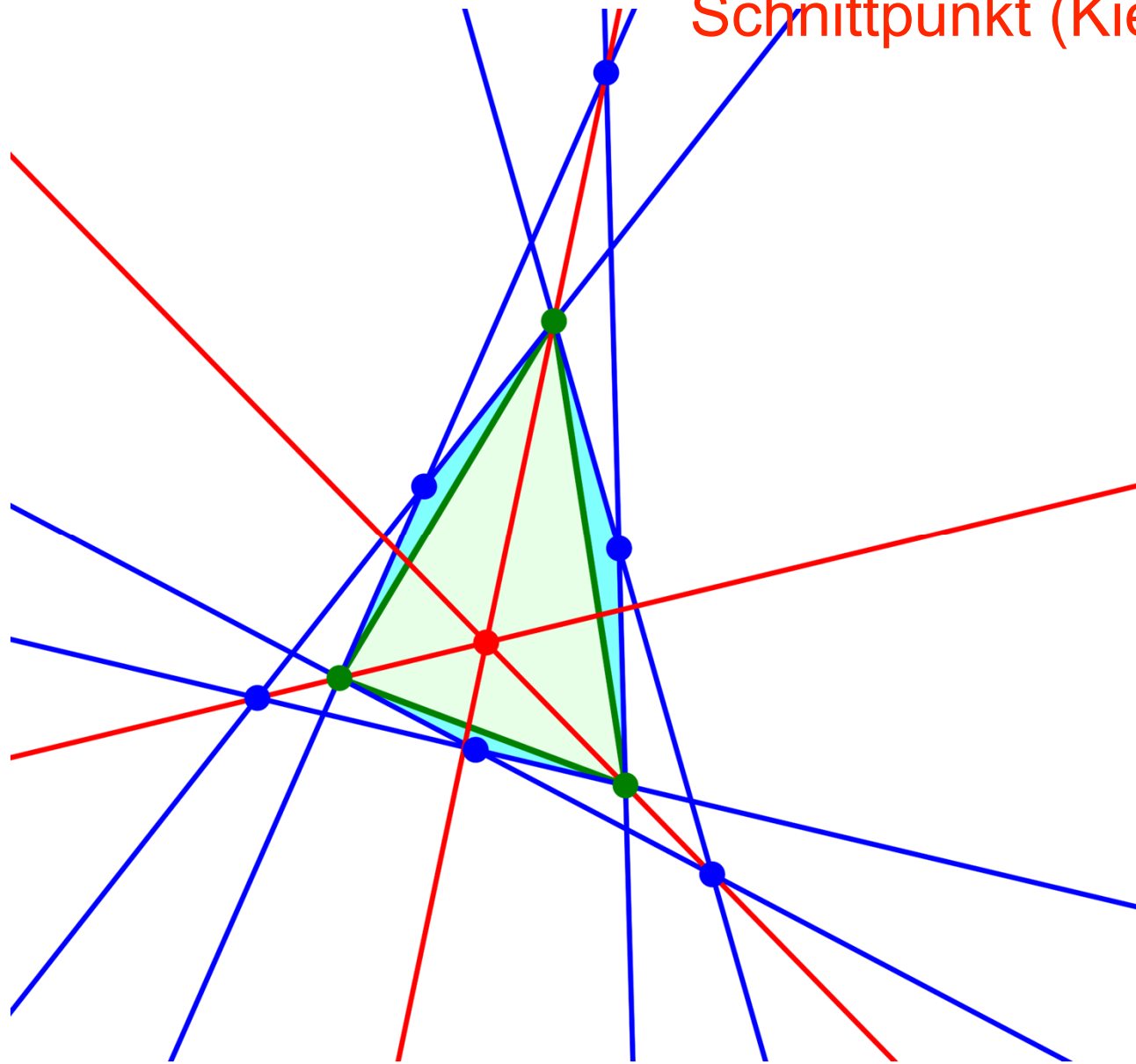
Streckensymmetralen



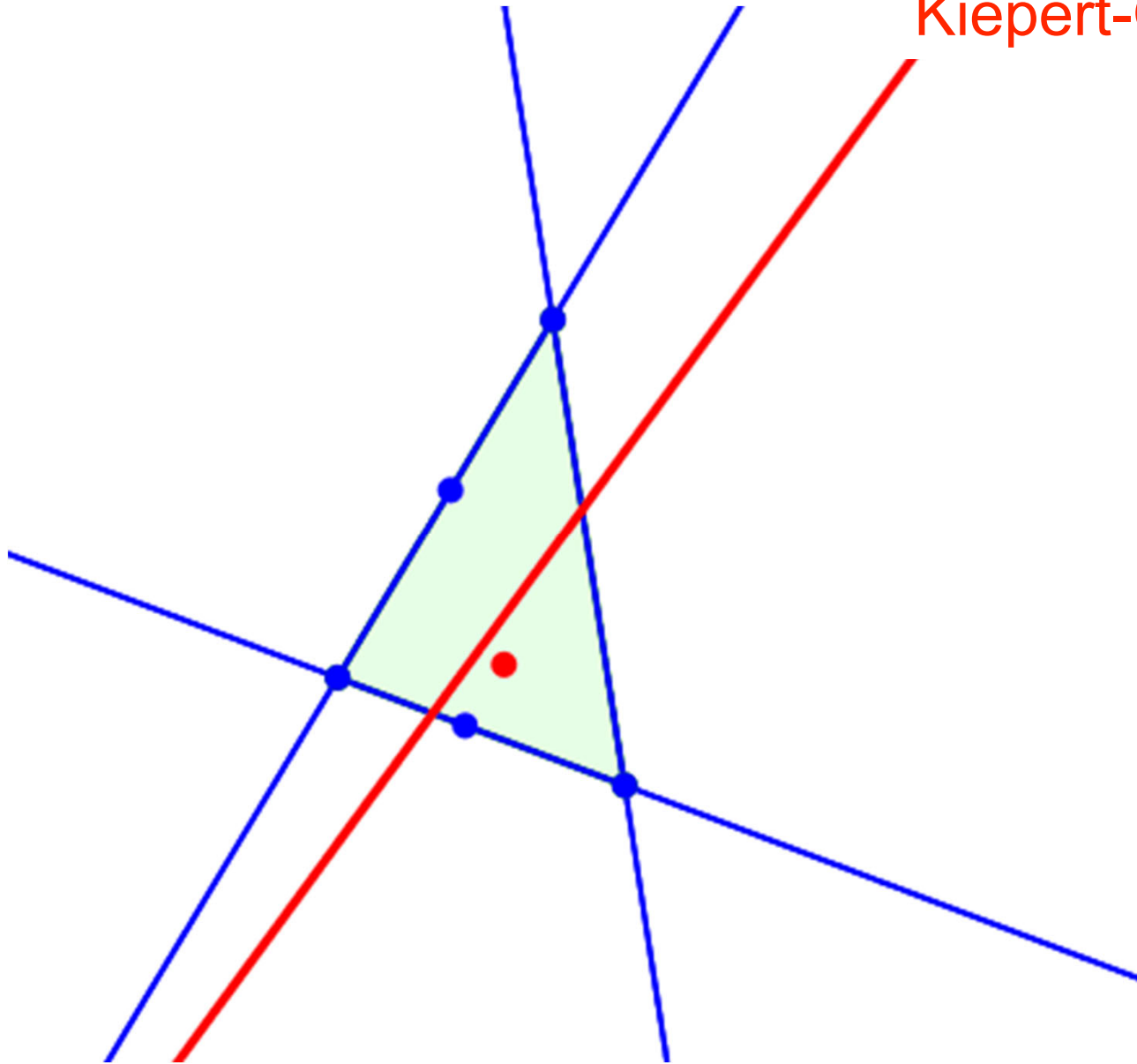
Weitere Punkte



Schnittpunkt (Kiepert 2)

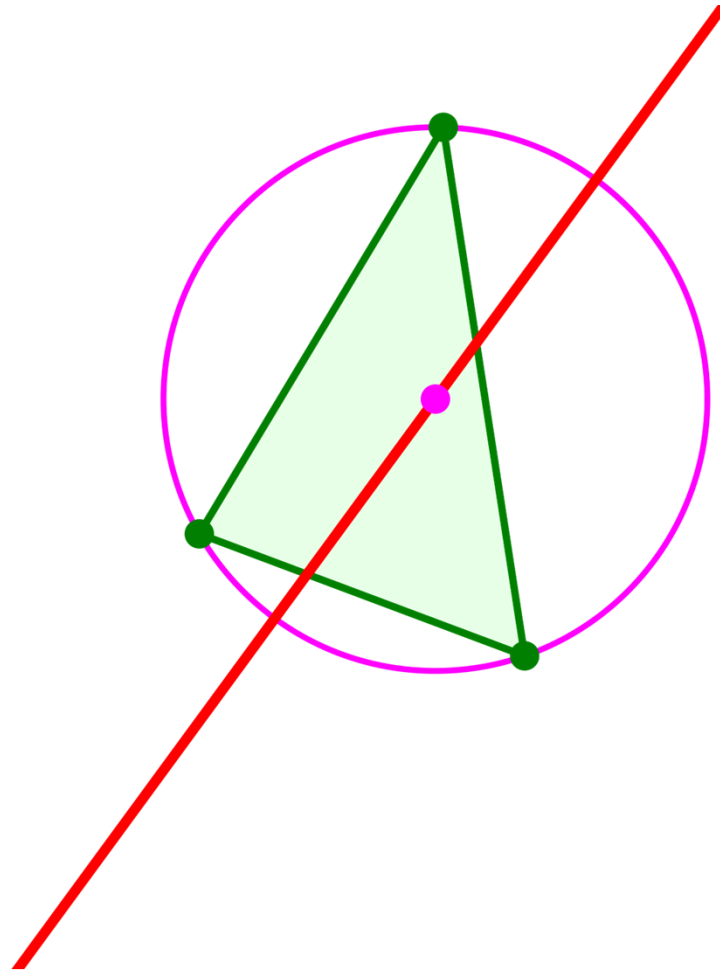


Kiepert-Gerade

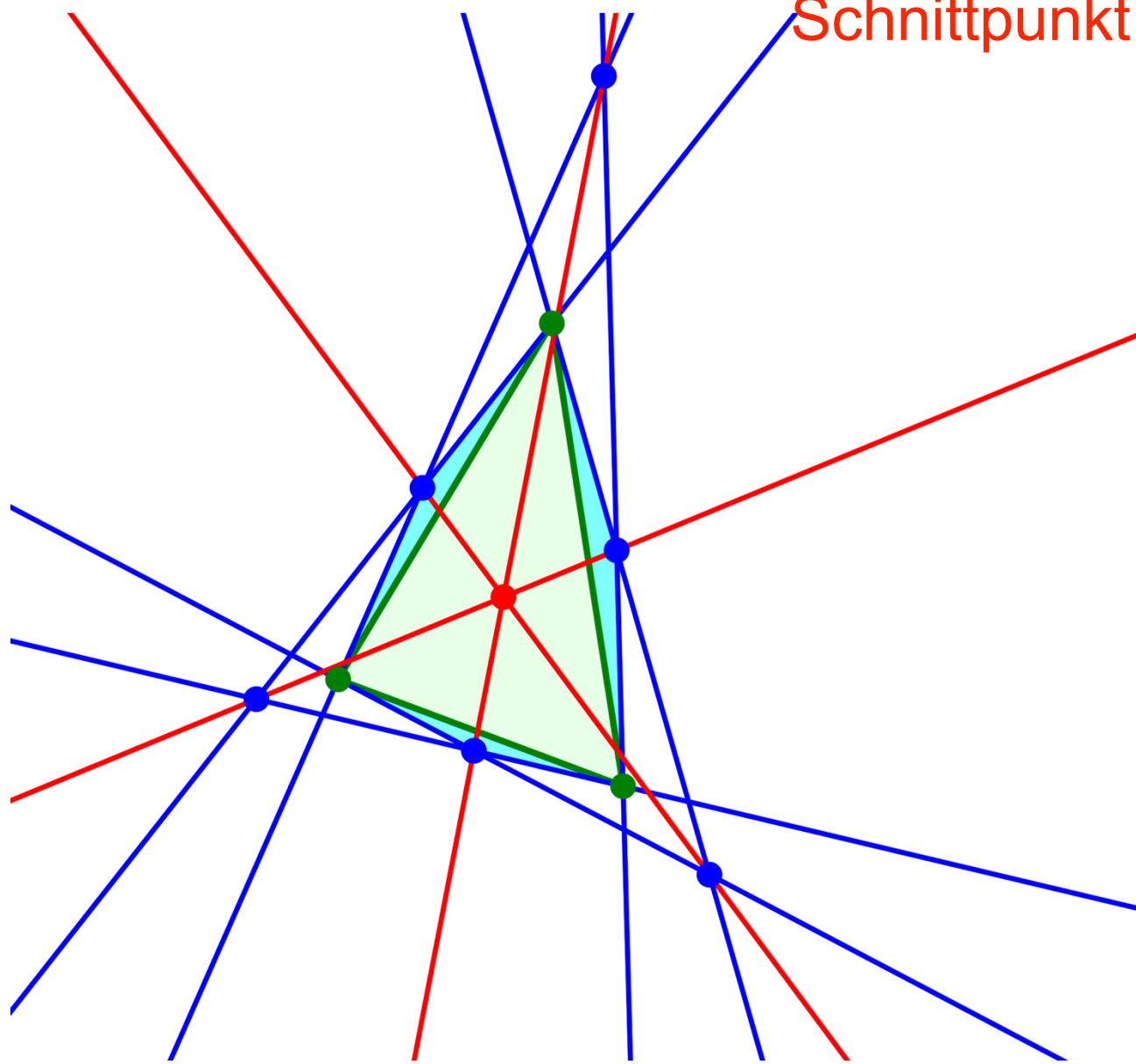


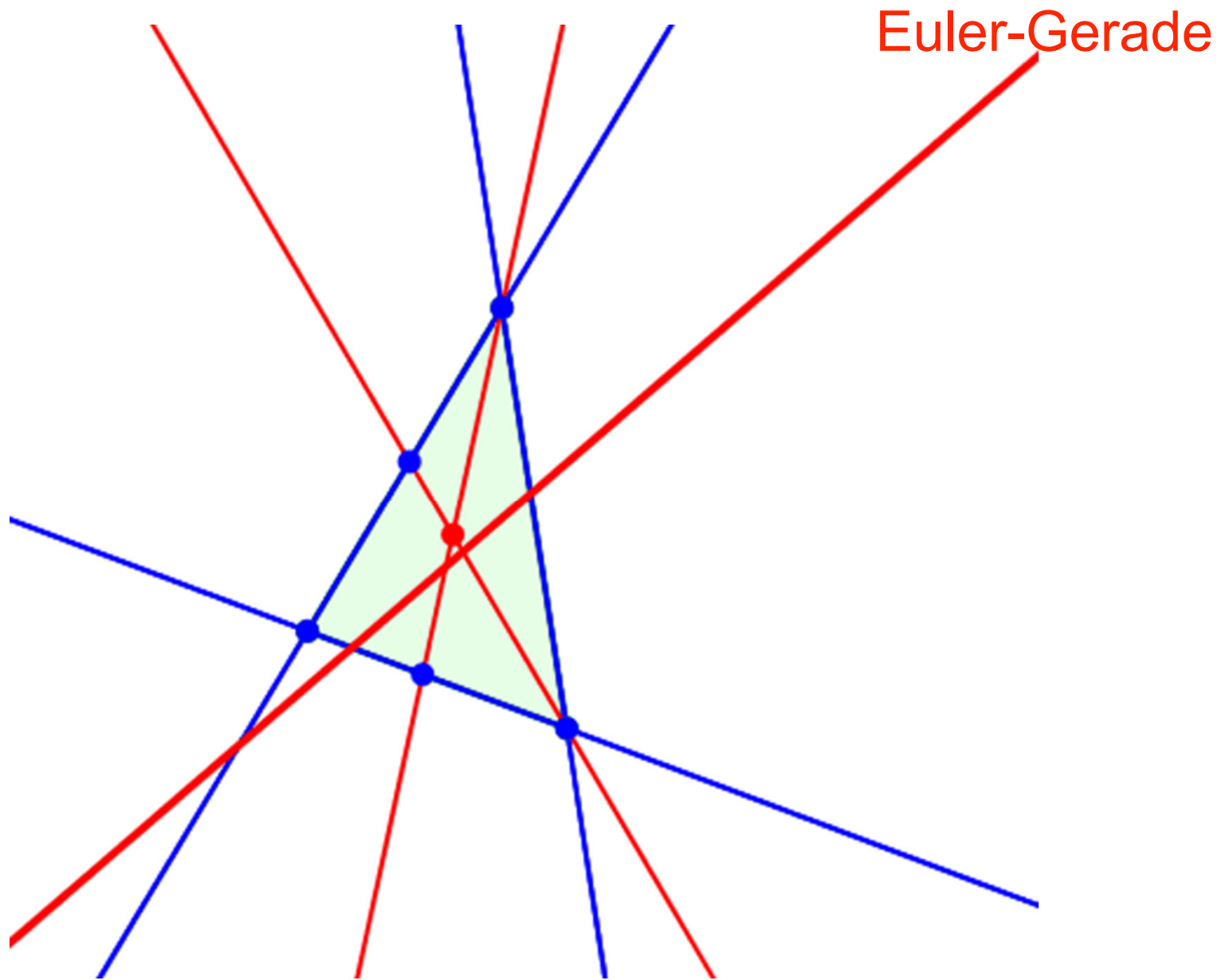
# Kiepert-Gerade

- Umkreismittelpunkt



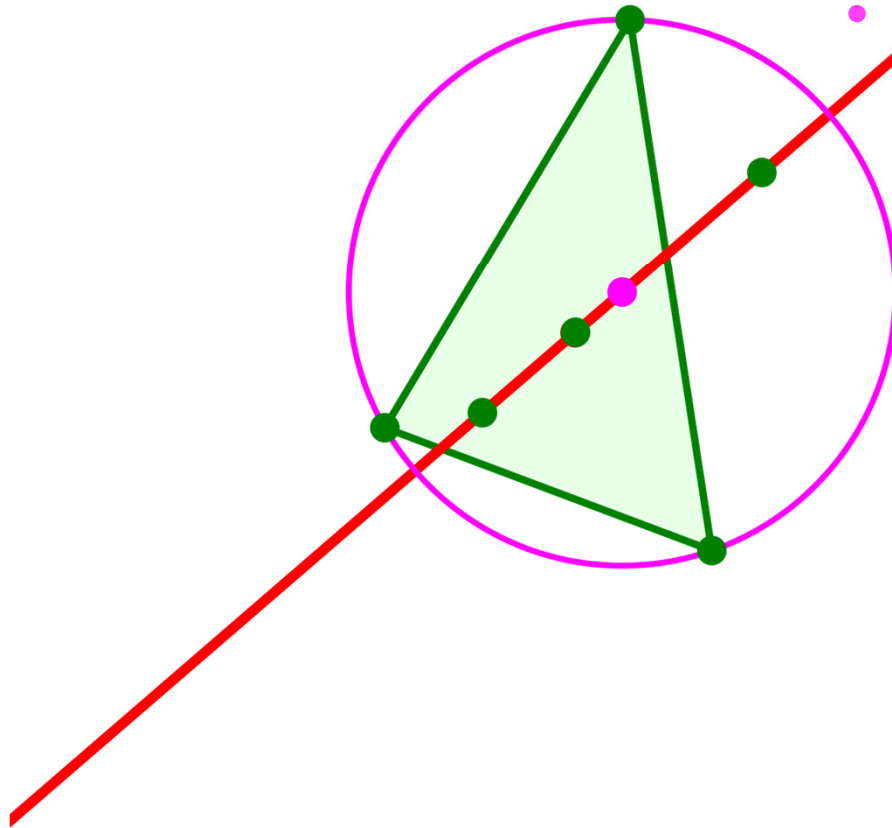
Schnittpunkt (Euler)





## Euler-Gerade

- Höhenschnittpunkt
- Schwerpunkt
- Umkreismittelpunkt
  - Gespiegelter Höhenschnittpunkt



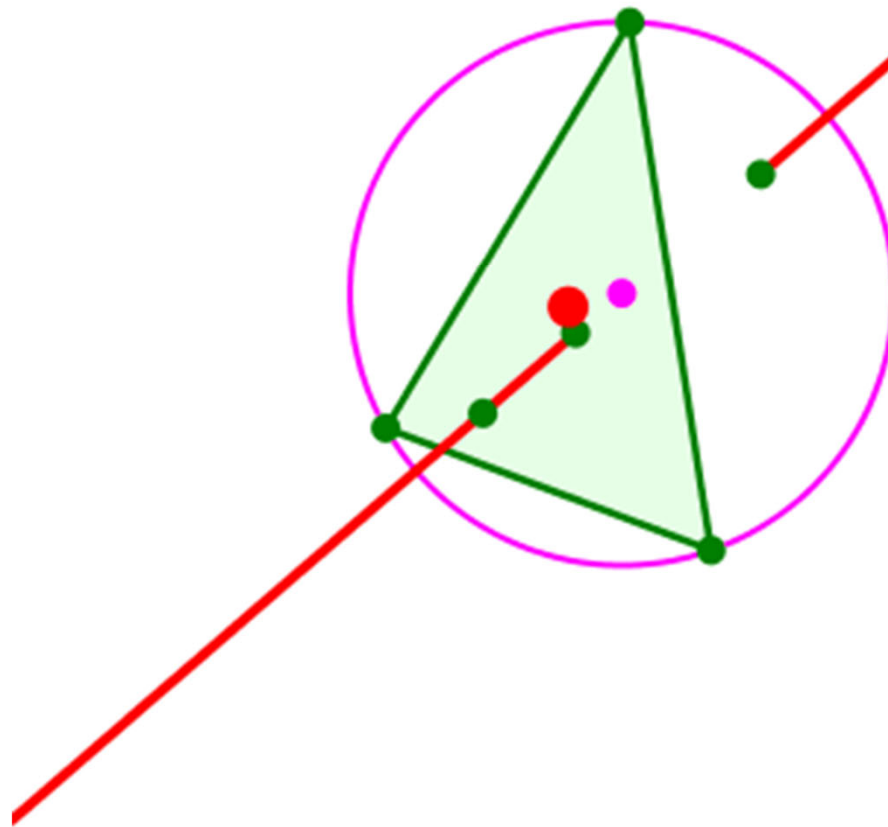
Euler-Gerade

Umkehrpunkte

• Schwerpunkt

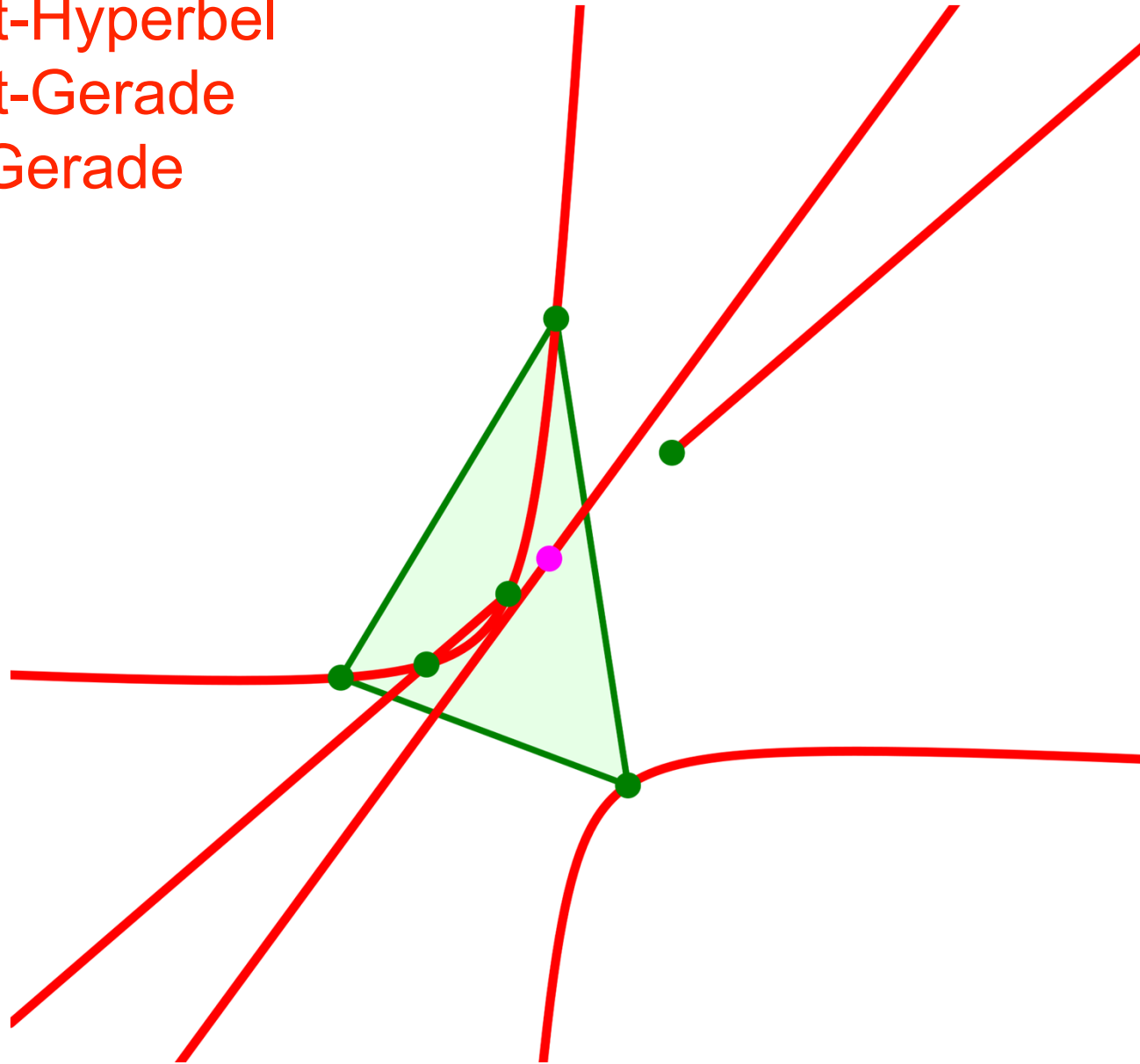
• Gespiegelter

Höhenschnittpunkt



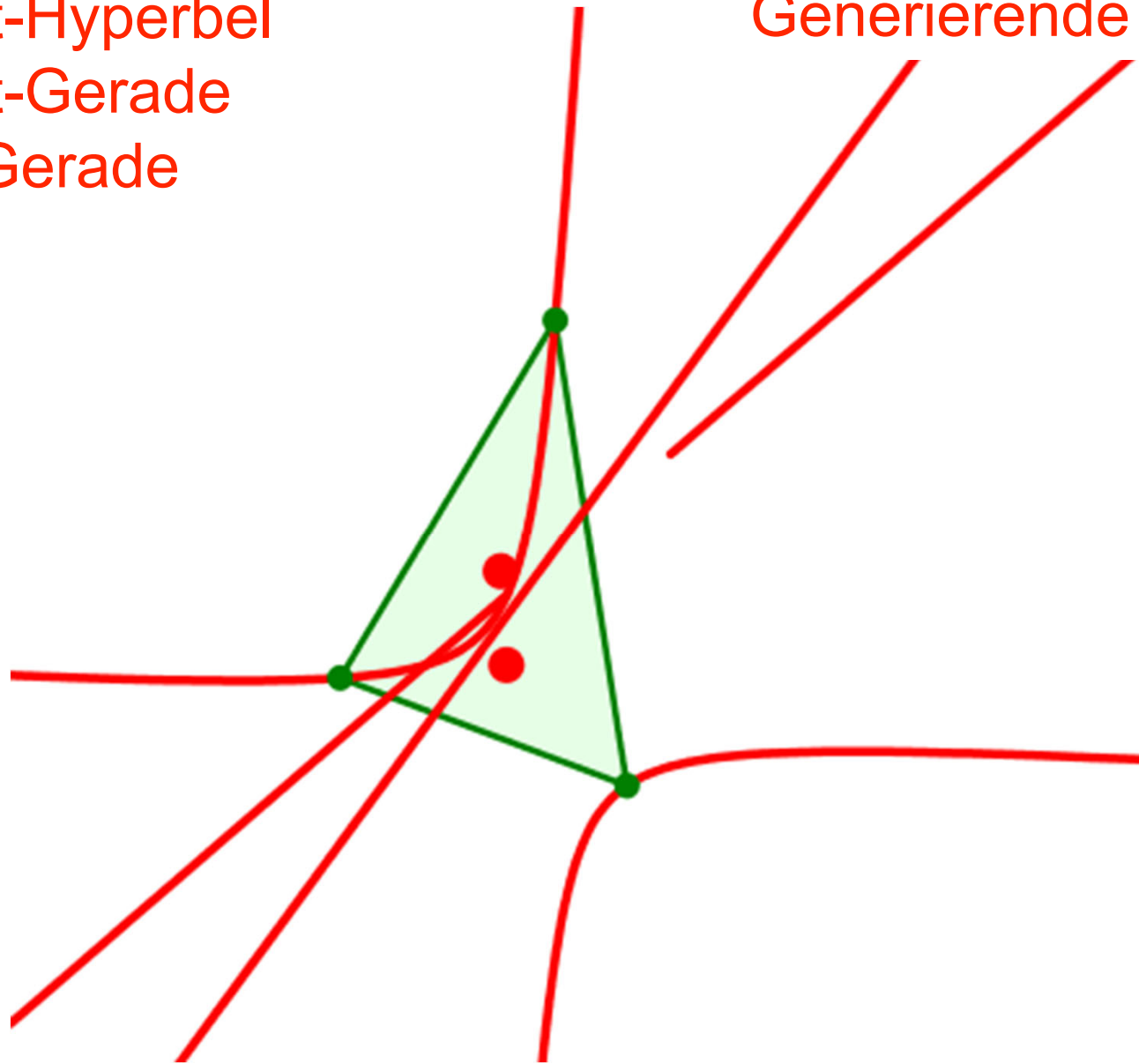


Kiepert-Hyperbel  
Kiepert-Gerade  
Euler-Gerade



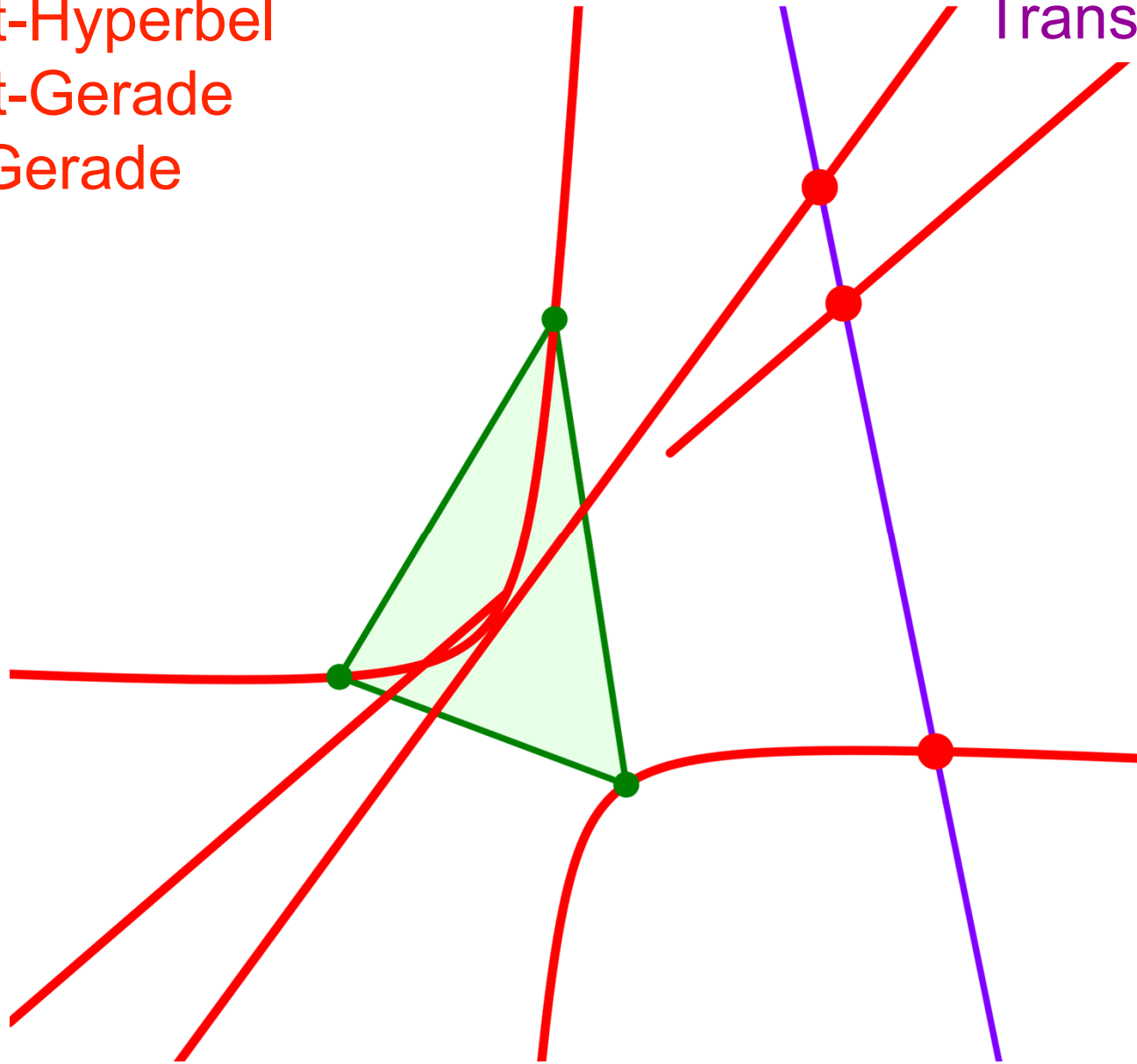
Kiepert-Hyperbel  
Kiepert-Gerade  
Euler-Gerade

Generierende Punkte



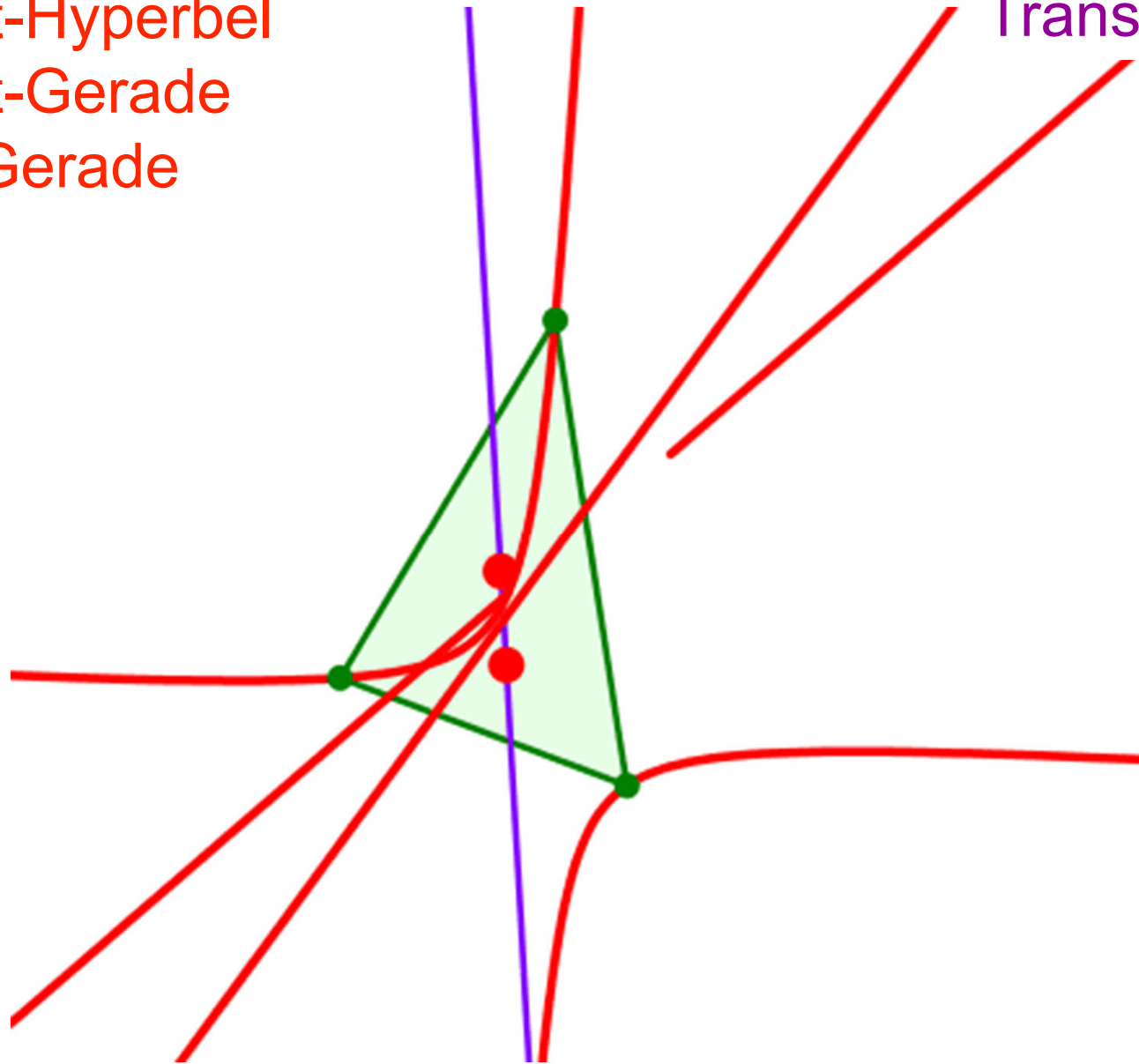
Kiepert-Hyperbel  
Kiepert-Gerade  
Euler-Gerade

Transversale



Kiepert-Hyperbel  
Kiepert-Gerade  
Euler-Gerade

Transversale



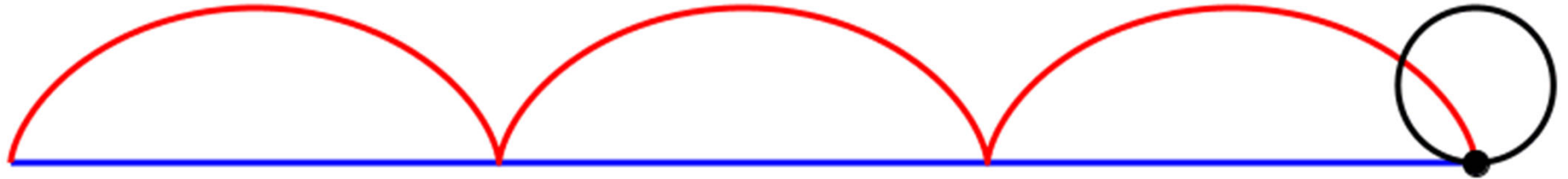
# Zykloide



# Zykloide

Christopher Wren 1632-1723

Bogenlänge =  $8r$



Spannweite =  $2\pi r$

# Kreisbewegung



# Bewegte Kamera





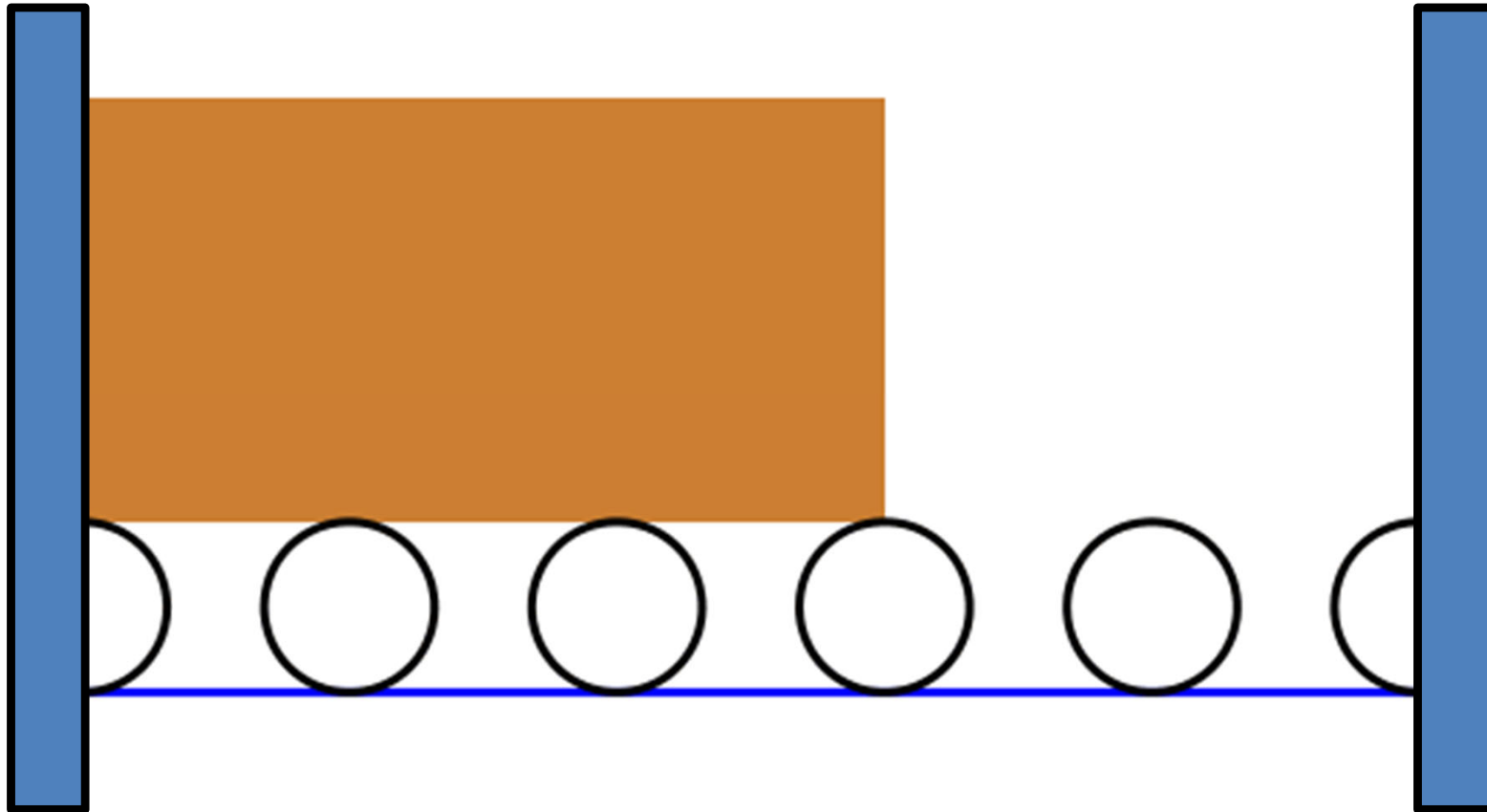
# Bewegte Kamera



und in die andere Richtung



# Rollenlager



# Gedrehte Kamera



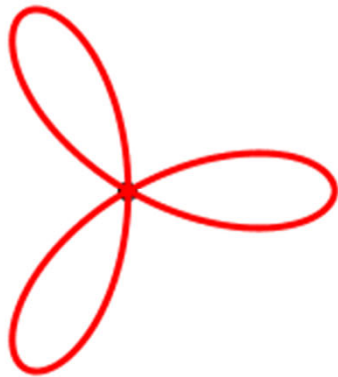
# Rosette



$$r(\varphi) = \cos(3\varphi), \quad \varphi \in [0, \pi]$$

Kreis und Rosette flächengleich

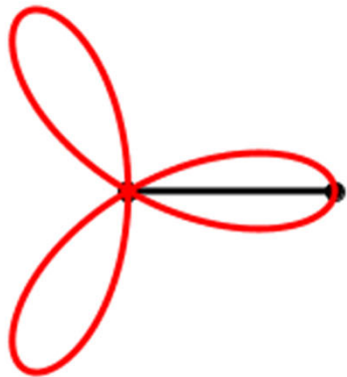
# Rosette



$$r(\varphi) = \cos(3\varphi)$$

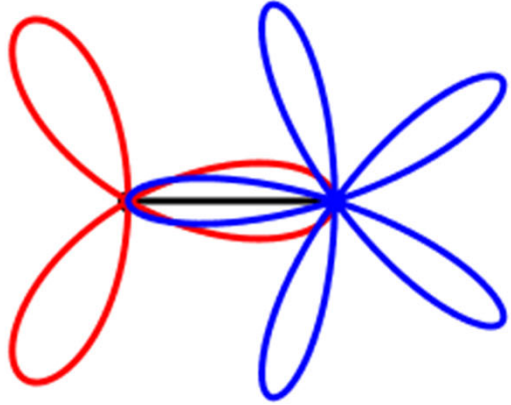
Kreis und Rosette flächengleich

## Rosette und Propeller



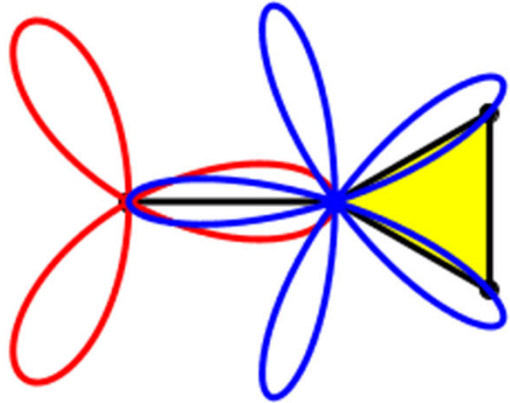
Der Propeller dreht halb so schnell wie der Punkt

# Rosettengetriebe

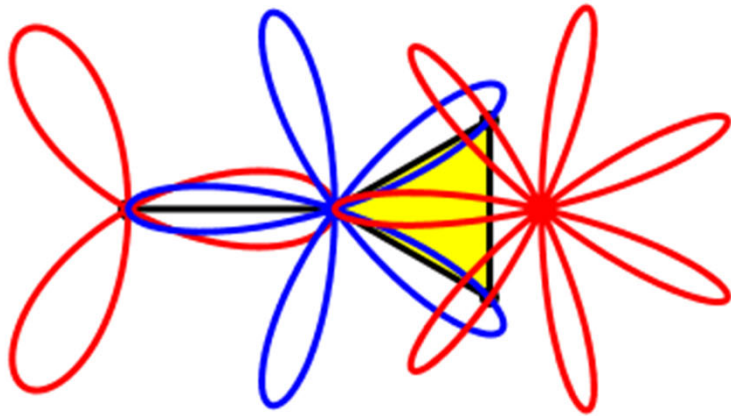




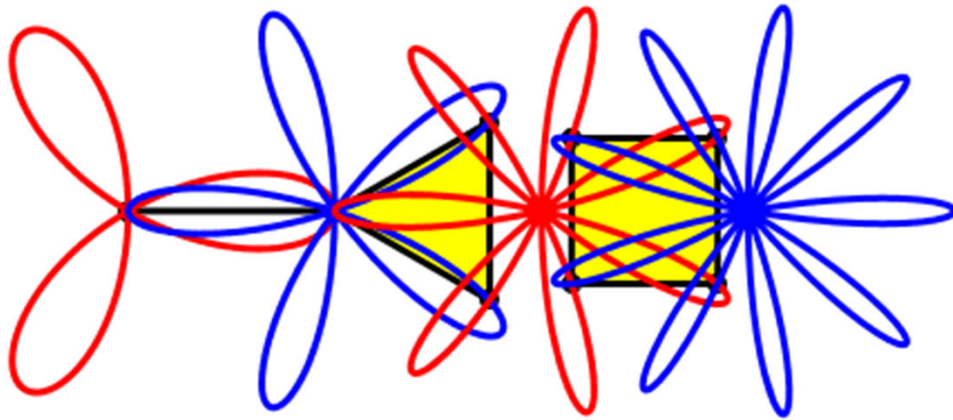
# Rosettengetriebe



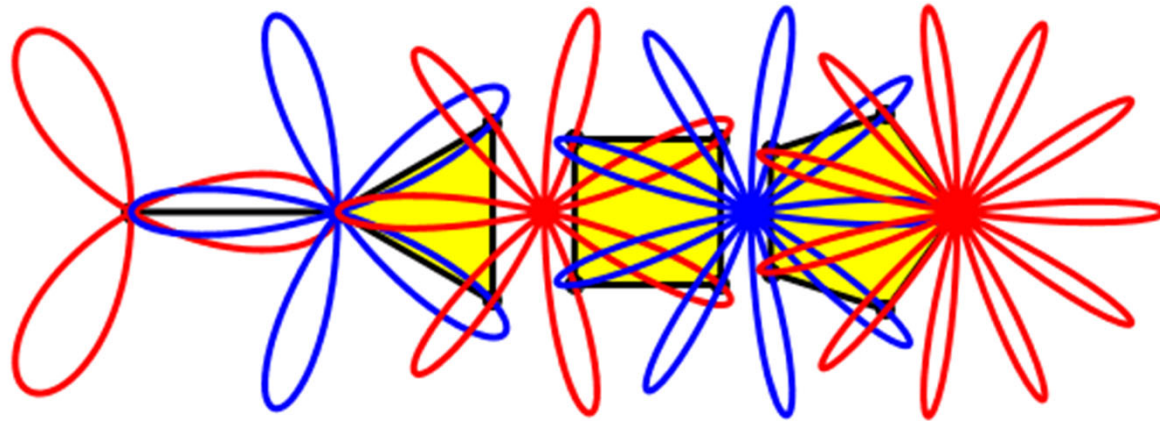
# Rosettengetriebe



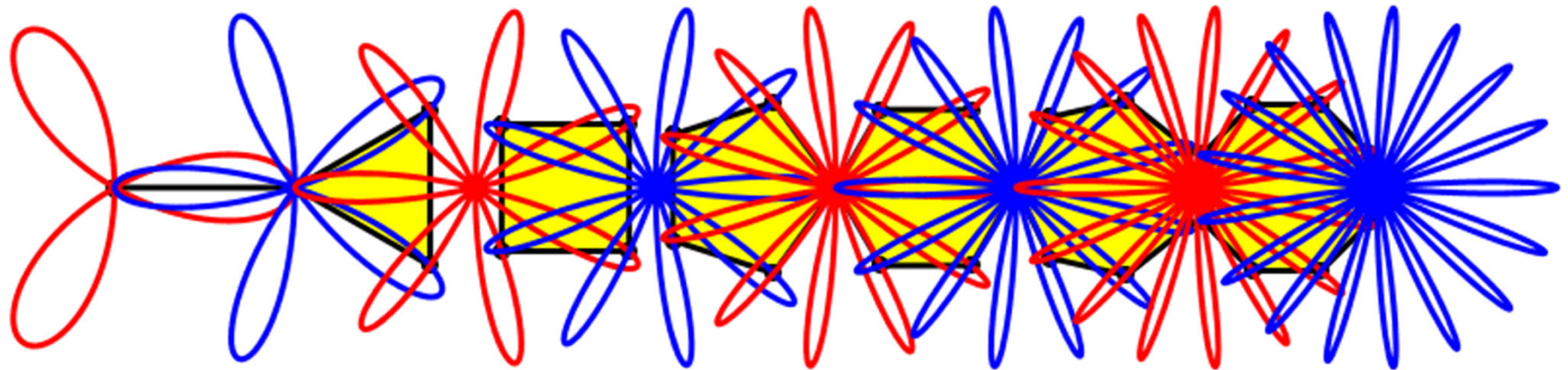
# Rosettengetriebe



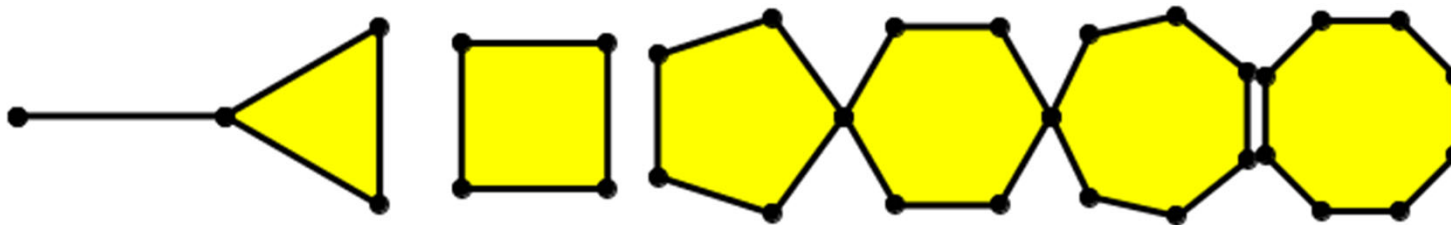
# Rosettengetriebe



# Rosettengetriebe



# Harmonische Drehgeschwindigkeiten



$$1 : \frac{1}{2} : \frac{1}{3} : \frac{1}{4} : \frac{1}{5} : \frac{1}{6} : \frac{1}{7} : \frac{1}{8}$$

# Danke



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