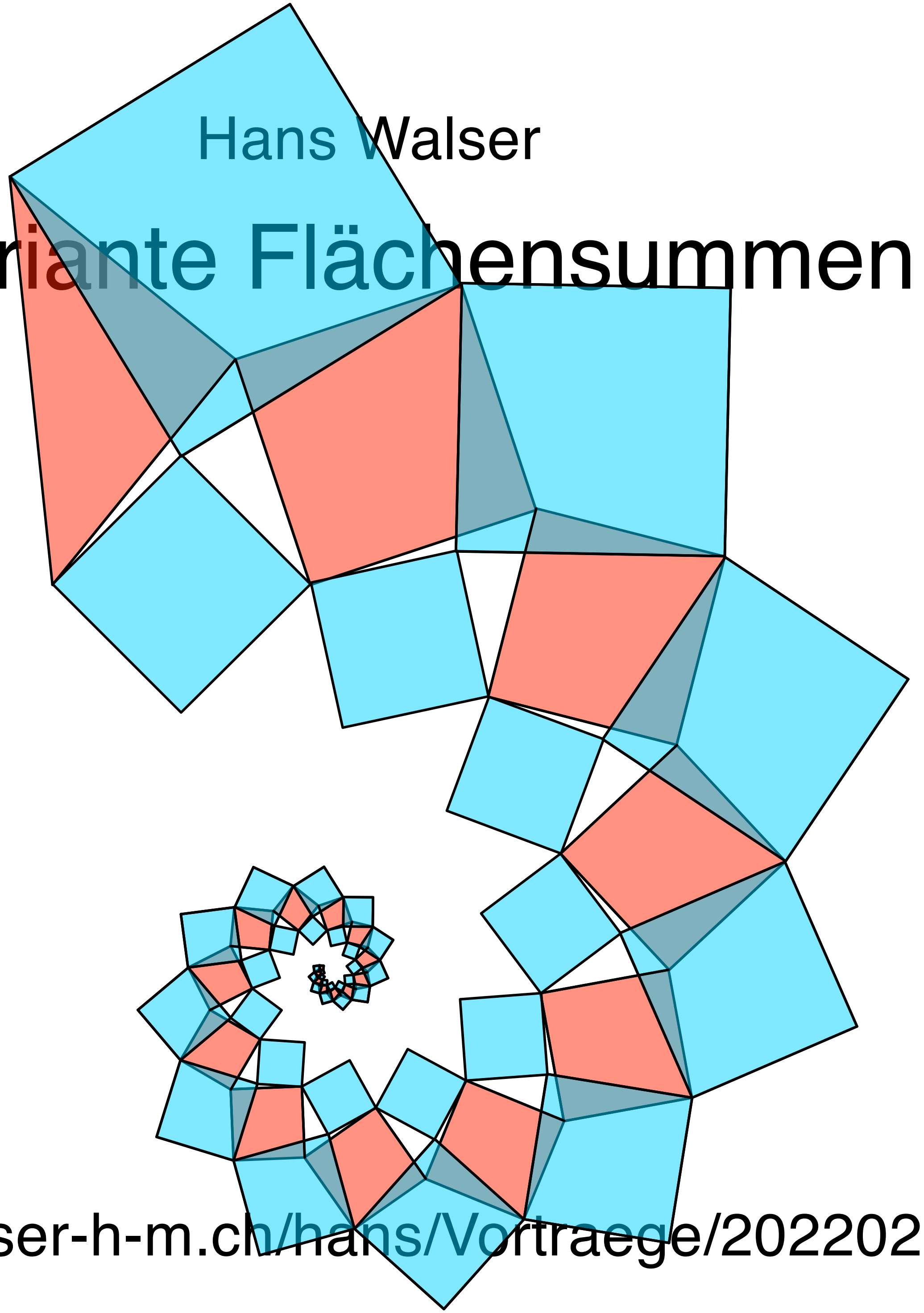
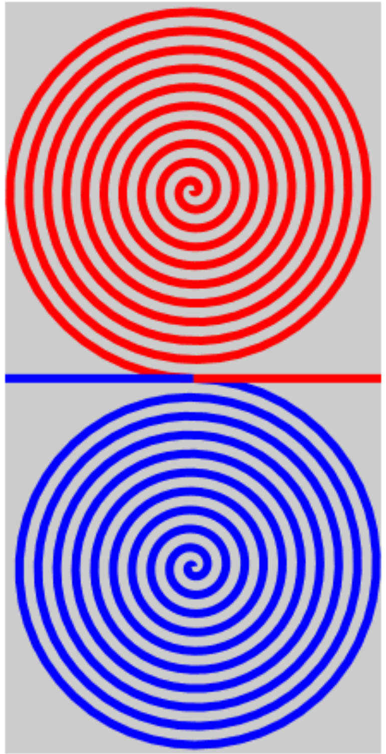
Hans Walser

Invariante Flächensummen



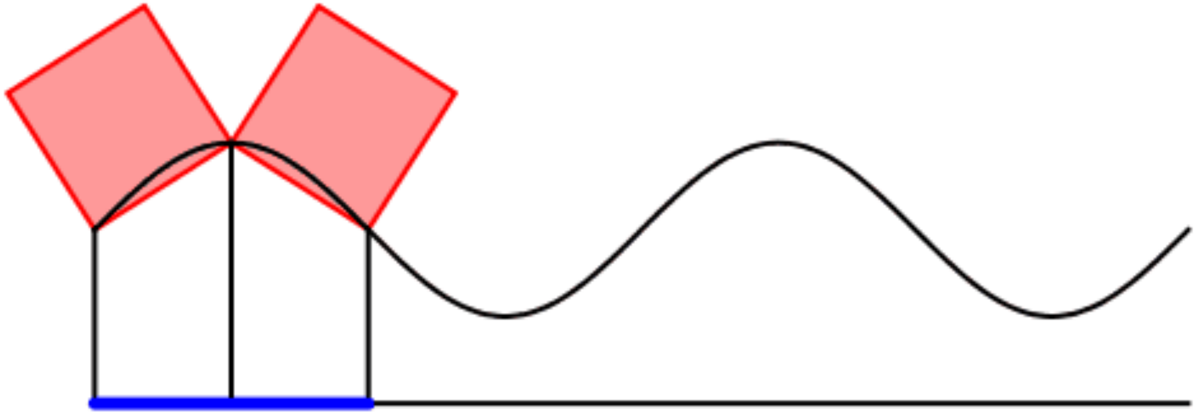
www.walser-h-m.ch/hans/Vortraege/20220203

Lakritze



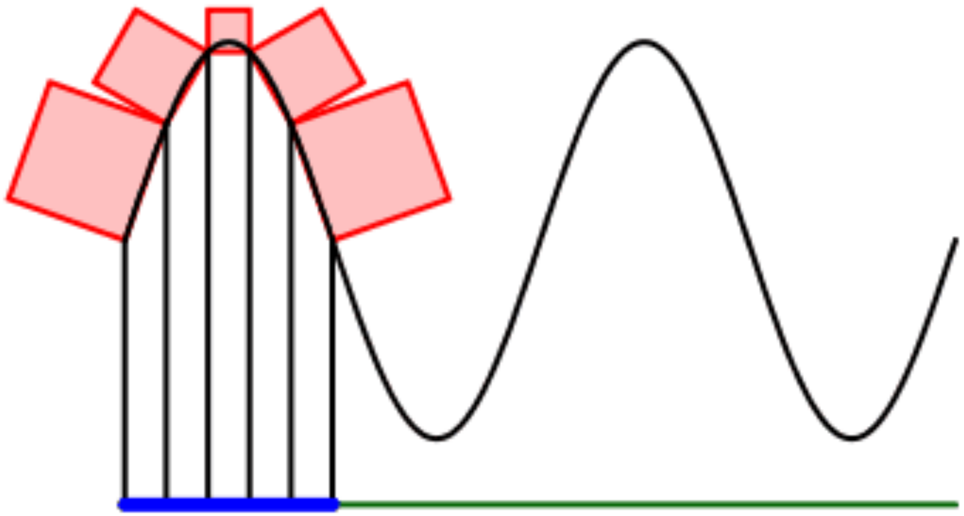
www.walser-h-m.ch/hans/Vortraege/20220203

Invariante Flächensumme



Quadratflächen = {3.47, 3.47}
Flächensumme = 6.93

Als ich das erste Mal auf dem Dampfwagen saß

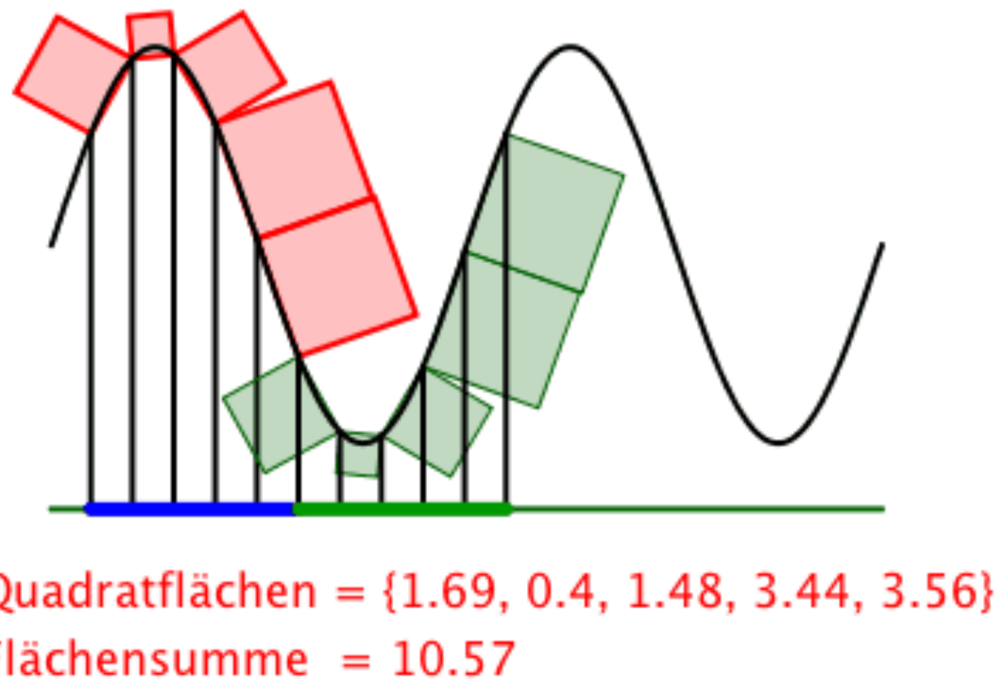


Quadratflächen = {3.5, 1.58, 0.39, 1.58, 3.5}
Flächensumme = 10.57

www.walser-h-m.ch/hans/Vortraege/20220203

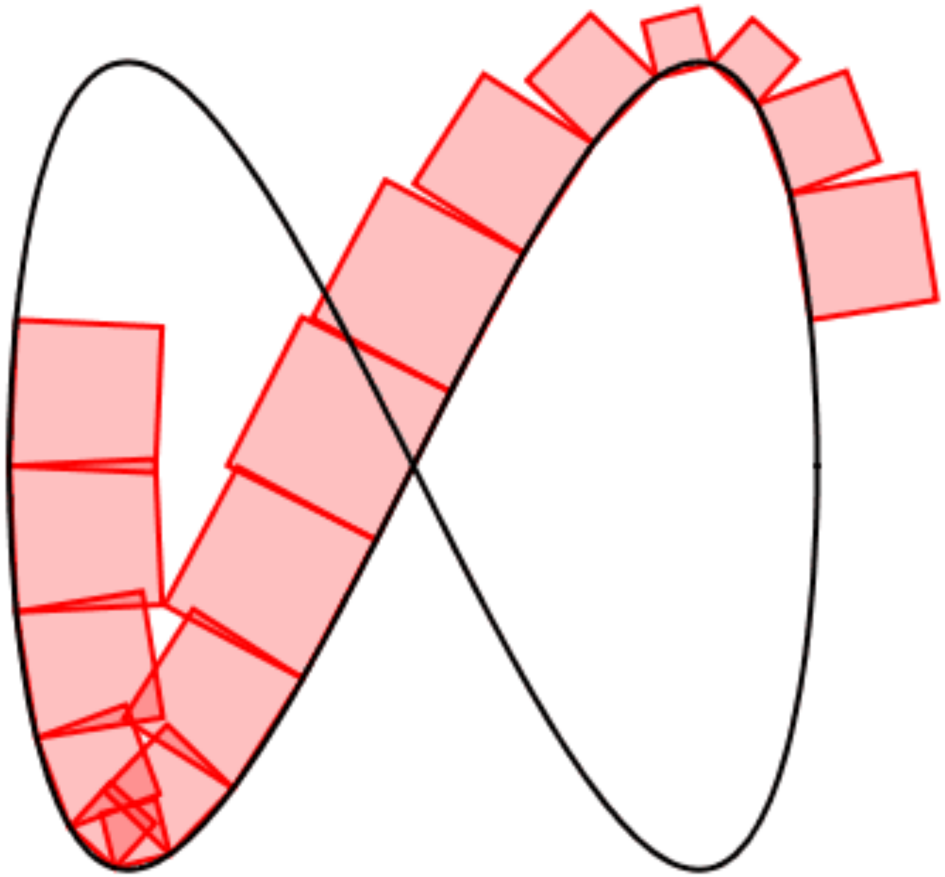
Als ich das erste Mal auf dem Dampfwagen saß

Ganze Periodenlänge. Schubspiegelsymmetrie



www.walser-h-m.ch/hans/Vortraege/20220203

Achterbahn (Lissajous-Kurve)

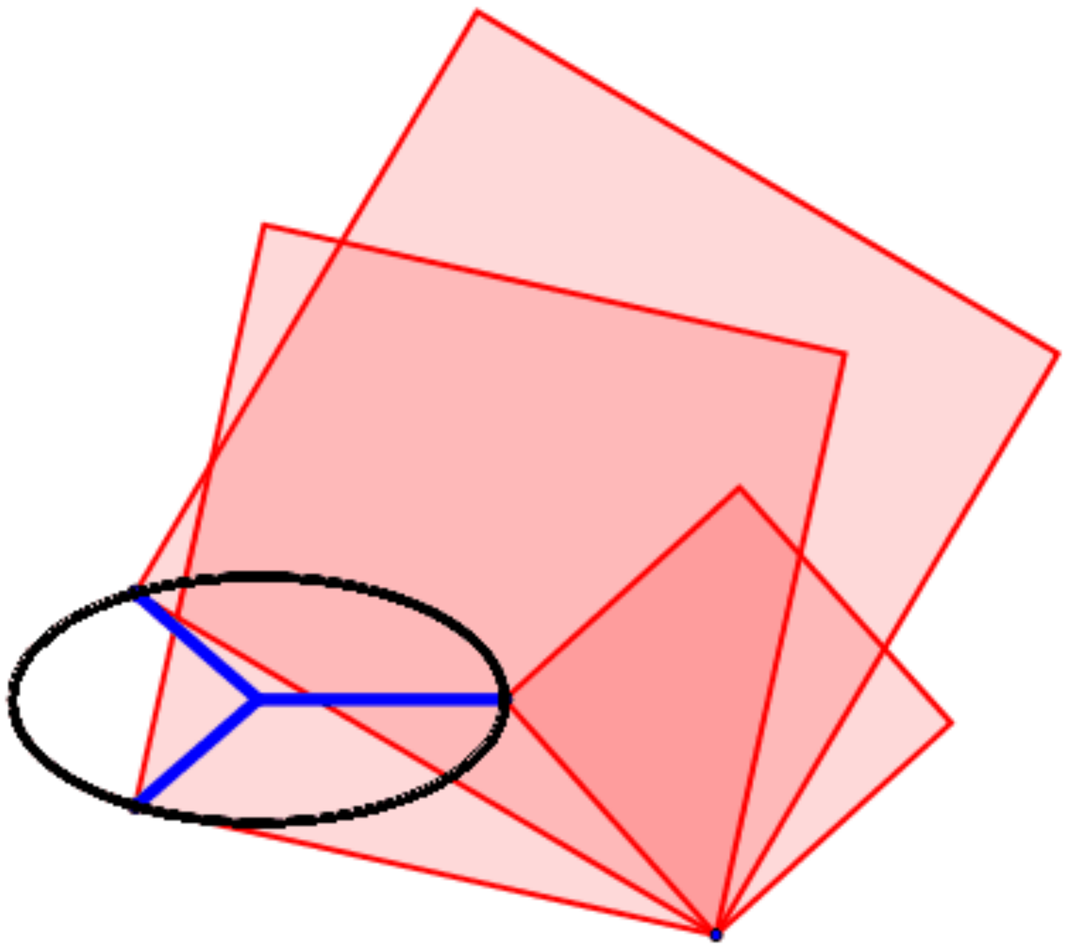


Quadratflächen = {0.1, 0.06, 0.02, 0.02, 0.05, 0.1, 0.15, 0.17, 0.15, 0.1, 0.05, 0.02, 0.02, 0.06, 0.1, 0.13, 0.13}

Flächensumme = 1.44

www.walser-h-m.ch/hans/Vortraege/20220203

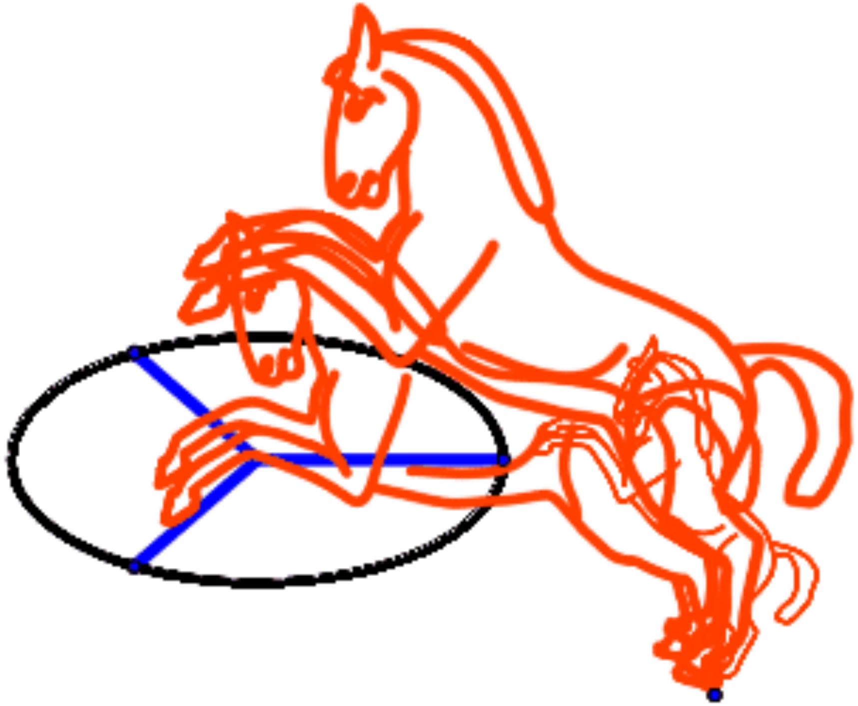
Externer Pivot



Quadratflächen = {23.39, 6.62, 30.01}

Flächensumme = 60.02

Externer Pivot
Тройка



Beweis?

Schlüsselformeln

$$\sum_{k=1}^{n} \cos\left(t + k\frac{2\pi}{n}\right) = 0$$

$$\sum_{k=1}^{n} \sin\left(t + k\frac{2\pi}{n}\right) = 0$$

$$\sum_{k=1}^{n} \cos^2\left(t + k\frac{2\pi}{n}\right) = \frac{n}{2}$$

$$\sum_{k=1}^{n} \sin^2\left(t + k\frac{2\pi}{n}\right) = \frac{n}{2}$$

Beweis?

Schlüsselformeln

$$\sum_{k=1}^{n} \cos\left(t + k\,\frac{2\pi}{n}\right) = 0$$

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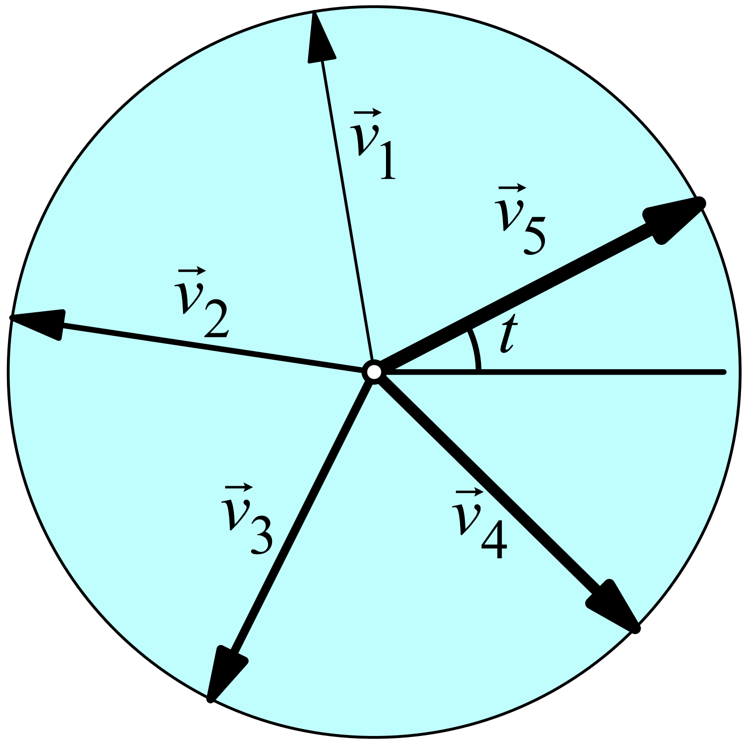
Beweis: regelmäßiges n-Eck

Schlüsselformeln

$$\sum_{k=1}^{n} \cos\left(t + k\,\frac{2\pi}{n}\right) = 0$$

$$\sum_{k=1}^{n} \sin\left(t + k\,\frac{2\pi}{n}\right) = 0$$

$$\vec{v}_k = \begin{bmatrix} \cos\left(t + k\,\frac{2\pi}{n}\right) \\ \sin\left(t + k\,\frac{2\pi}{n}\right) \end{bmatrix}$$



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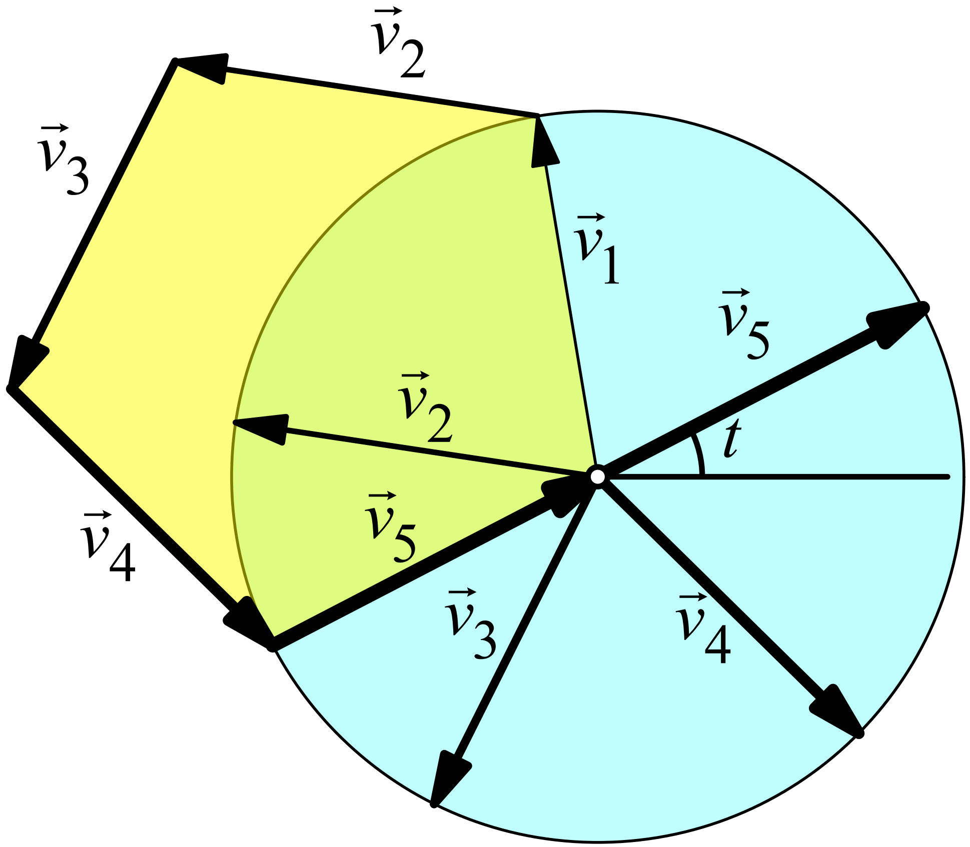
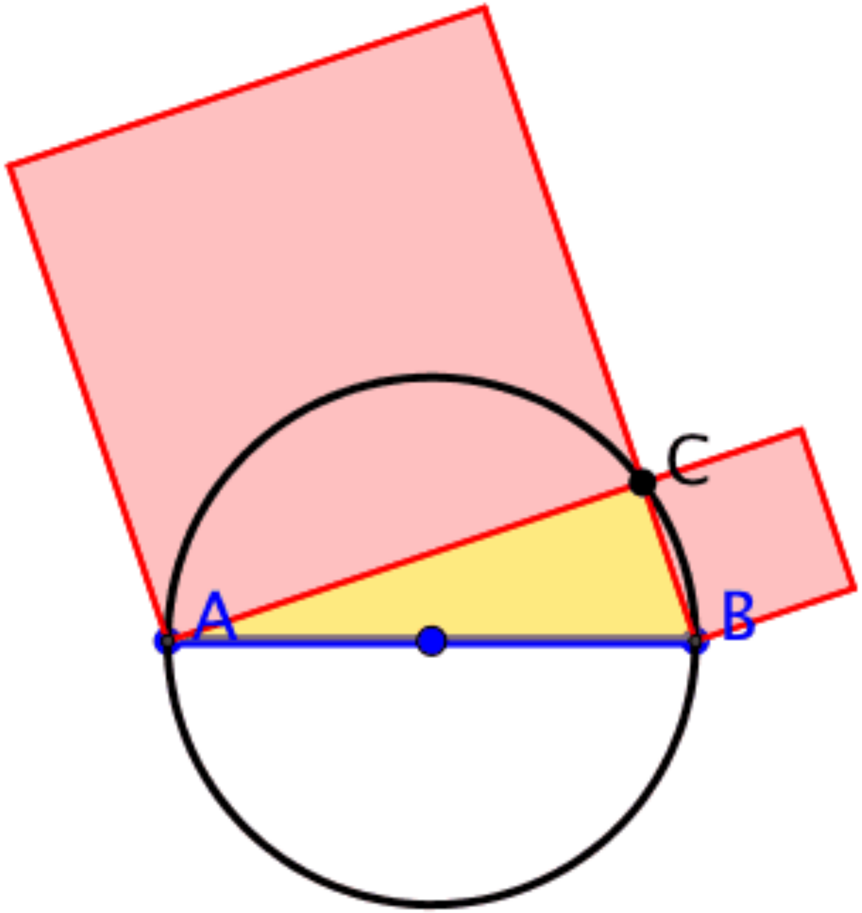


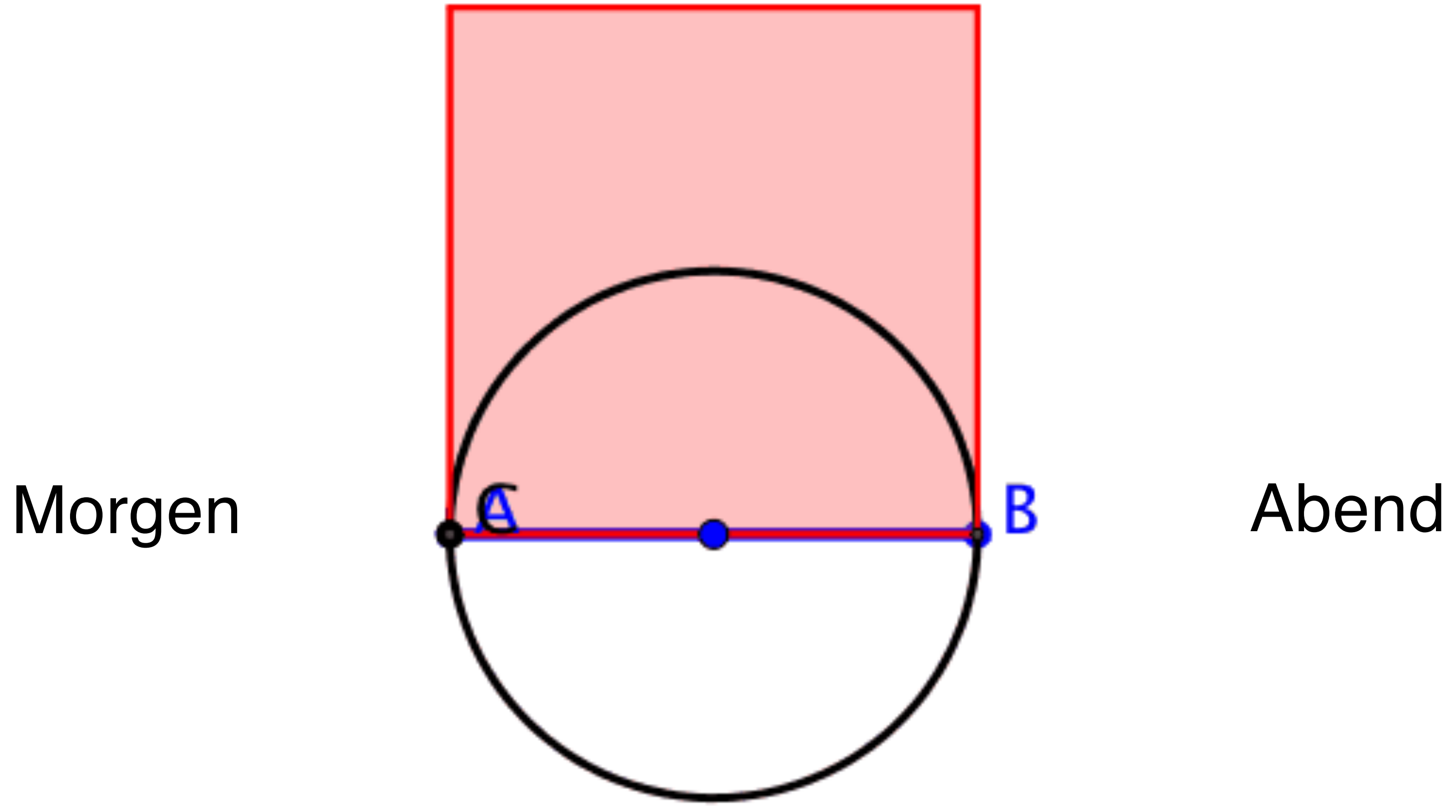
Pythagoras



Quadratflächen = {0.4, 3.6}

Flächensumme = 4

Pythagoras

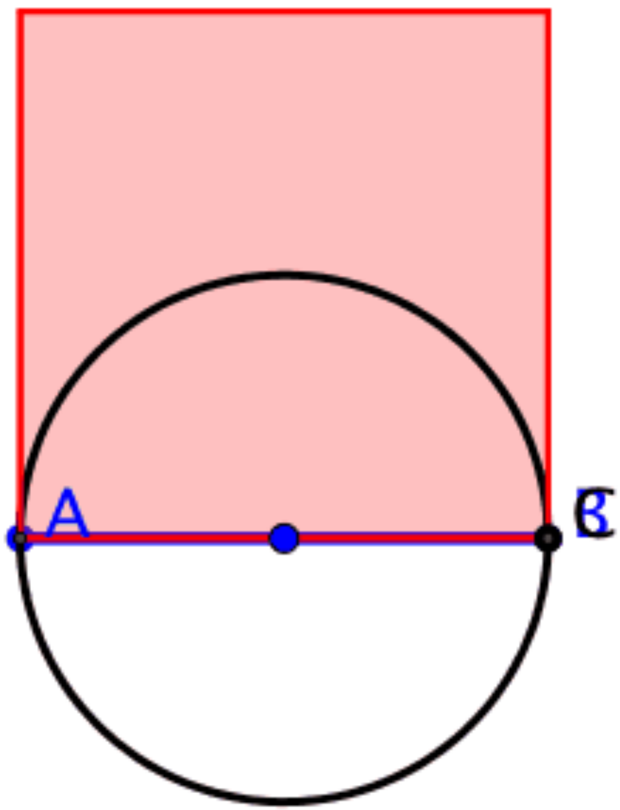


Quadratflächen = {4, 0}

Flächensumme = 4

Pythagoras bei Nacht

Morgen

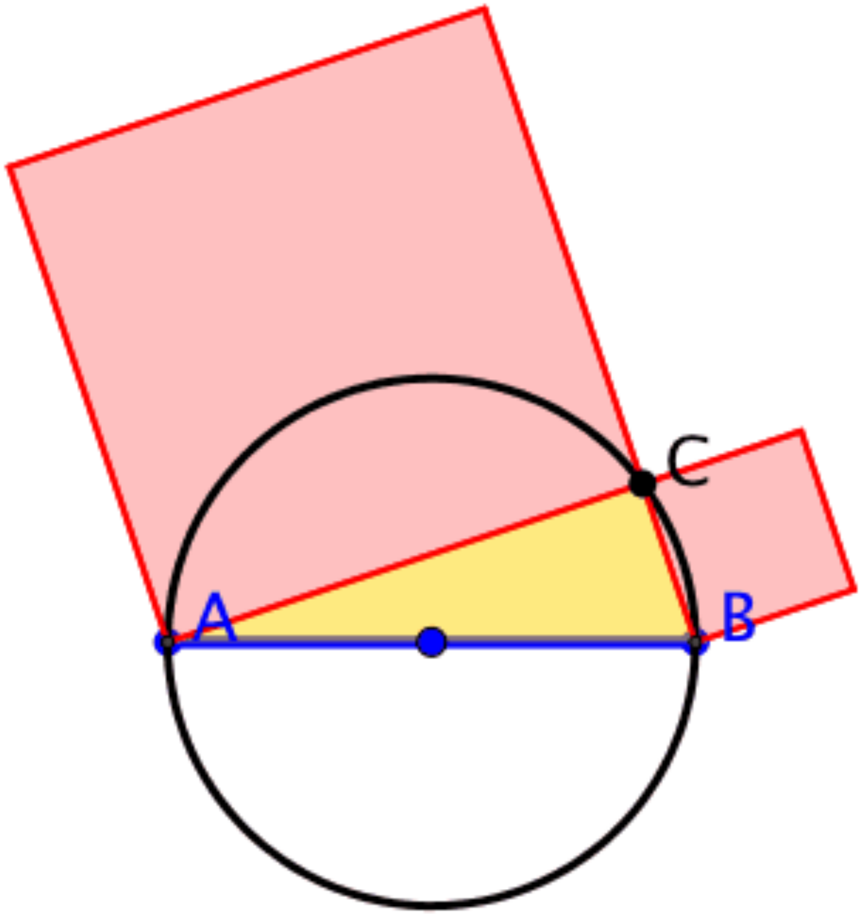


Abend

Quadratflächen = {0, 4}

Flächensumme = 4

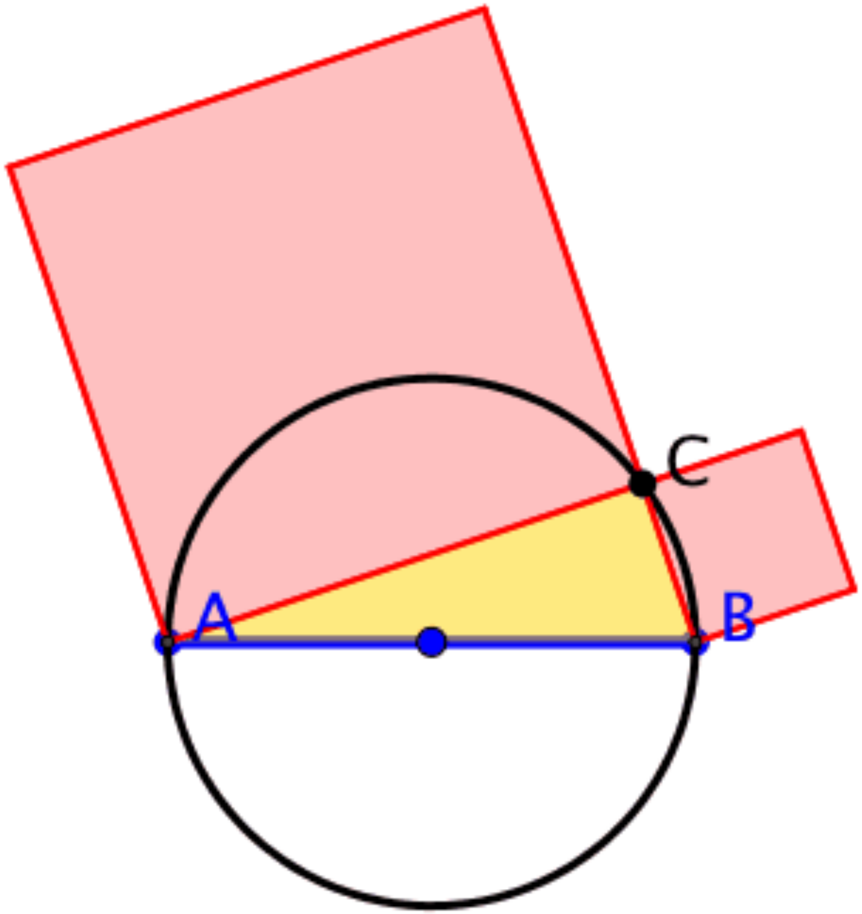
Kopernikus



Quadratflächen = {0.4, 3.6}

Flächensumme = 4

Kopernikus



Quadratflächen = {0.4, 3.6}

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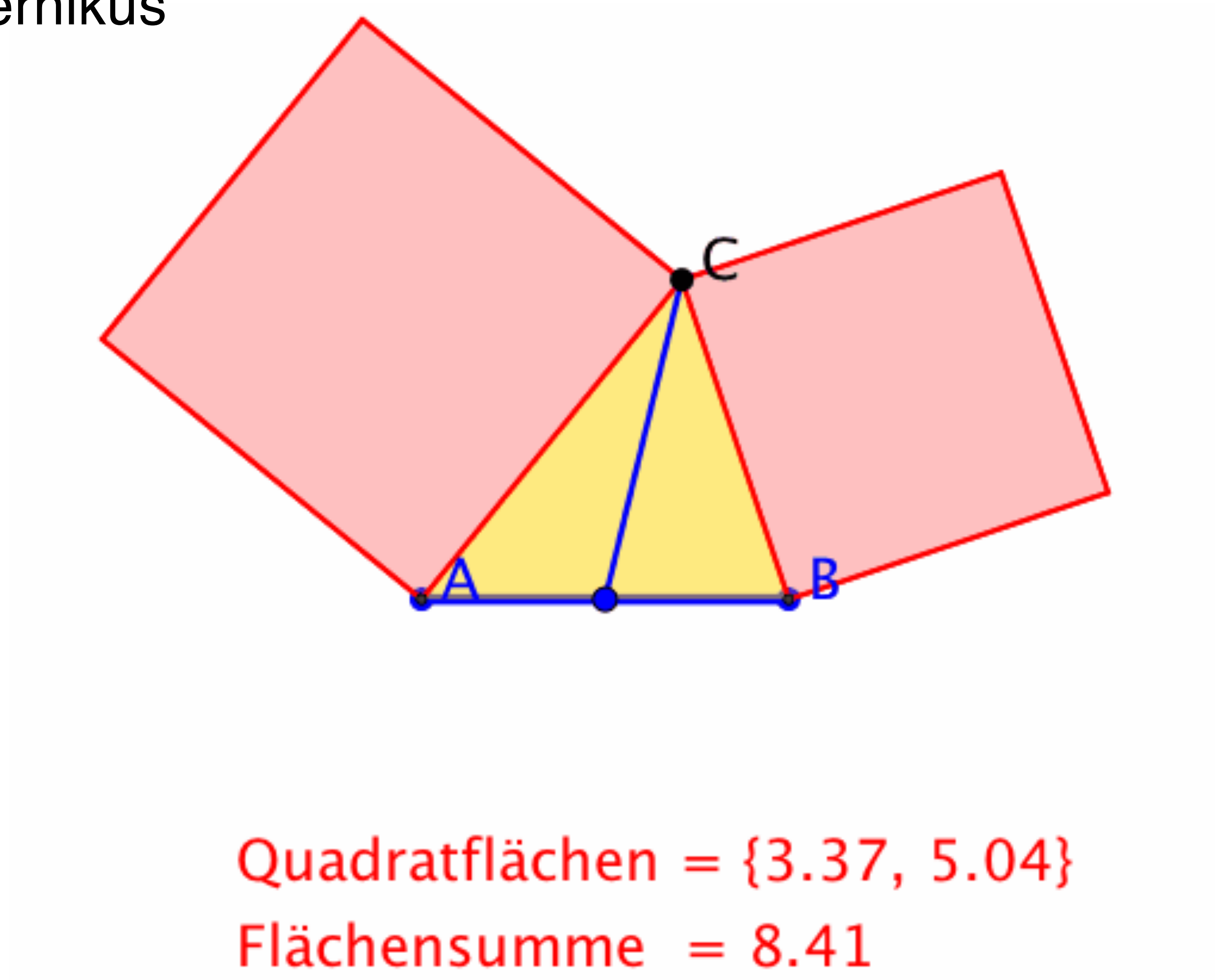
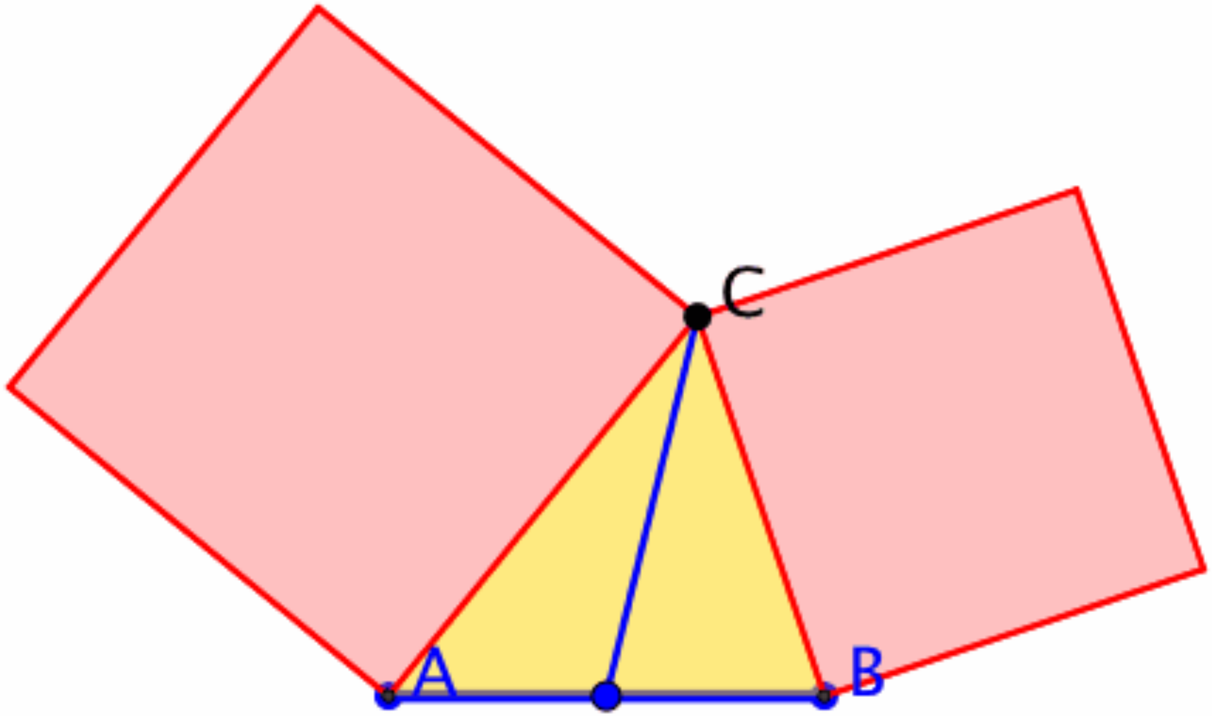
Kopernikus

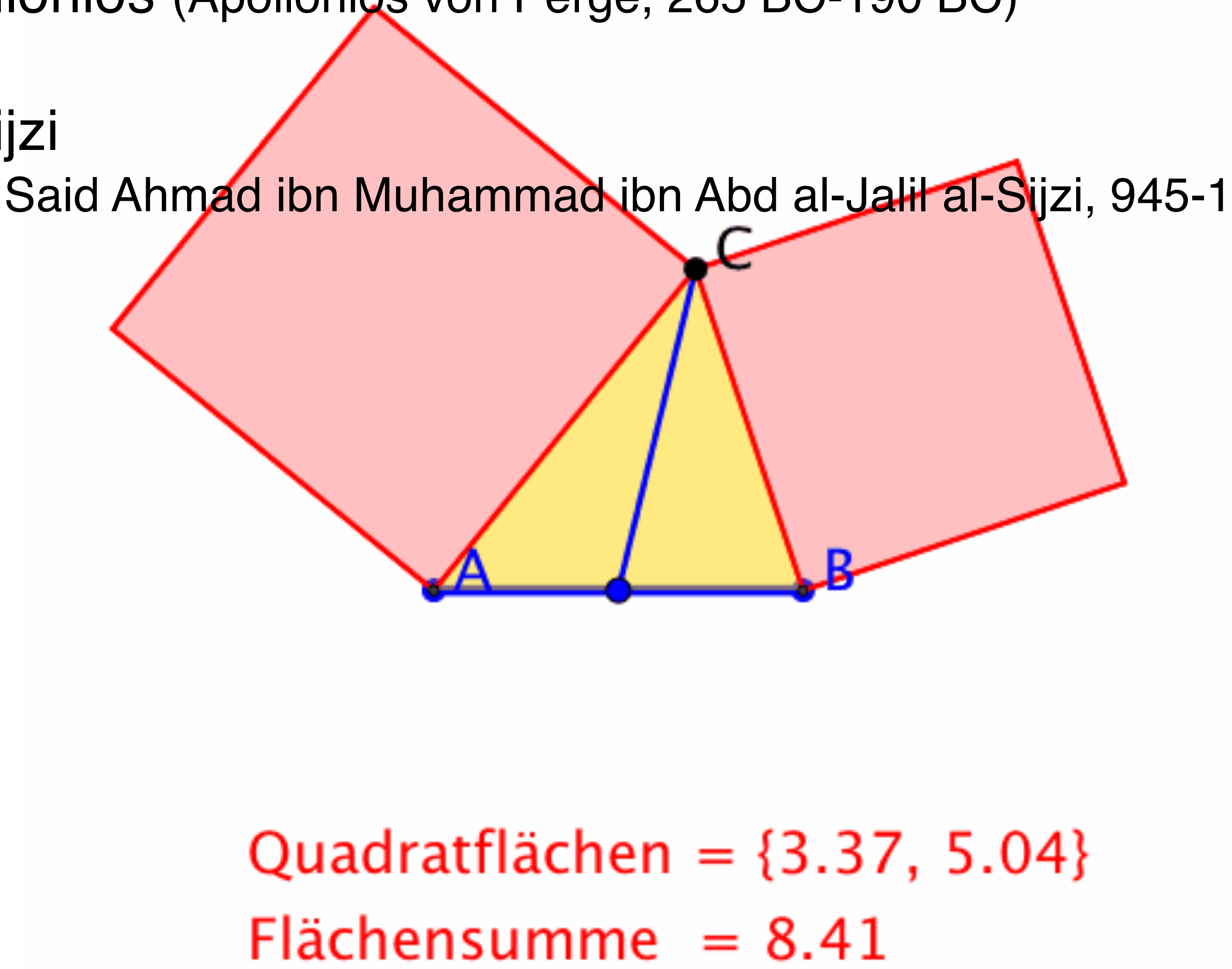




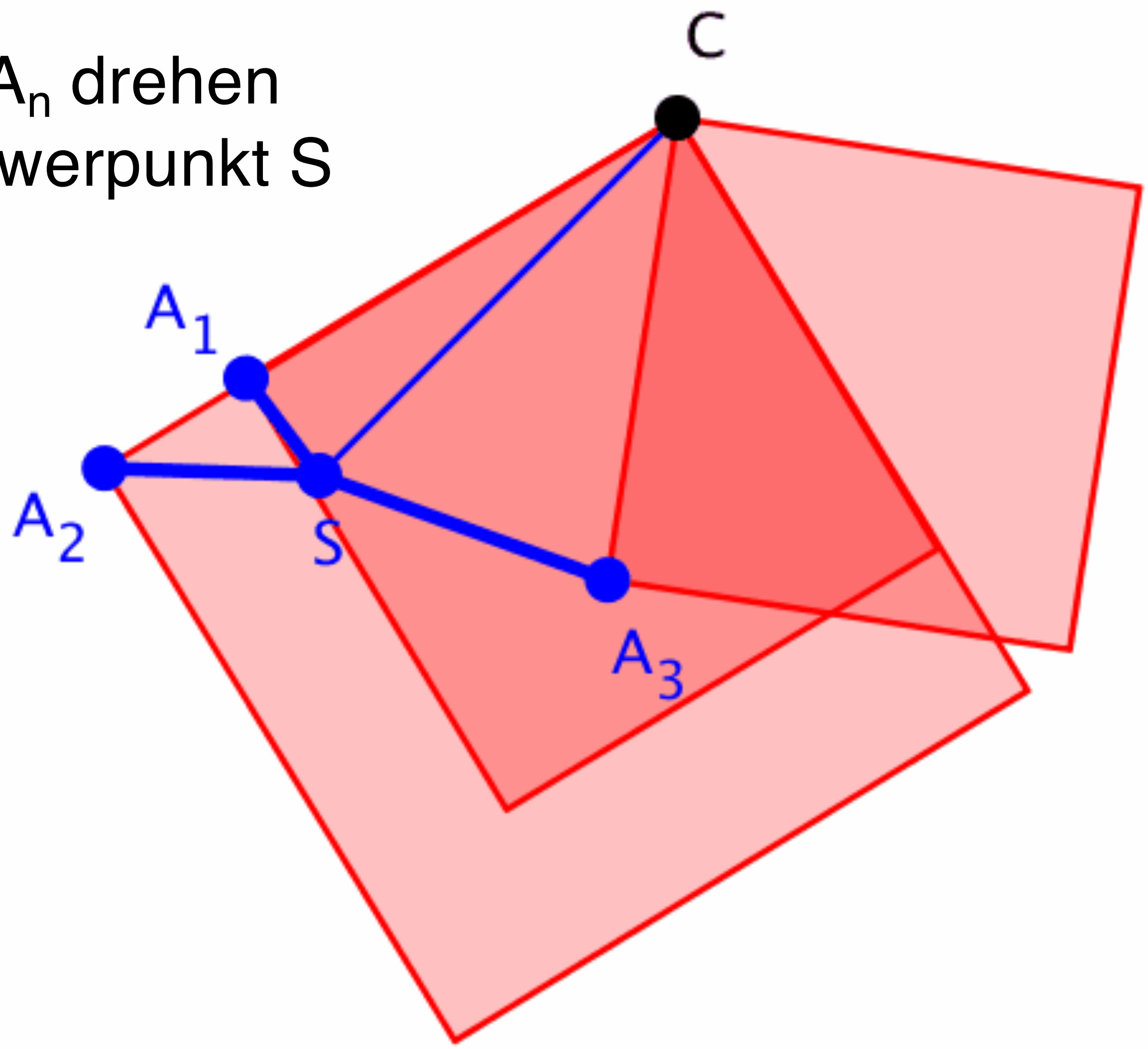
Quadratflächen = {3.37, 5.04}
Flächensumme = 8.41

Apollonios (Apollonios von Perge, 265 BC-190 BC)

al-Sijzi
(Abu Said Ahmad ibn Muhammad ibn Abd al-Jalil al-Sijzi, 945-1020)

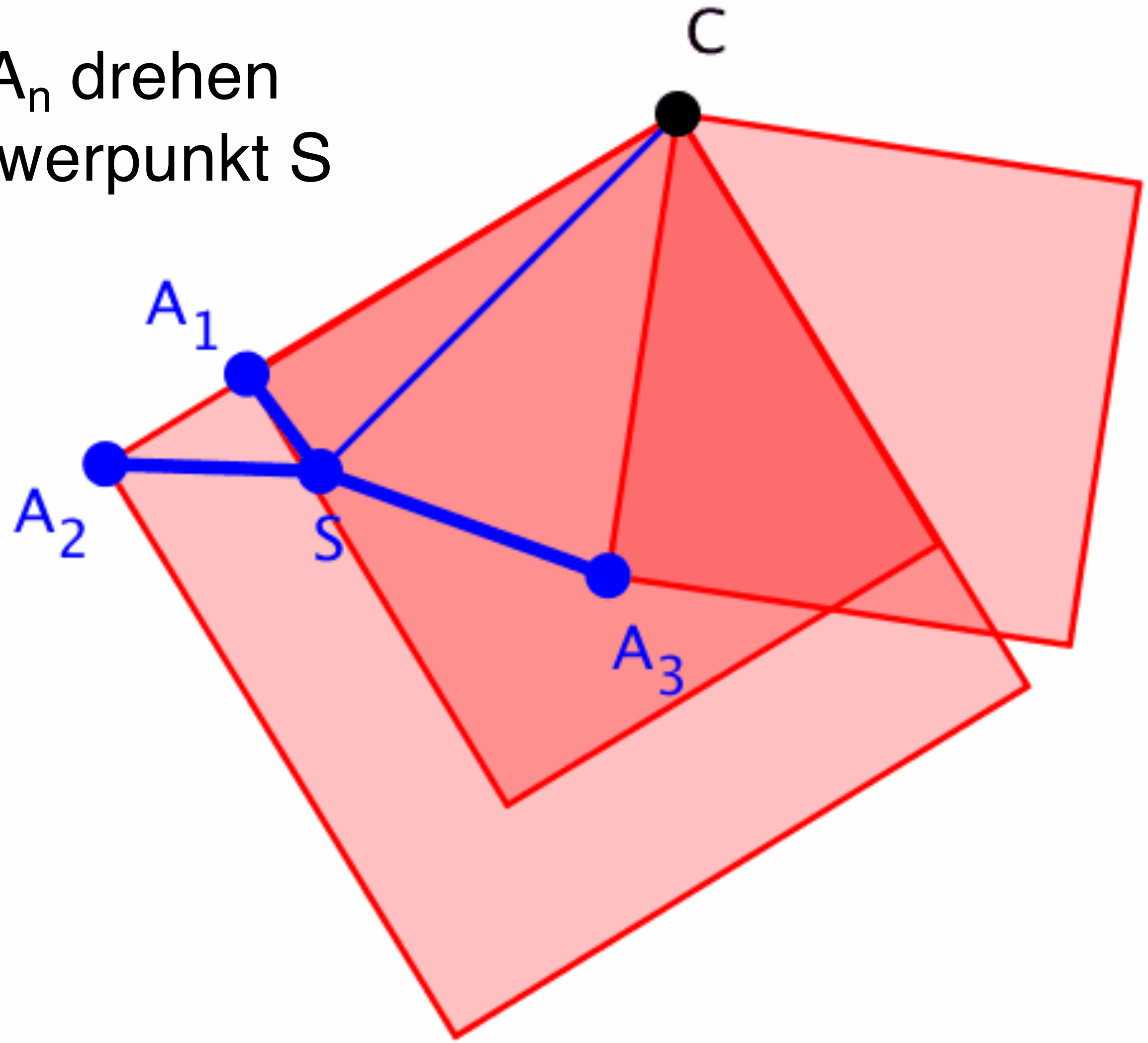


$A_1, \ldots, A_n$ drehen
um Schwerpunkt S



Quadratflächen = {522.64, 253.12, 294.23}
Flächensumme = 1069.99

$A_1, \ldots, A_n$ drehen
um Schwerpunkt S



Quadratflächen = {522.64, 253.12, 294.23}
Flächensumme = 1069.99

$A_1, \ldots, A_n$ drehen um Schwerpunkt S

C ein externer Punkt

Summe der Quadrate der Abstände von C zu den Punkten $A_1, \ldots, A_n$ invariant.

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Summe der Quadrate der Abstände von C zu den Punkten $A_1, \ldots, A_n$ invariant.

Beweis:

Ursprung in den Schwerpunkt S

$A_1, \ldots, A_n$ drehen um Schwerpunkt S

C ein externer Punkt

Summe der Quadrate der Abstände von C zu den Punkten $A_1, \ldots, A_n$ invariant.

Beweis:

Ursprung in den Schwerpunkt S

$$A_k\left(x_k, y_k\right), \quad k = 1, \ldots, n \qquad \sum_{k=1}^{n} x_k = 0, \quad \sum_{k=1}^{n} y_k = 0$$

$A_1, \ldots, A_n$ drehen um Schwerpunkt S

C ein externer Punkt

Summe der Quadrate der Abstände von C zu den Punkten $A_1, \ldots, A_n$ invariant.

Beweis:

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$$A_k(x_k, y_k), \quad k = 1, \ldots, n \qquad \sum_{k=1}^{n} x_k = 0, \quad \sum_{k=1}^{n} y_k = 0$$

$$C(x_C, y_C)$$

$A_1, \ldots, A_n$ drehen um Schwerpunkt S

C ein externer Punkt

$$\sum_{k=1}^{n} d\left(C, A_k\right)^2 = \sum_{k=1}^{n}\left(\left(x_k - x_C\right)^2 + \left(y_k - y_C\right)^2\right)$$

$A_1, \ldots, A_n$ drehen um Schwerpunkt S

C ein externer Punkt

$$\sum_{k=1}^{n} d\left(C, A_k\right)^2 = \sum_{k=1}^{n}\left(\left(x_k - x_C\right)^2 + \left(y_k - y_C\right)^2\right)$$

$$= \sum_{k=1}^{n}\left(x_k^2 + y_k^2\right) - 2x_C \sum_{k=1}^{n} x_k - 2y_C \sum_{k=1}^{n} y_k + n\left(x_C^2 + y_C^2\right)$$

$A_1, \ldots, A_n$ drehen um Schwerpunkt S

C ein externer Punkt

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$$= \sum_{k=1}^{n}\left(x_k^2 + y_k^2\right) - 2x_C \underbrace{\sum_{k=1}^{n} x_k}_{=0} - 2y_C \underbrace{\sum_{k=1}^{n} y_k}_{=0} + n\left(x_C^2 + y_C^2\right)$$

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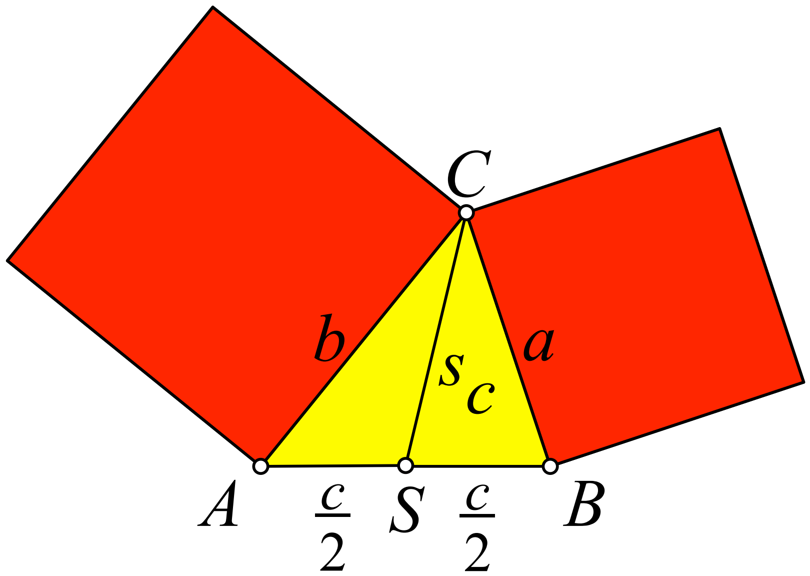
$$= \sum_{k=1}^{n} \left(x_k^2 + y_k^2 \right) - 2x_C \underbrace{\sum_{k=1}^{n} x_k}_{=0} - 2y_C \underbrace{\sum_{k=1}^{n} y_k}_{=0} + n\left(x_C^2 + y_C^2 \right)$$

$$= \underbrace{\sum_{k=1}^{n} d(S, A_k)^2}_{\text{konstant}} + n \underbrace{d(SC)^2}_{\text{konstant}}$$

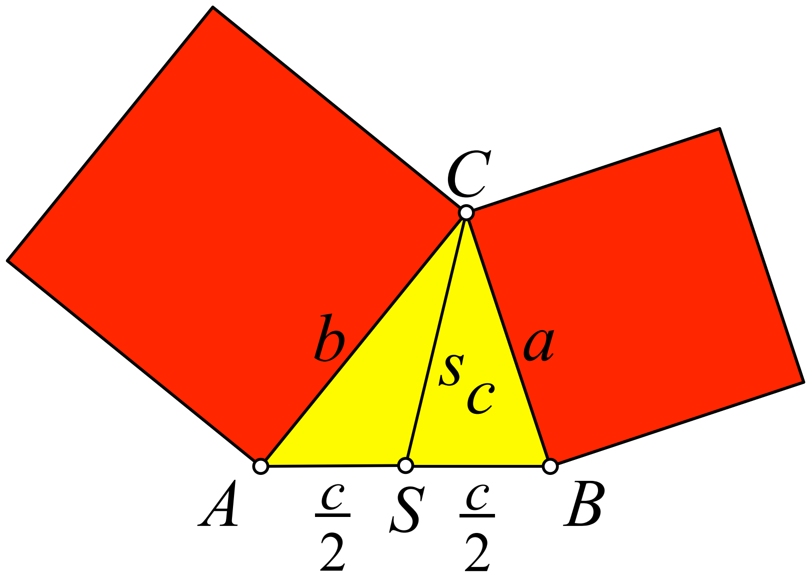
A_1, ... , A_n drehen um Schwerpunkt S

C ein externer Punkt



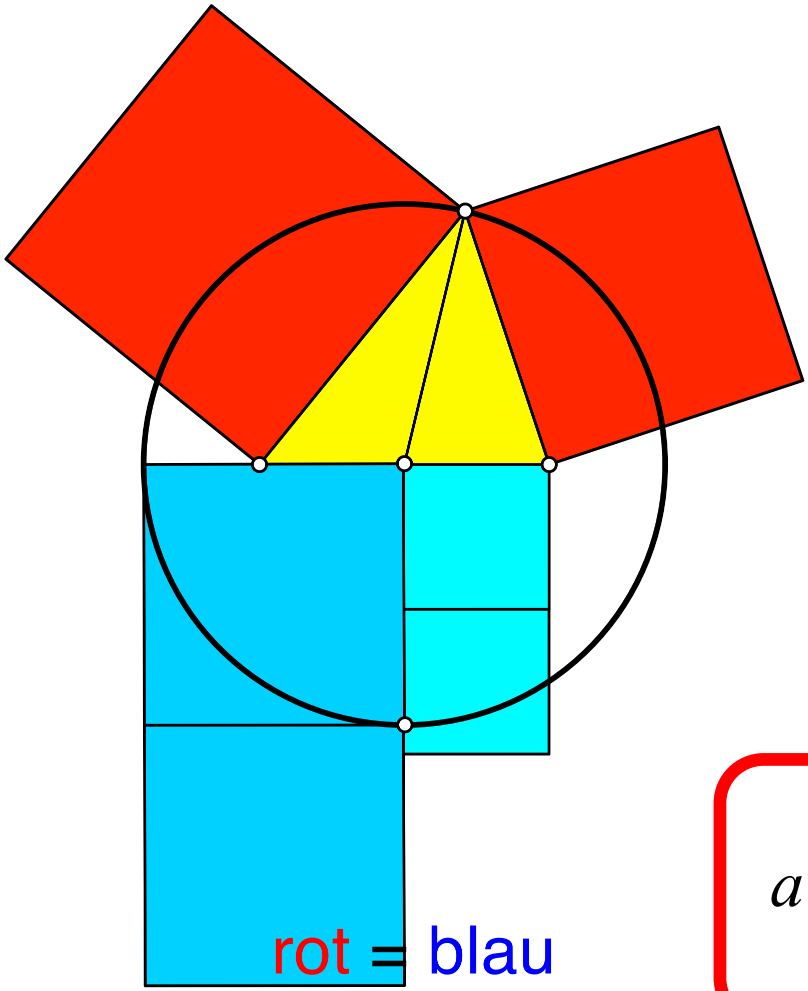
A_1, ... , A_n drehen um Schwerpunkt S

C ein externer Punkt



$$a^2 + b^2 = 2\left(\frac{c}{2}\right)^2 + 2s_c^2$$

Apollonios / al-Sijzi



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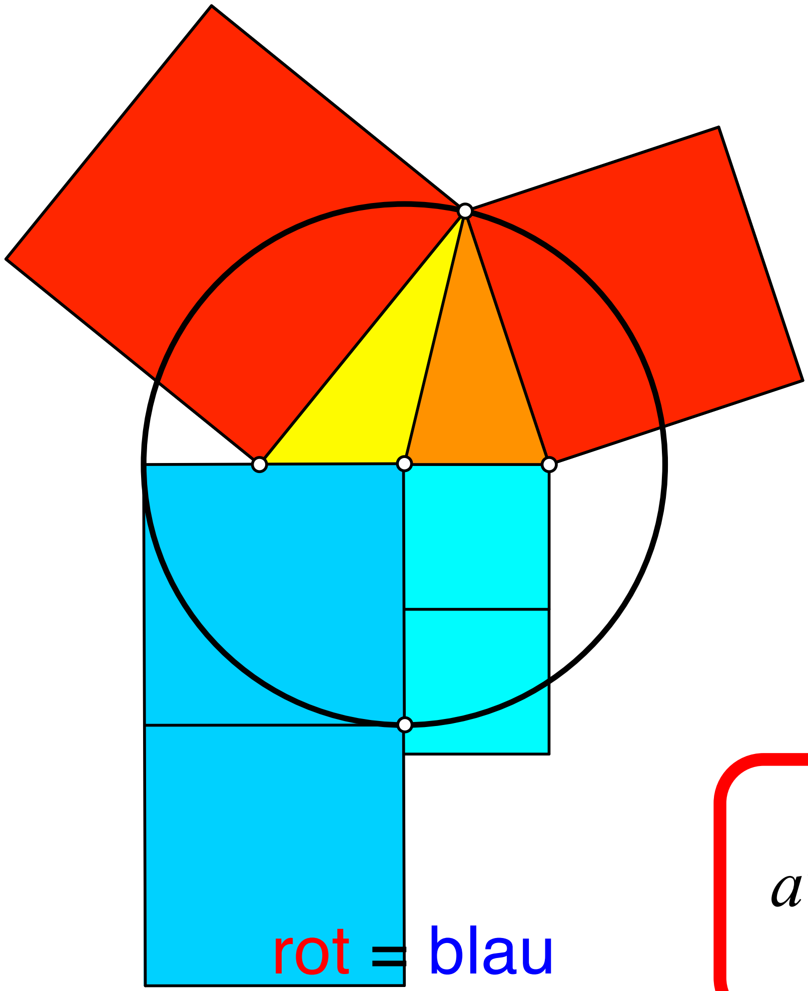

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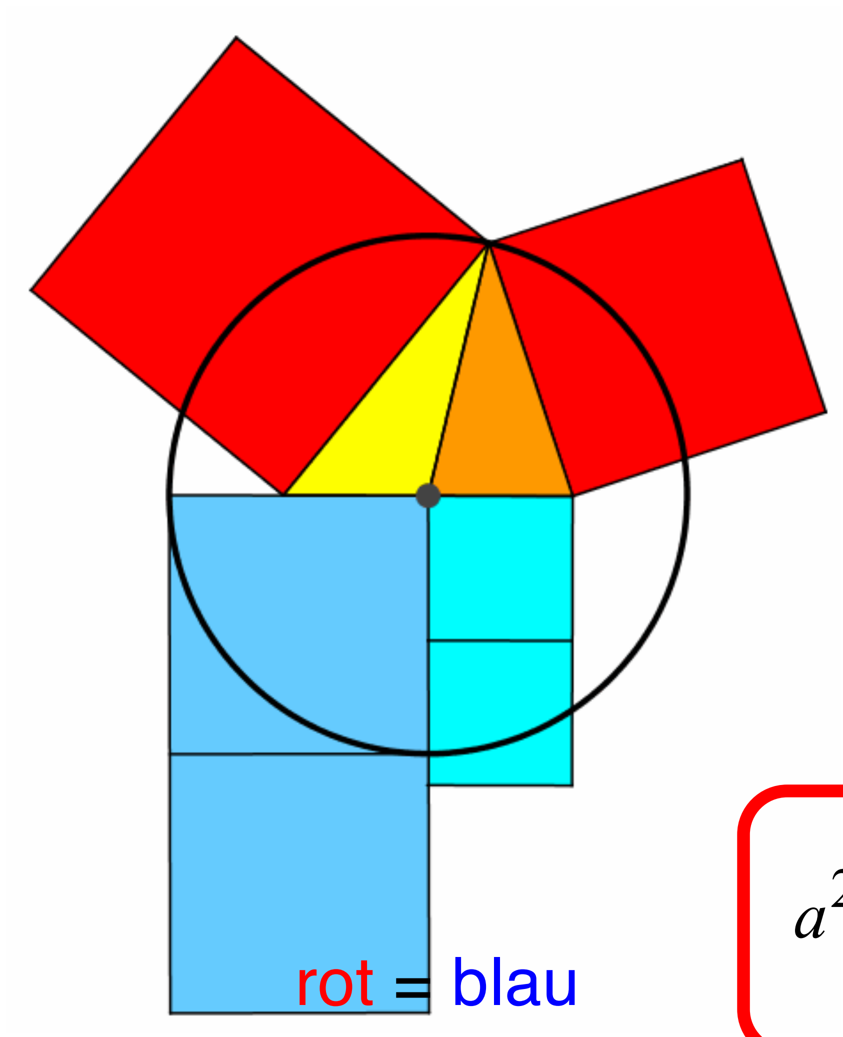
Apollonios / al-Sijzi



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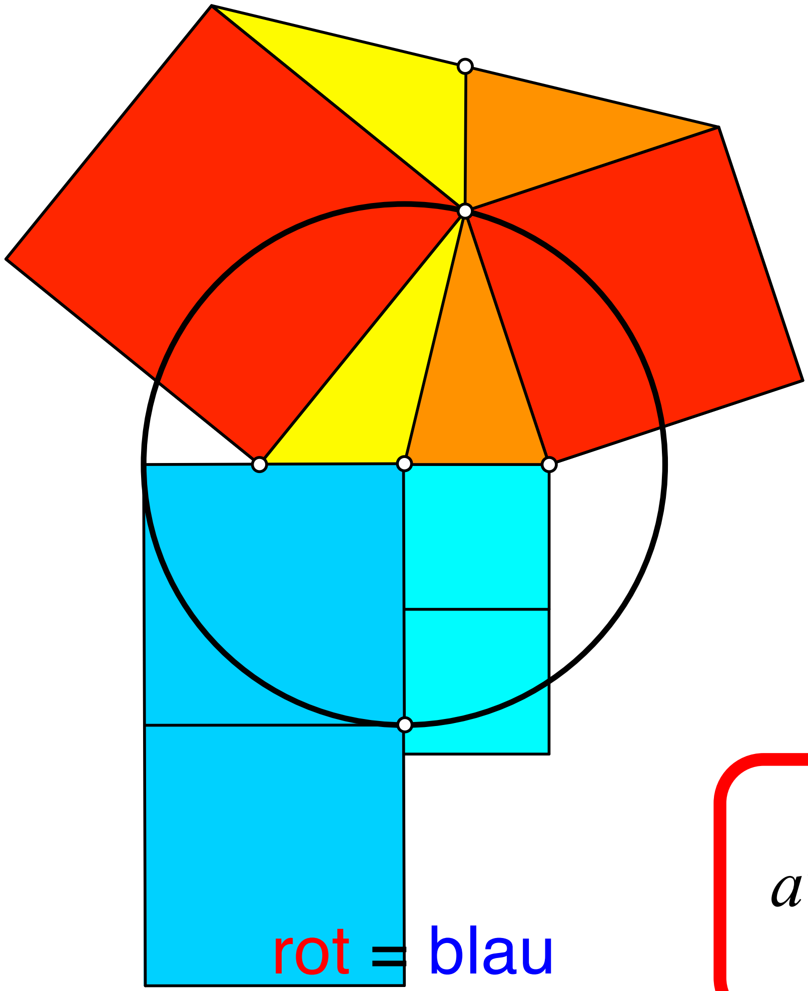

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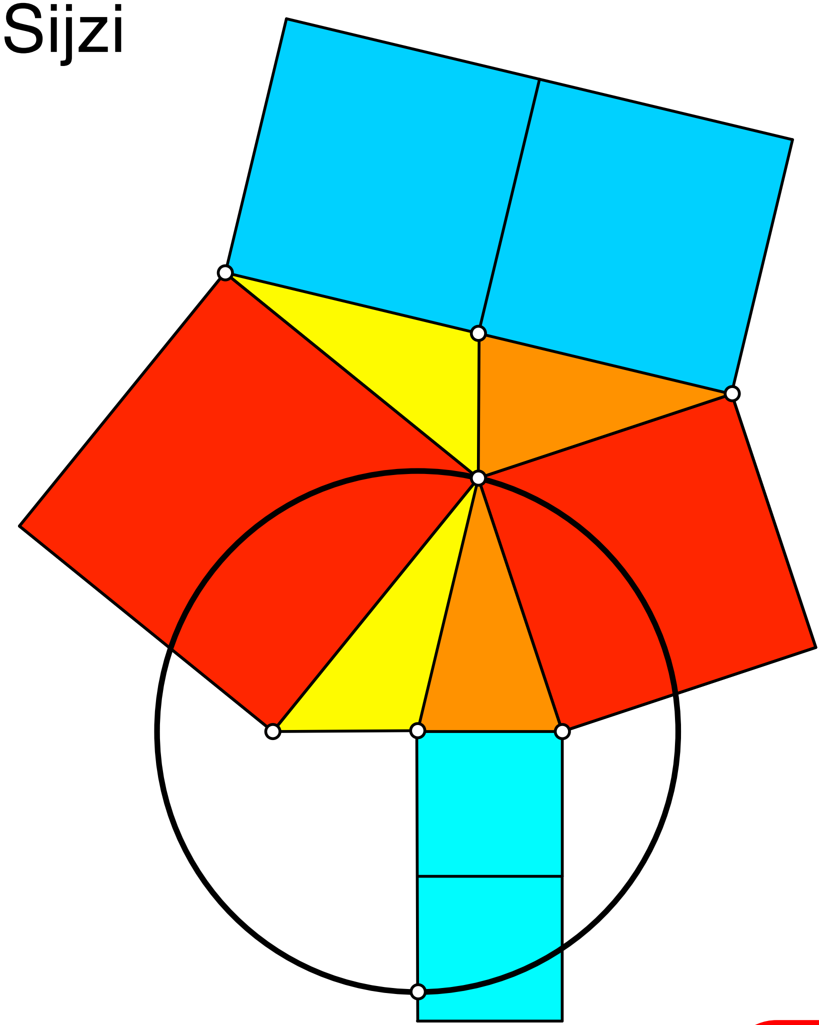
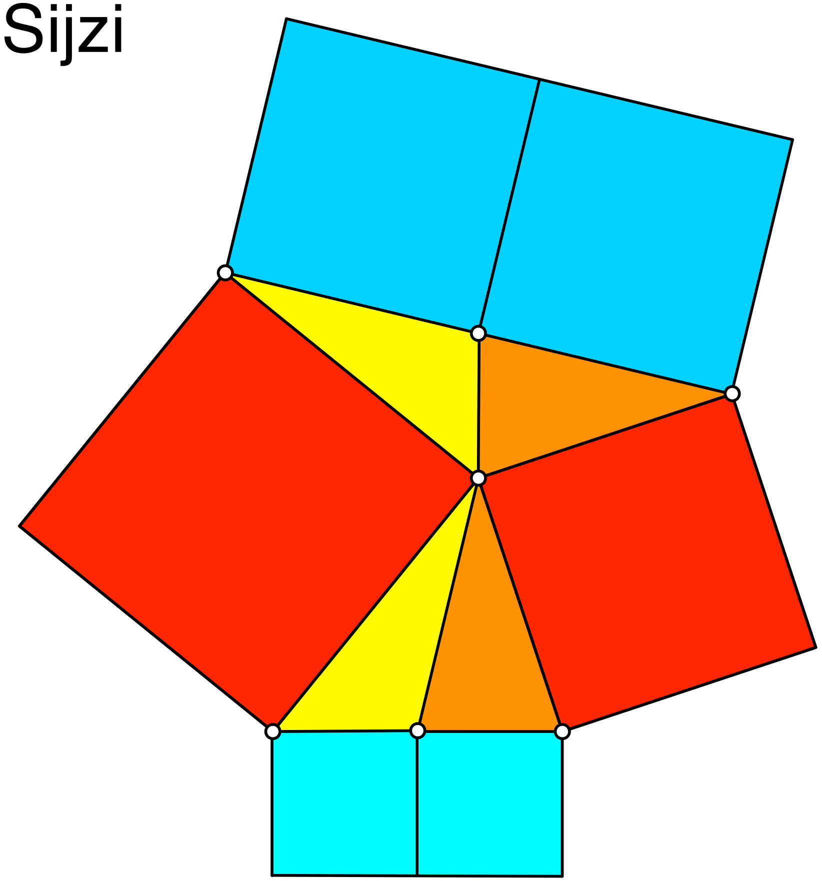
Apollonios / al-Sijzi



rot = blau

$$a^2 + b^2 = 2\left(\frac{c}{2}\right)^2 + 2s_c^2$$

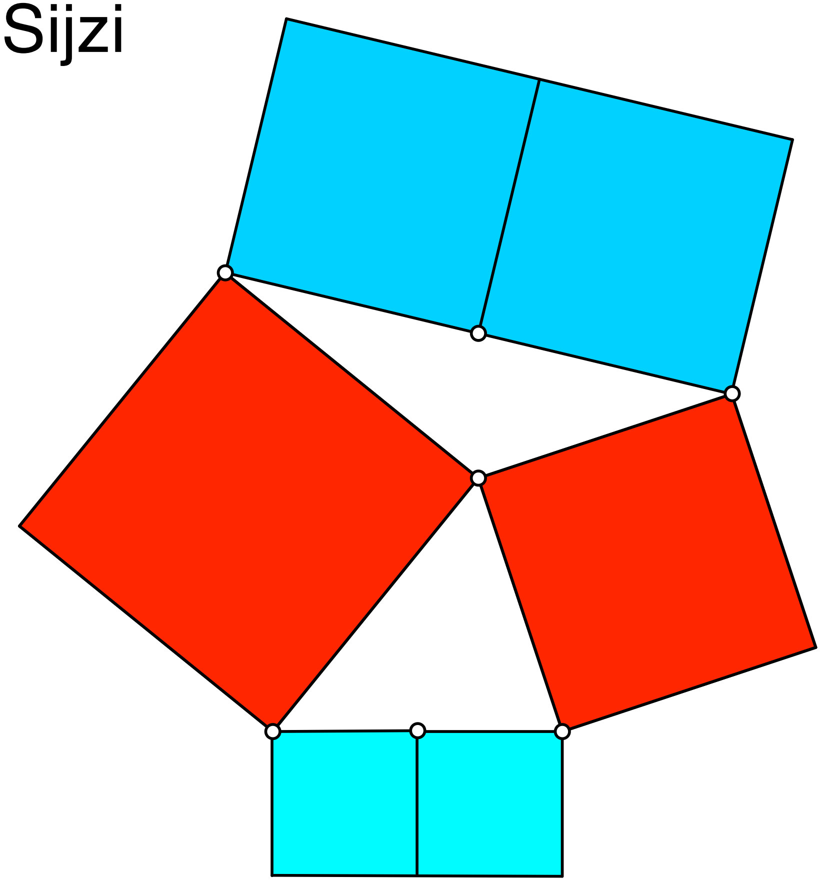
Apollonios / al-Sijzi



rot = blau

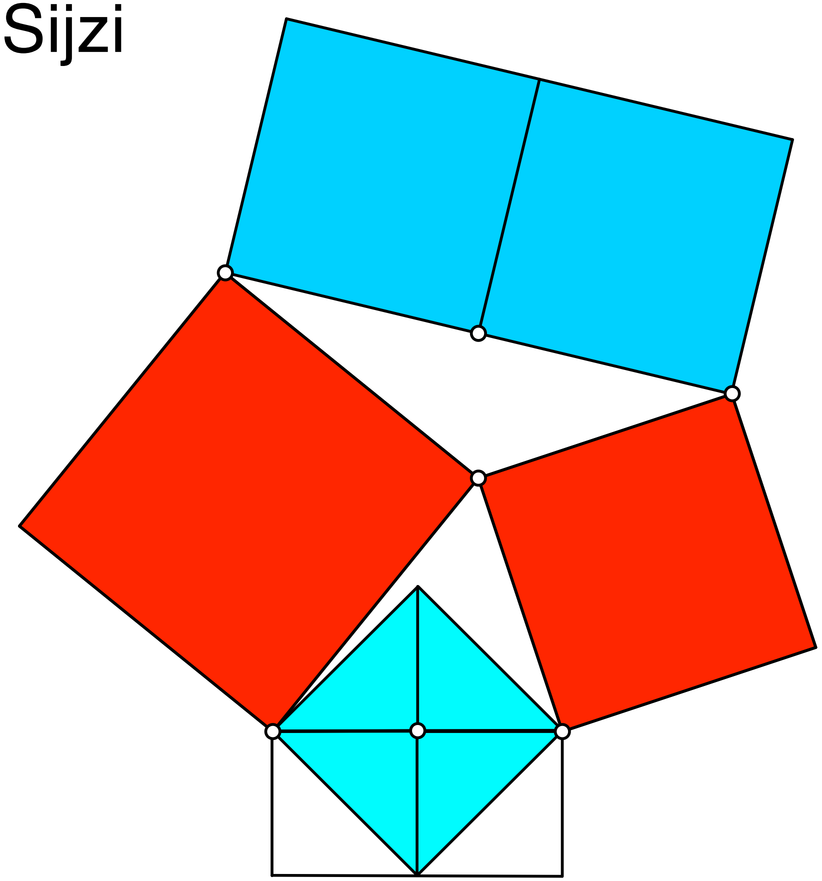
$$a^2 + b^2 = 2\left(\frac{c}{2}\right)^2 + 2s_c^2$$

Apollonios / al-Sijzi



rot = blau

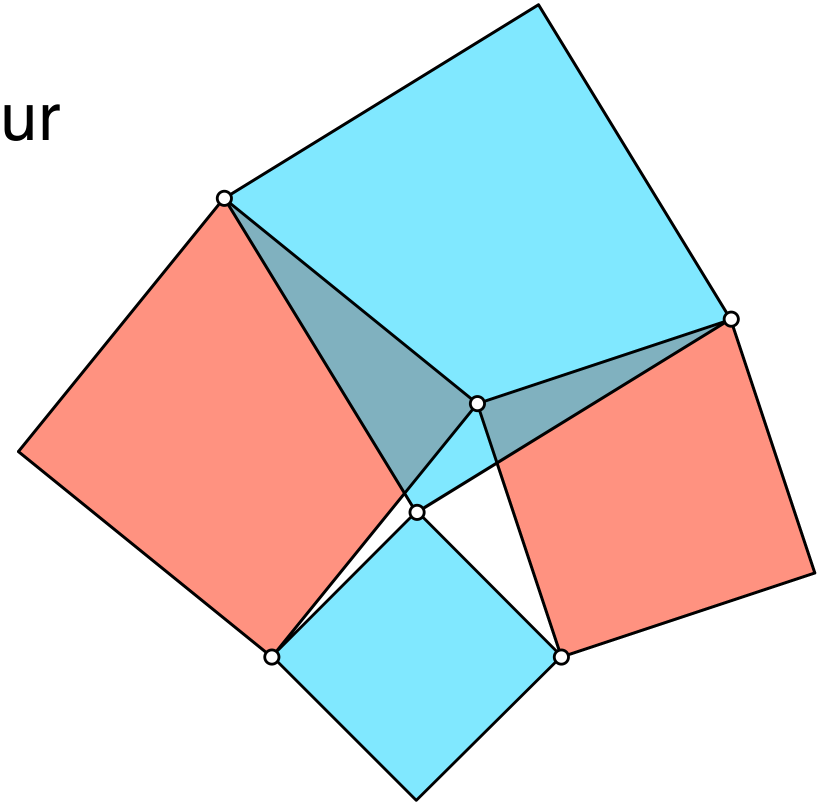
Apollonios / al-Sijzi



rot = blau

Papillon

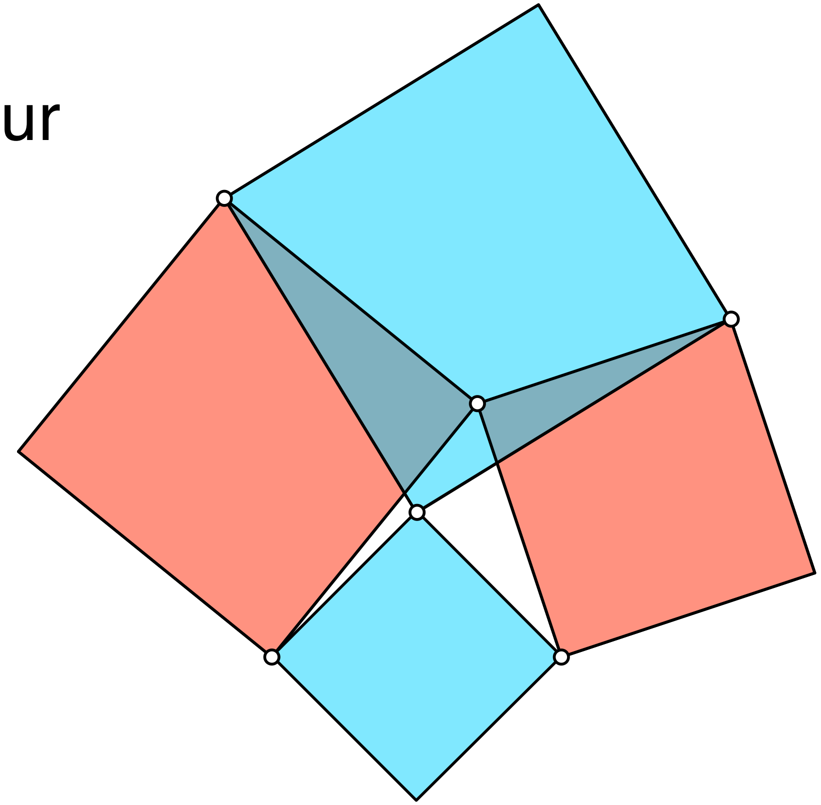
Schließungsfigur



rot = blau

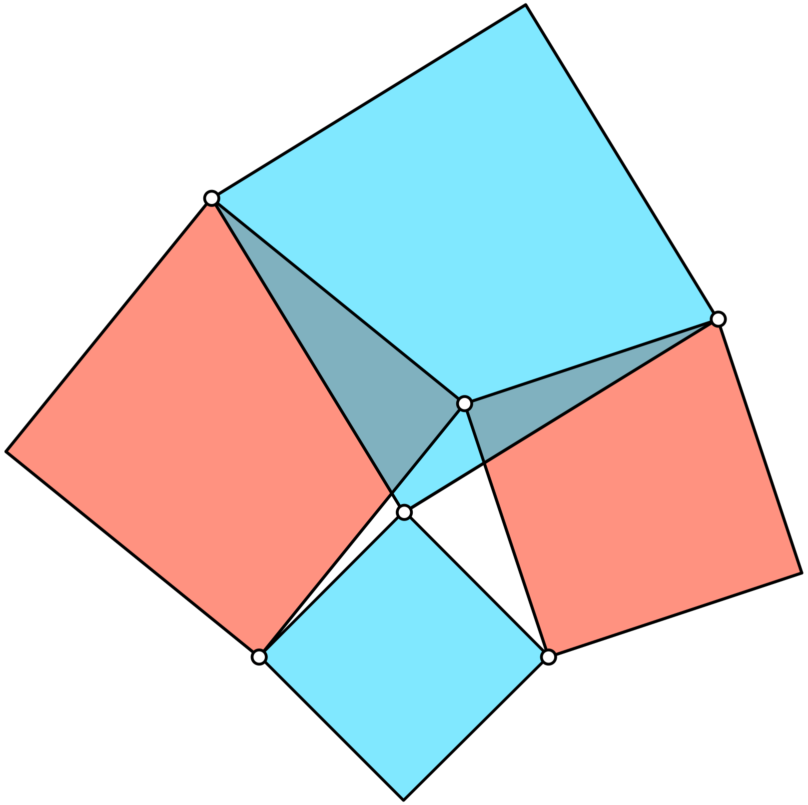
Papillon

Schließungsfigur



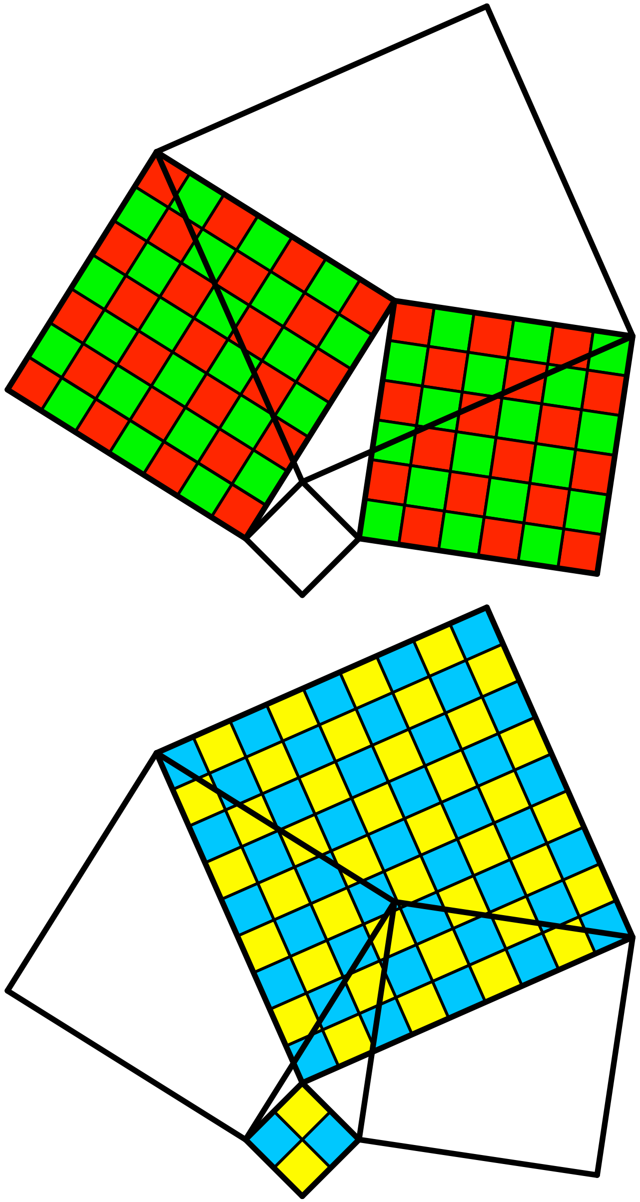
Beweis der Schließungseigenschaft:
Walser, H. (2021): Spiel mit Quadraten
MU Der Mathematikunterricht
Jahrgang 67. Heft 3. August 2021. 17-27

Papillon



rot = blau

4	7	1	8
5	5	1	7
5	10	2	11
6	**7**	**2**	**9**
6	13	3	14
7	9	3	11
7	11	1	13
8	9	1	12
8	11	4	13
9	13	5	15
10	11	5	14
11	12	3	16
11	13	1	17
13	13	7	17
13	14	2	19



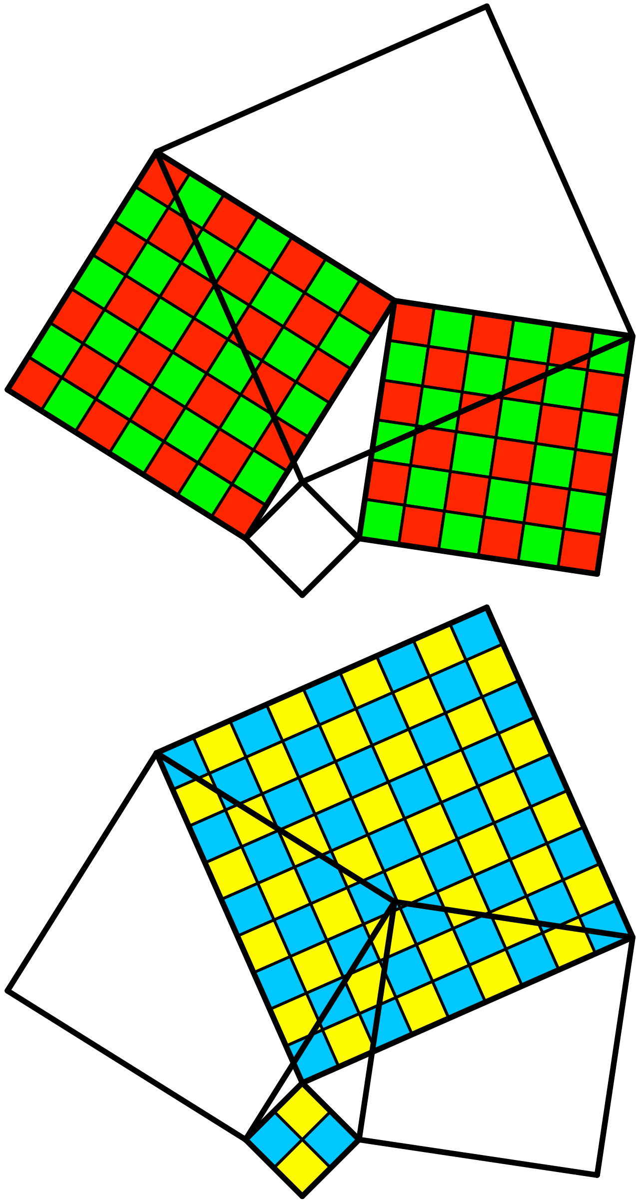
ganzzahlig

$$6^2 + 7^2 = 2^2 + 9^2$$

$$36 + 49 = 4 + 81 = 85$$

rot/grün = blau/gelb

4	7	1	8
5	5	1	7
5	10	2	11
6	**7**	**2**	**9**
6	13	3	14
7	9	3	11
7	11	1	13
8	9	1	12
8	11	4	13
9	13	5	15
10	11	5	14
11	12	3	16
11	13	1	17
13	13	7	17
13	14	2	19



rot/grün = blau/gelb

ganzzahlig

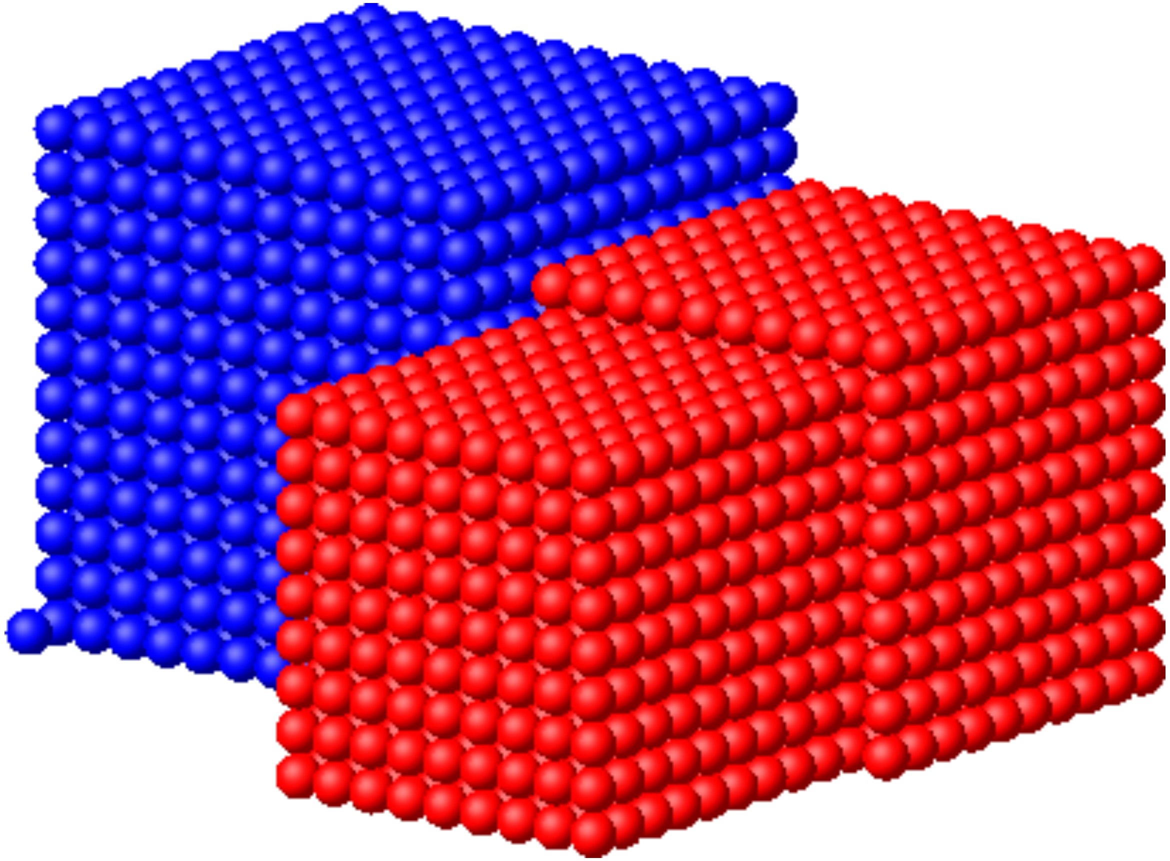
$$6^2 + 7^2 = 2^2 + 9^2$$

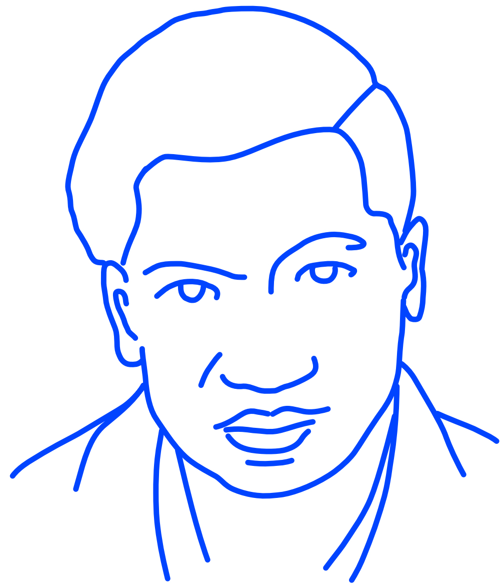
$$36 + 49 = 4 + 81 = 85$$

S. Ramanujan
1887-1920

$$9^3 + 10^3 = 1^3 + 12^3$$

$$729 + 1000 = 1 + 1728$$





S. Ramanujan
1887-1920

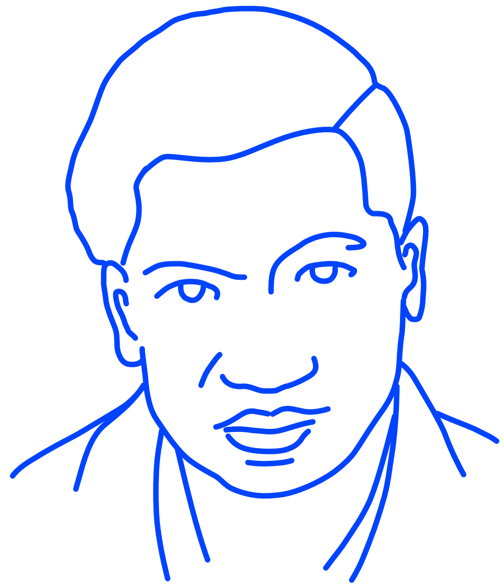
$$9^3 + 10^3 = 1^3 + 12^3$$

$$729 + 1000 = 1 + 1728$$

rot = blau

9	10	1	12	1729
9	15	2	16	4104
18	20	2	24	13832
15	33	2	34	39312
27	30	3	36	46683
18	30	4	32	32832
16	33	9	34	40033
19	24	10	27	20683
31	33	12	40	65728
26	36	17	39	64232

rot = blau



S. Ramanujan
1887-1920

$$9^3 + 10^3 = 1^3 + 12^3$$

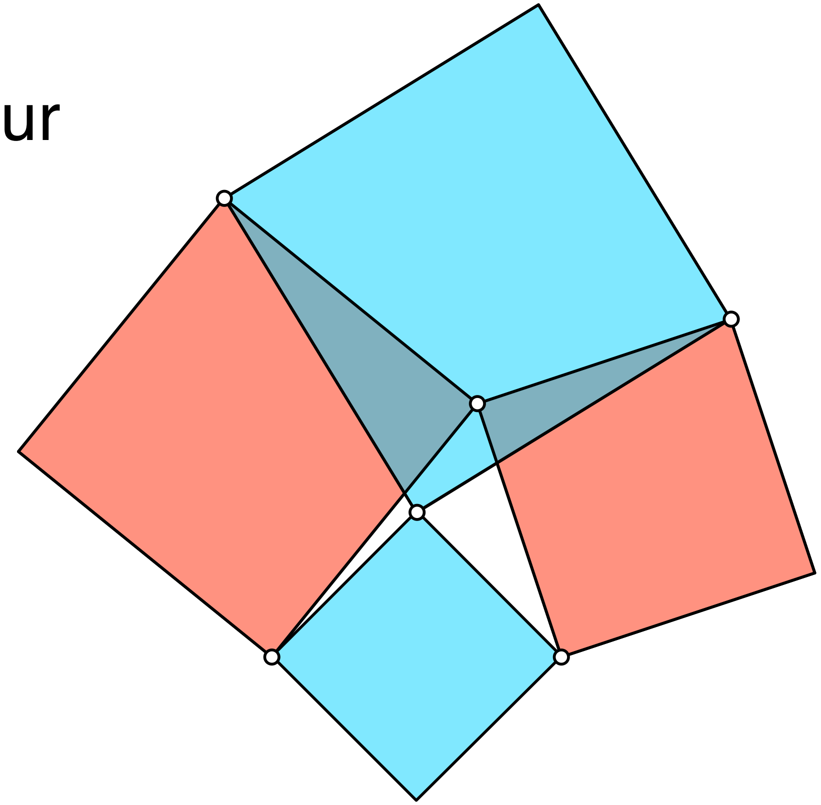
$$729 + 1000 = 1 + 1728$$

$$59^4 + 158^4 = 133^4 + 134^4 = 635'318'657$$

rot = blau

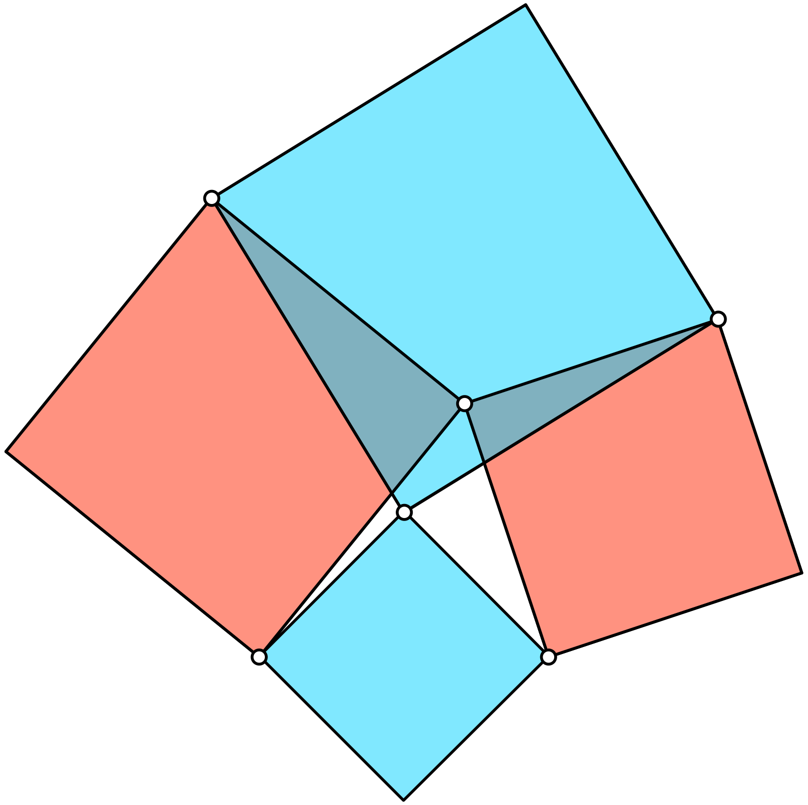
Papillon

Schließungsfigur



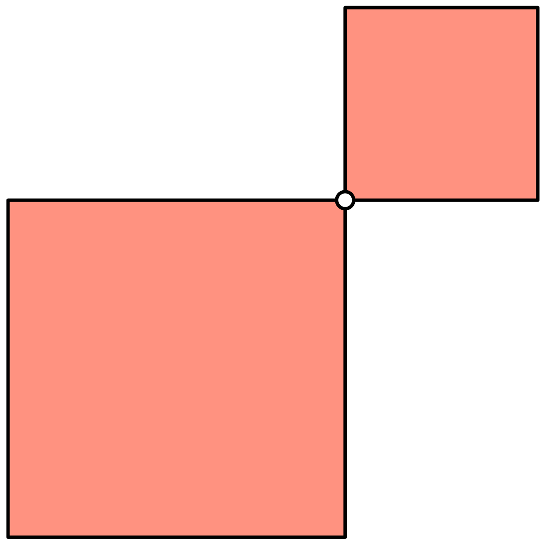
rot = blau

Zurück zu Pythagoras

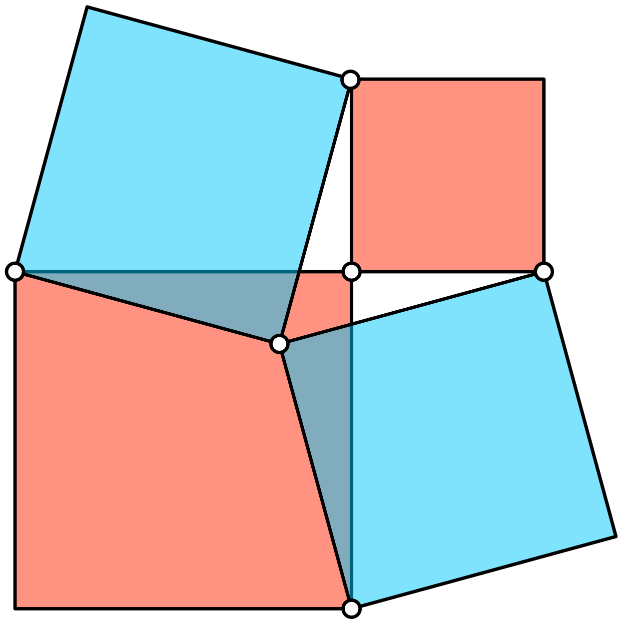


rot = blau

Zurück zu Pythagoras

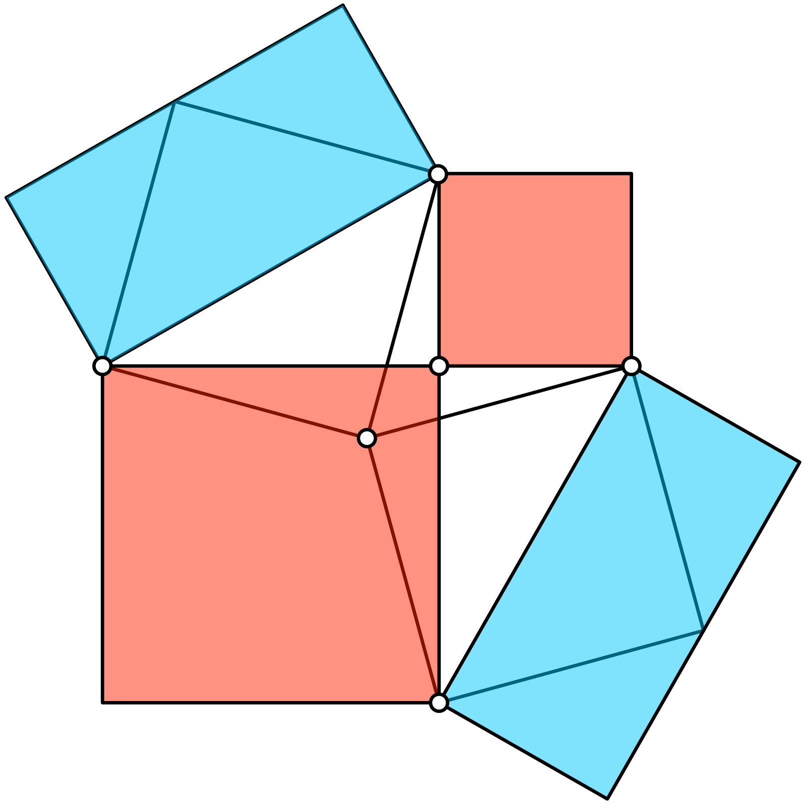


Zurück zu Pythagoras



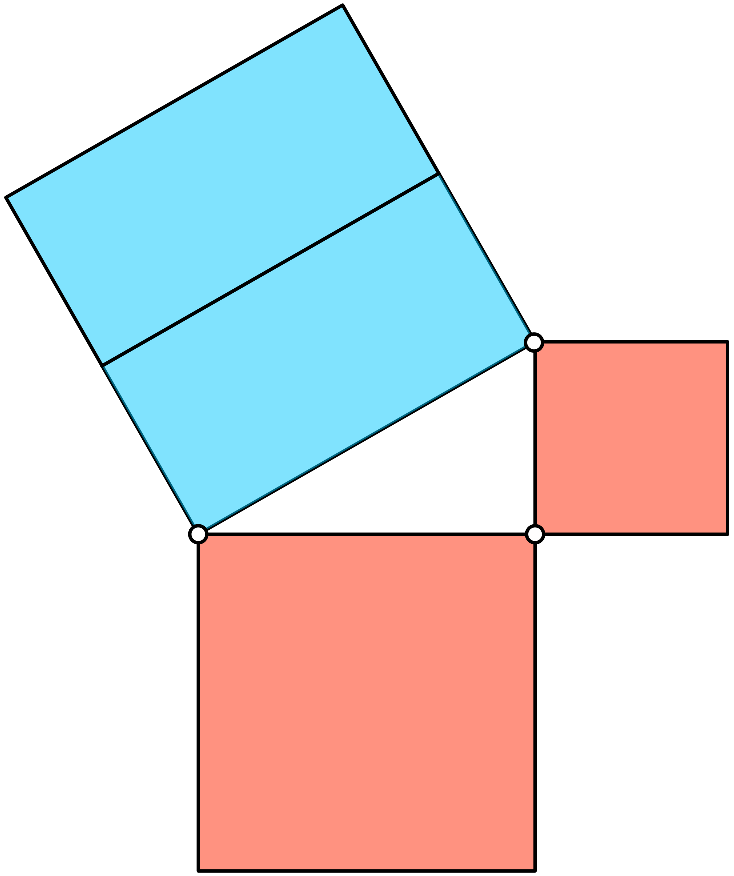
rot = blau

Zurück zu Pythagoras



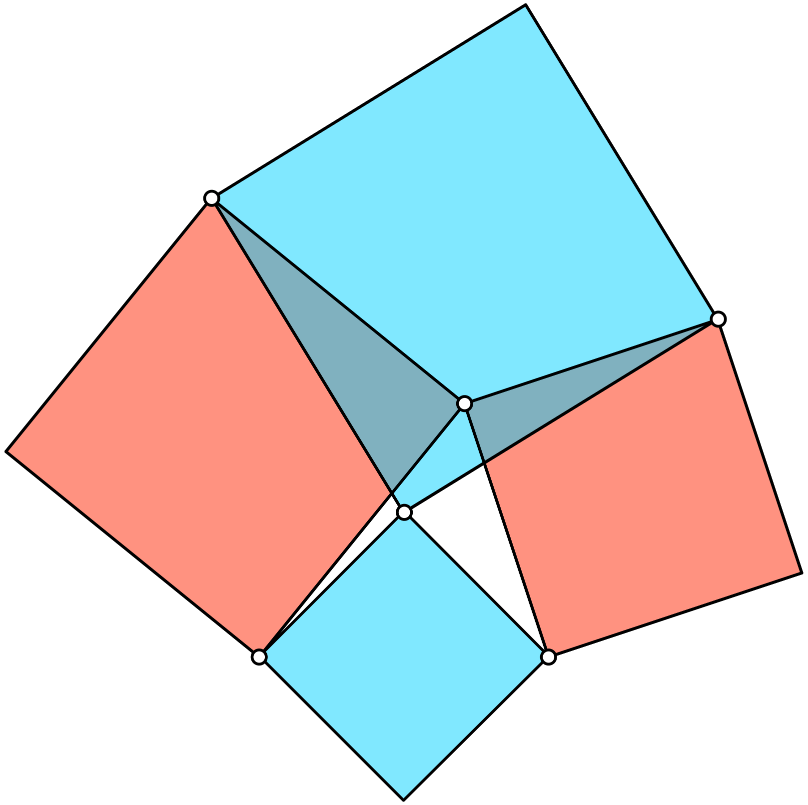
rot = blau

Zurück zu Pythagoras



rot = blau

Papillon



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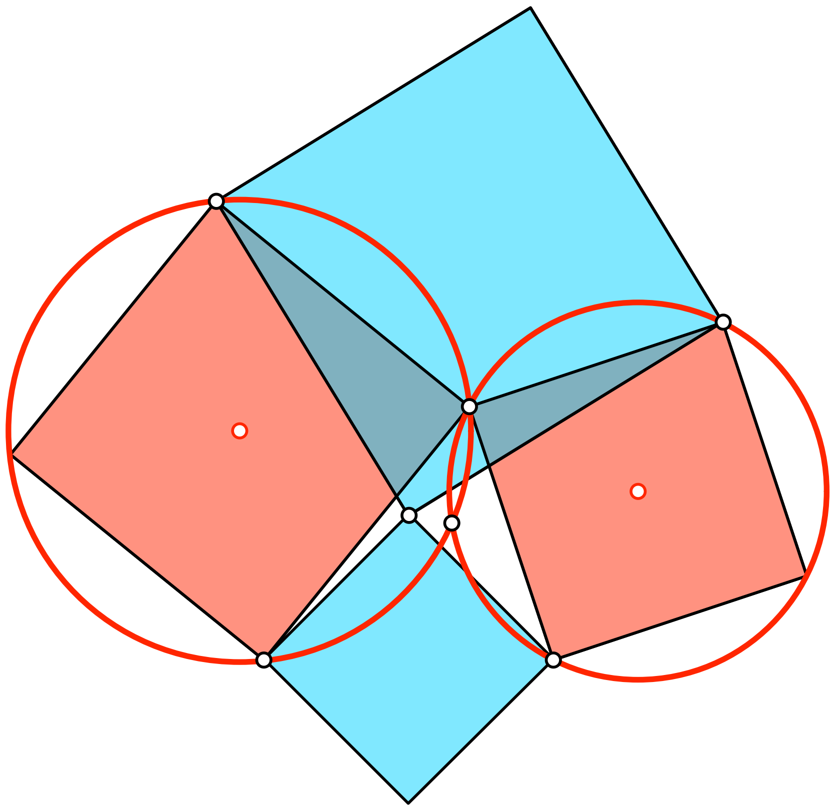
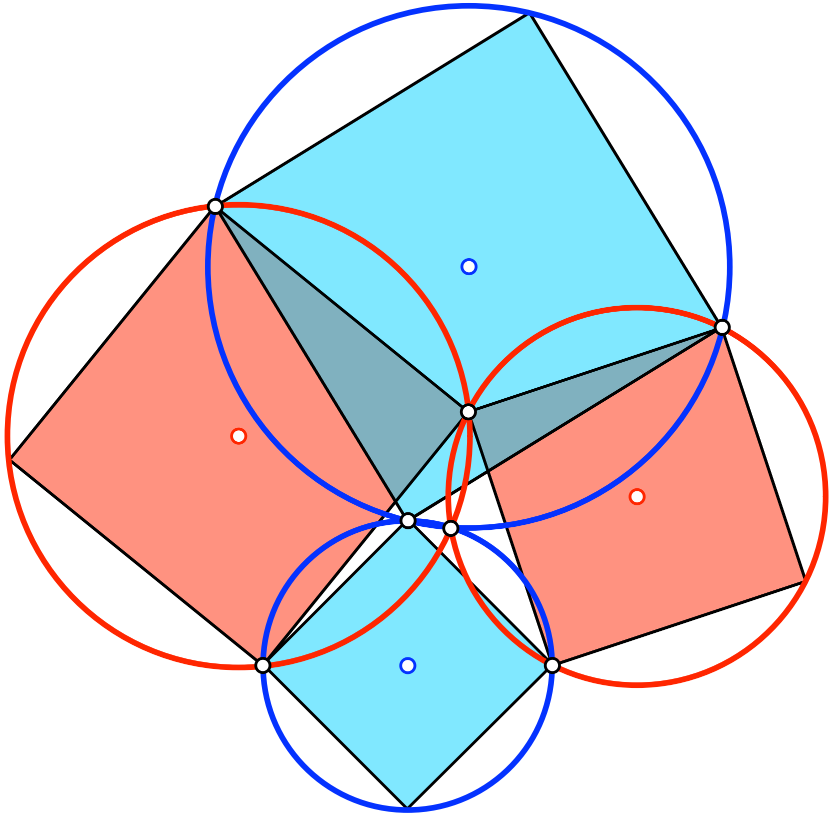


Umkreise

Papillon

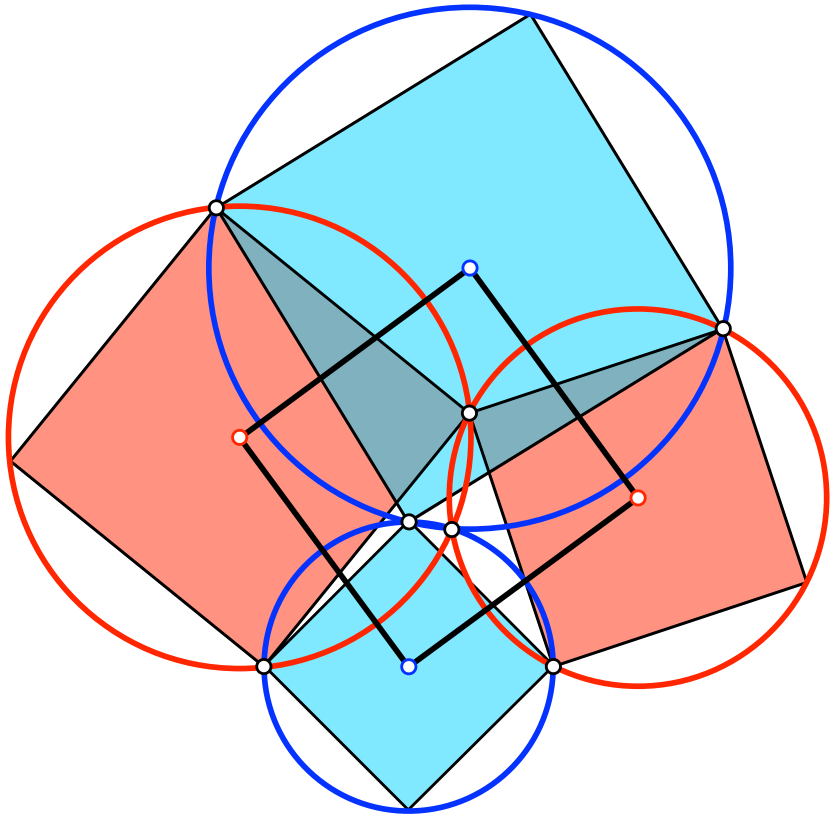
Gemeinsamer Schnittpunkt



Umkreise

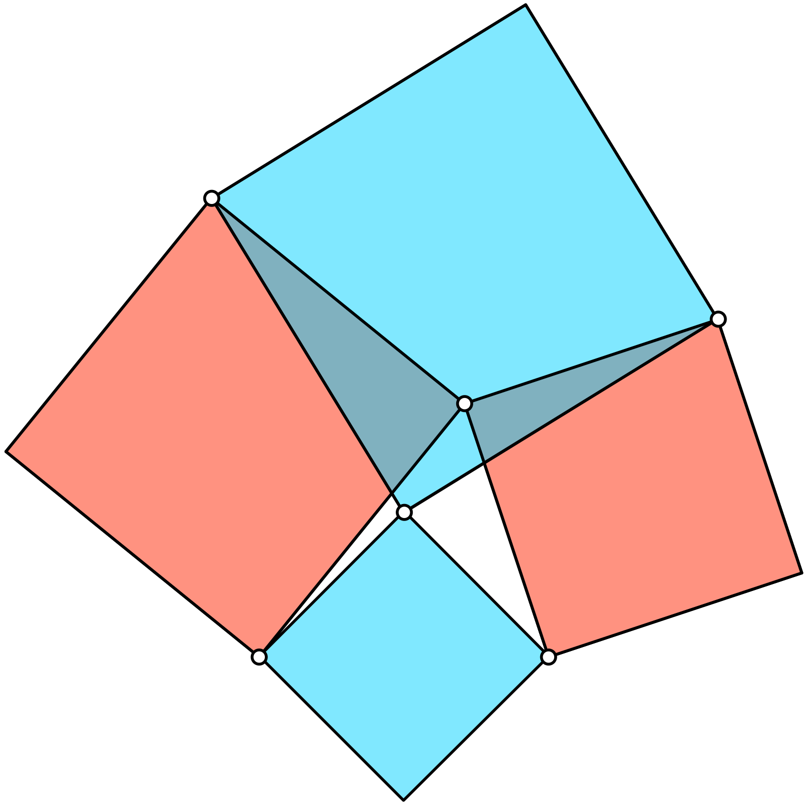
Papillon

Quadrat



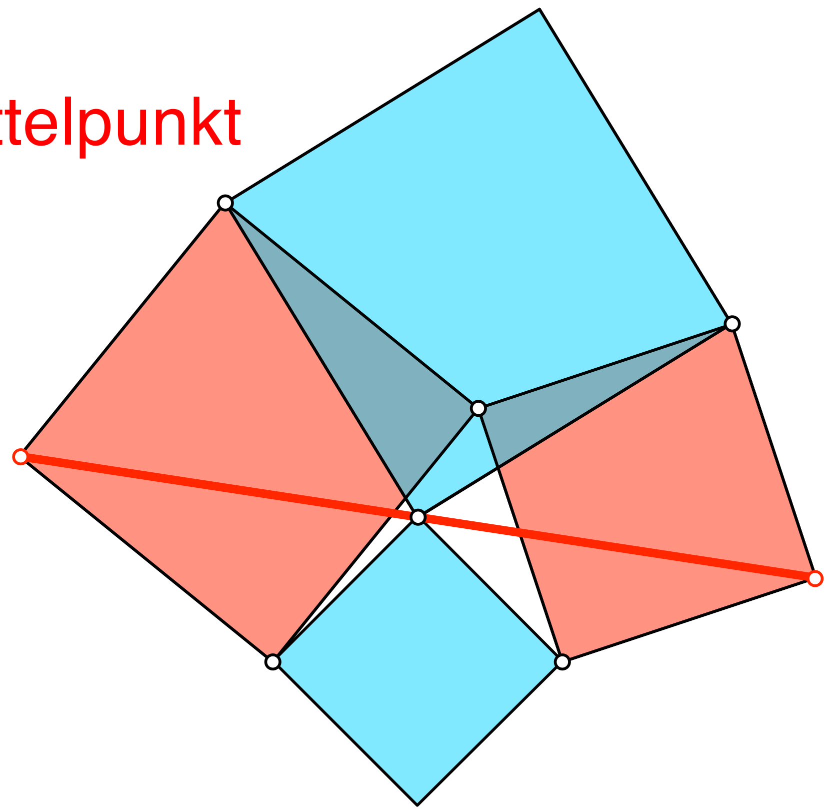
Umkreise

Papillon



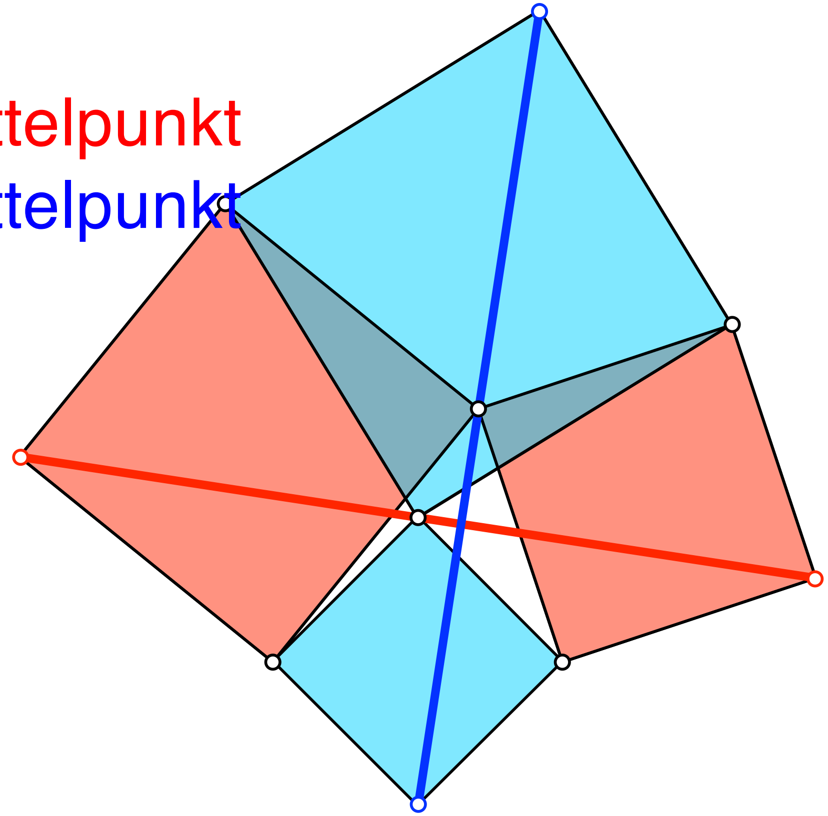
Papillon

Strecke mit Mittelpunkt



Papillon

Strecke mit Mittelpunkt
Strecke mit Mittelpunkt



rot = blau

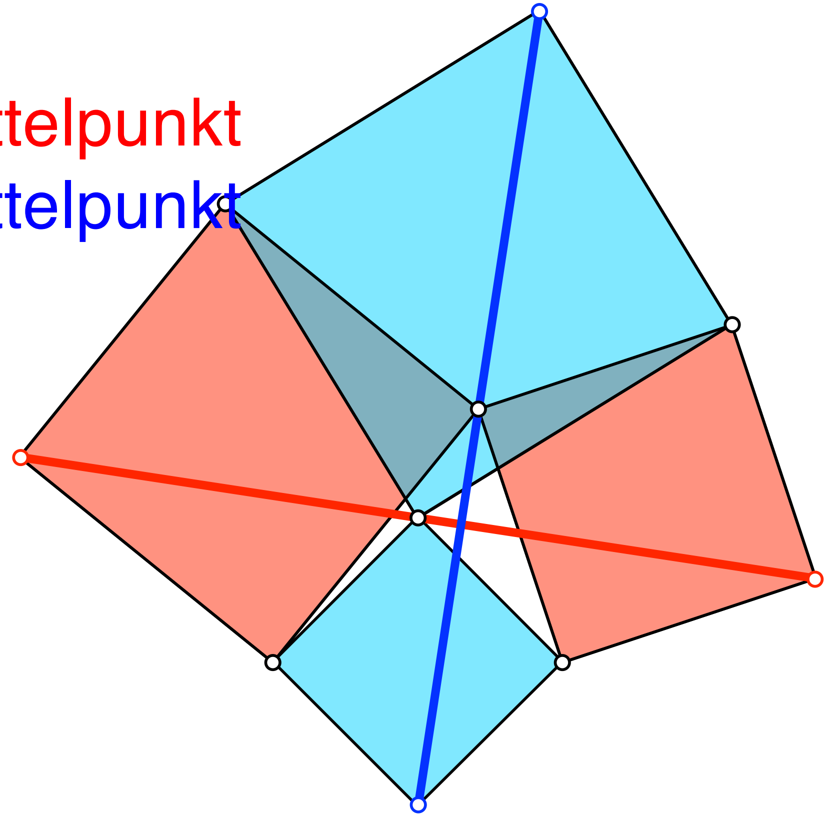
Papillon

Strecke mit Mittelpunkt

Strecke mit Mittelpunkt

orthogonal

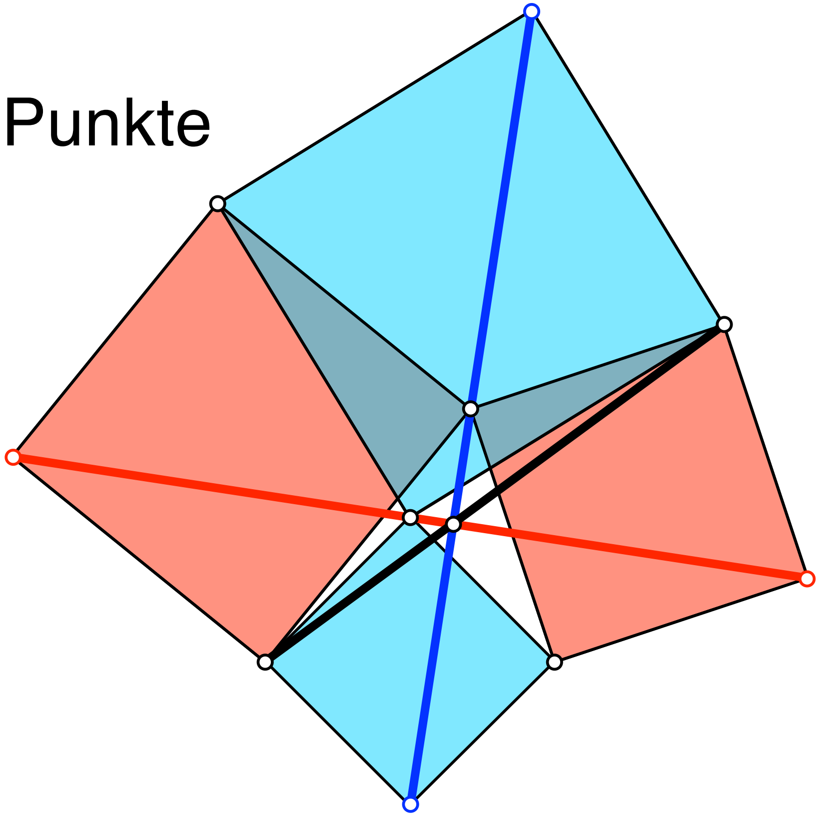
gleich lang



rot = blau

Papillon

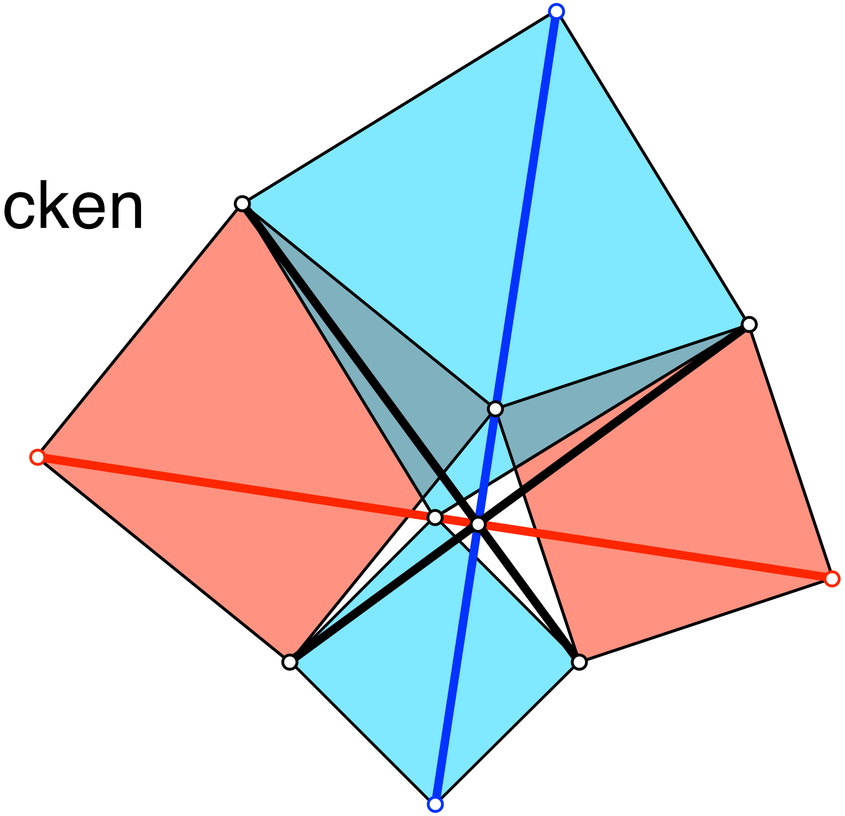
Drei kollineare Punkte



rot = blau

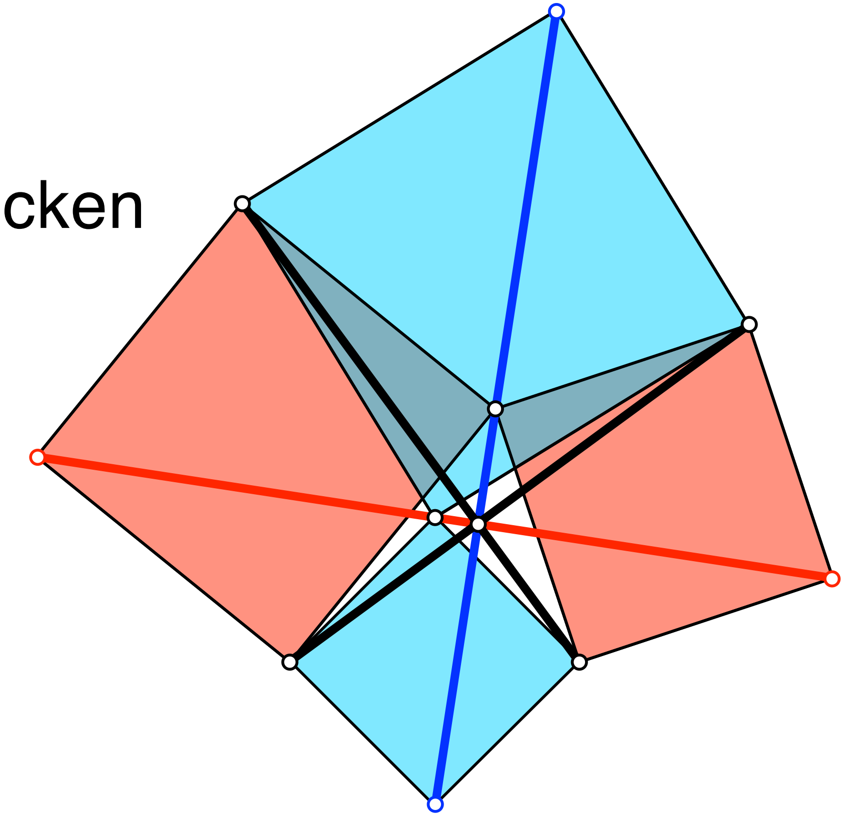
Papillon

gleich lange
schwarze Strecken
45°-Winkel



Papillon

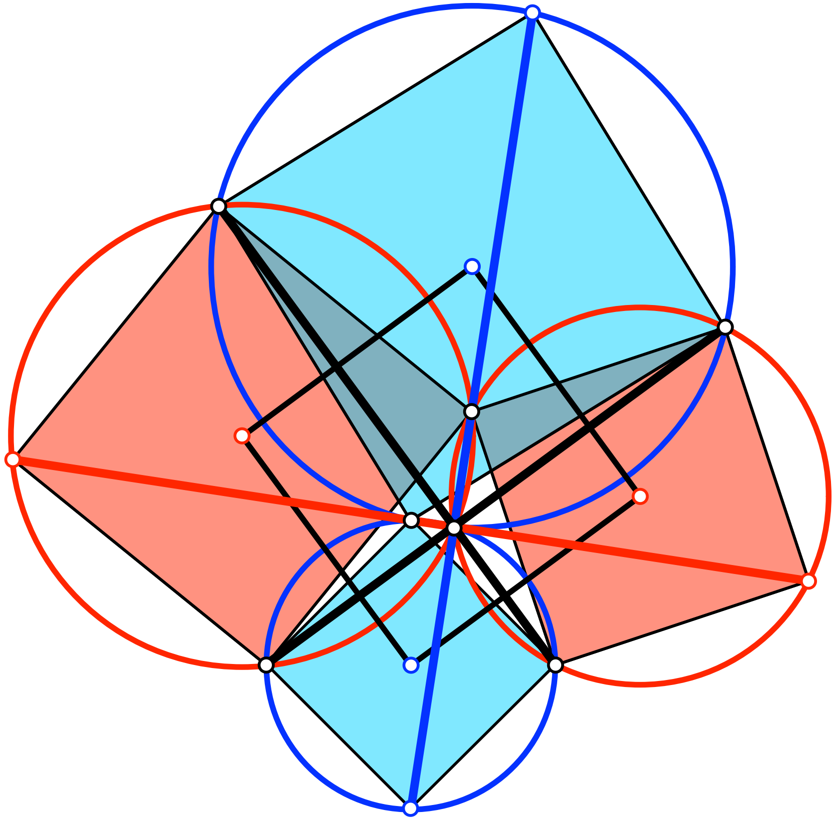
gleich lange
schwarze Strecken
45°-Winkel



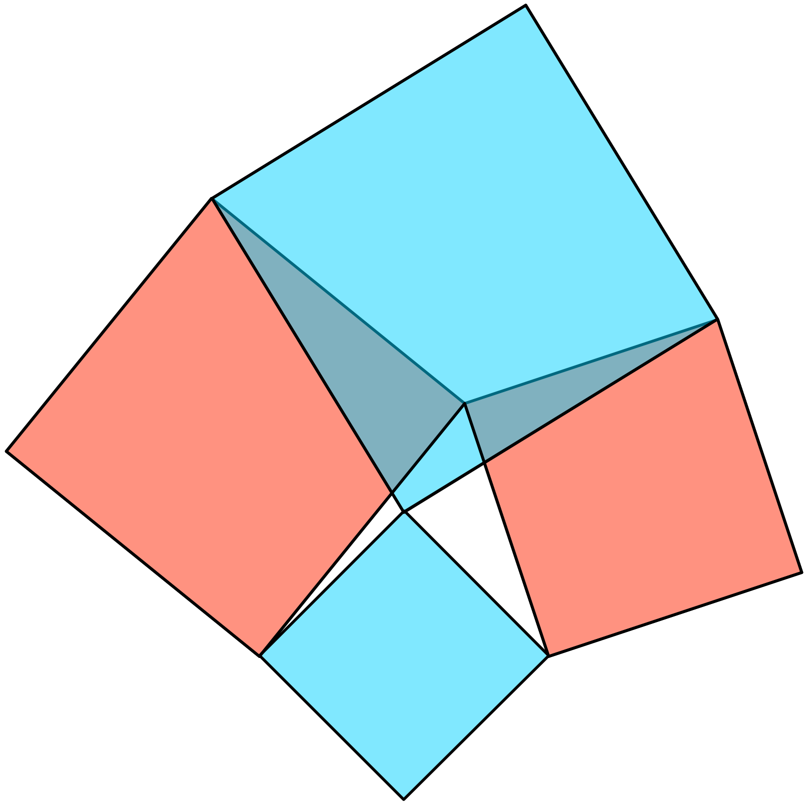
$\color{red}{\text{rot}} = \sqrt{2}\ \text{schwarz}$

$\color{blue}{\text{blau}} = \sqrt{2}\ \text{schwarz}$

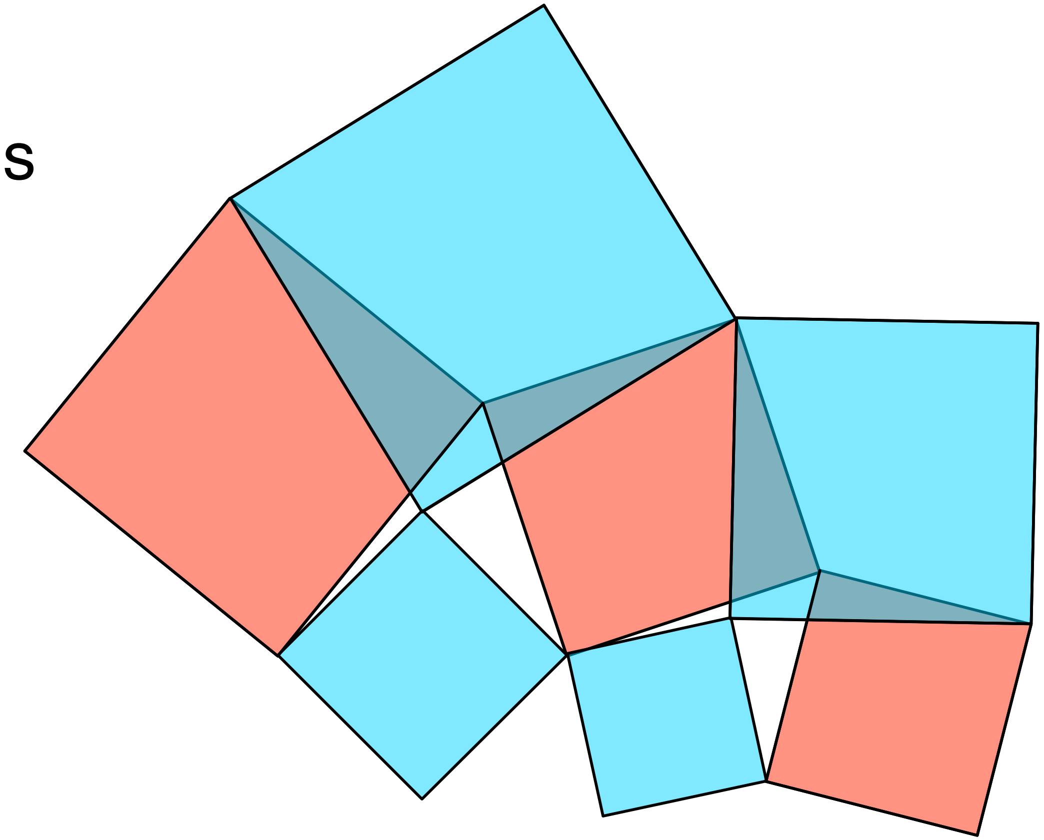
Papillon



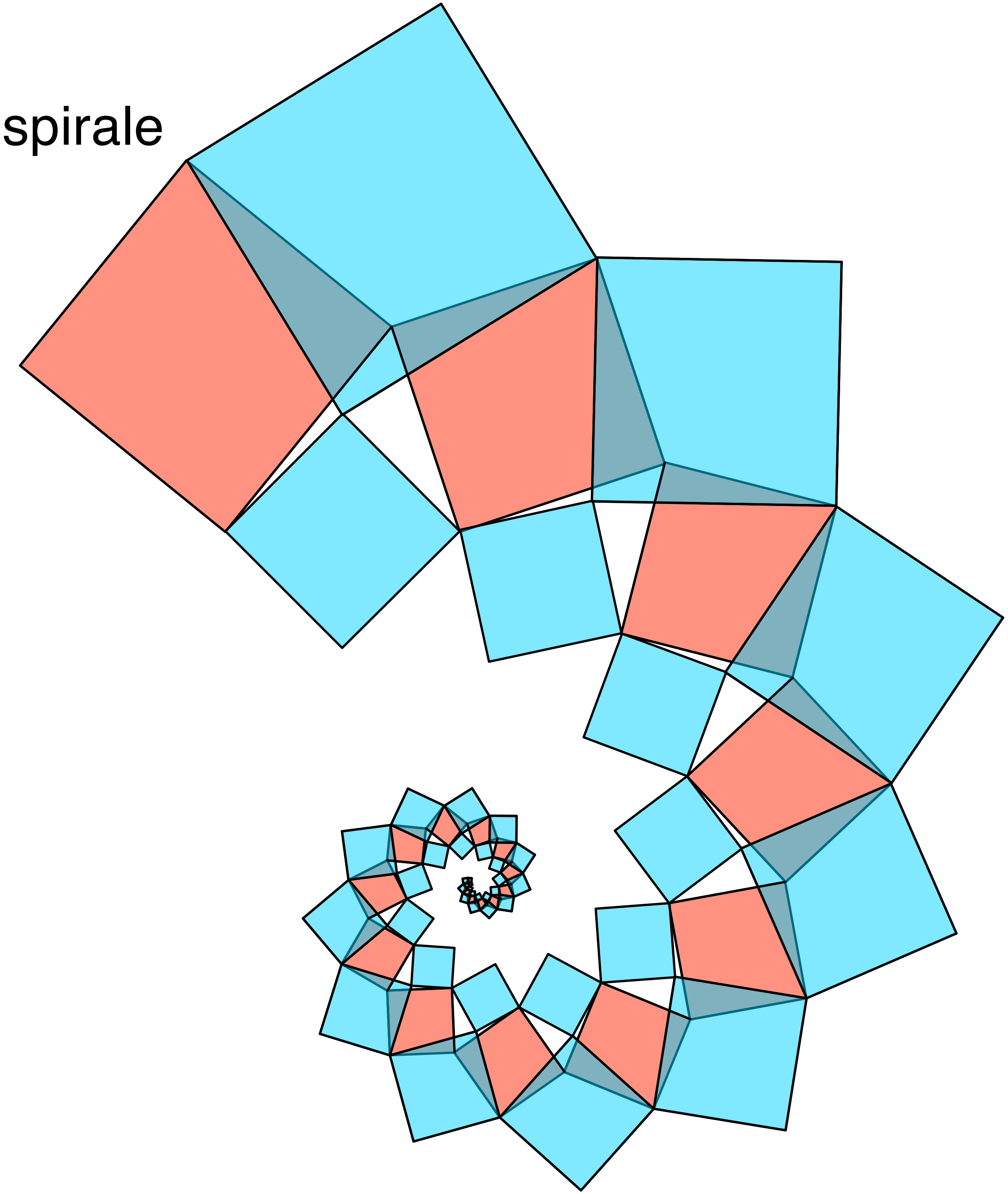
Papillon



Papillons

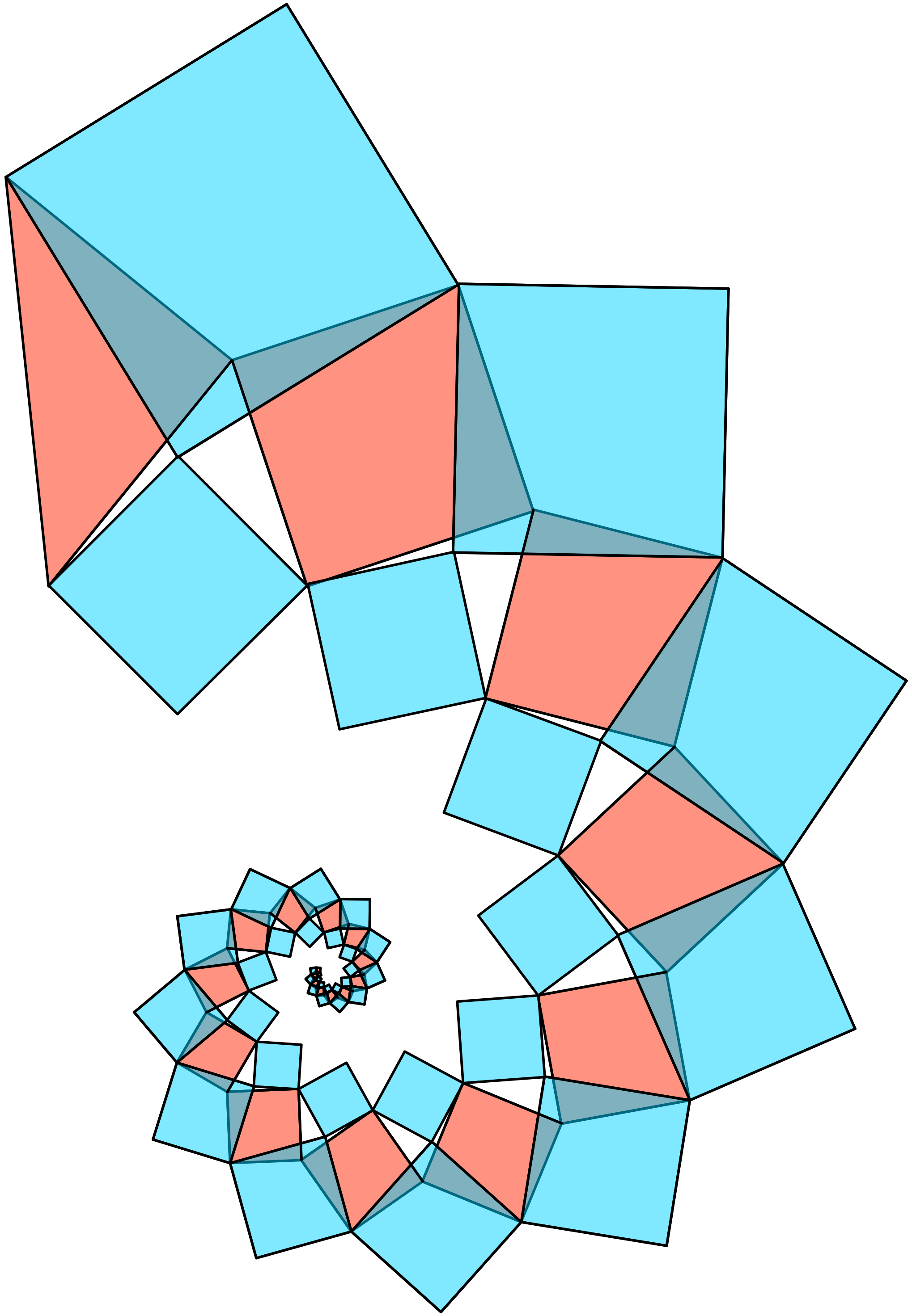


Papillonspirale

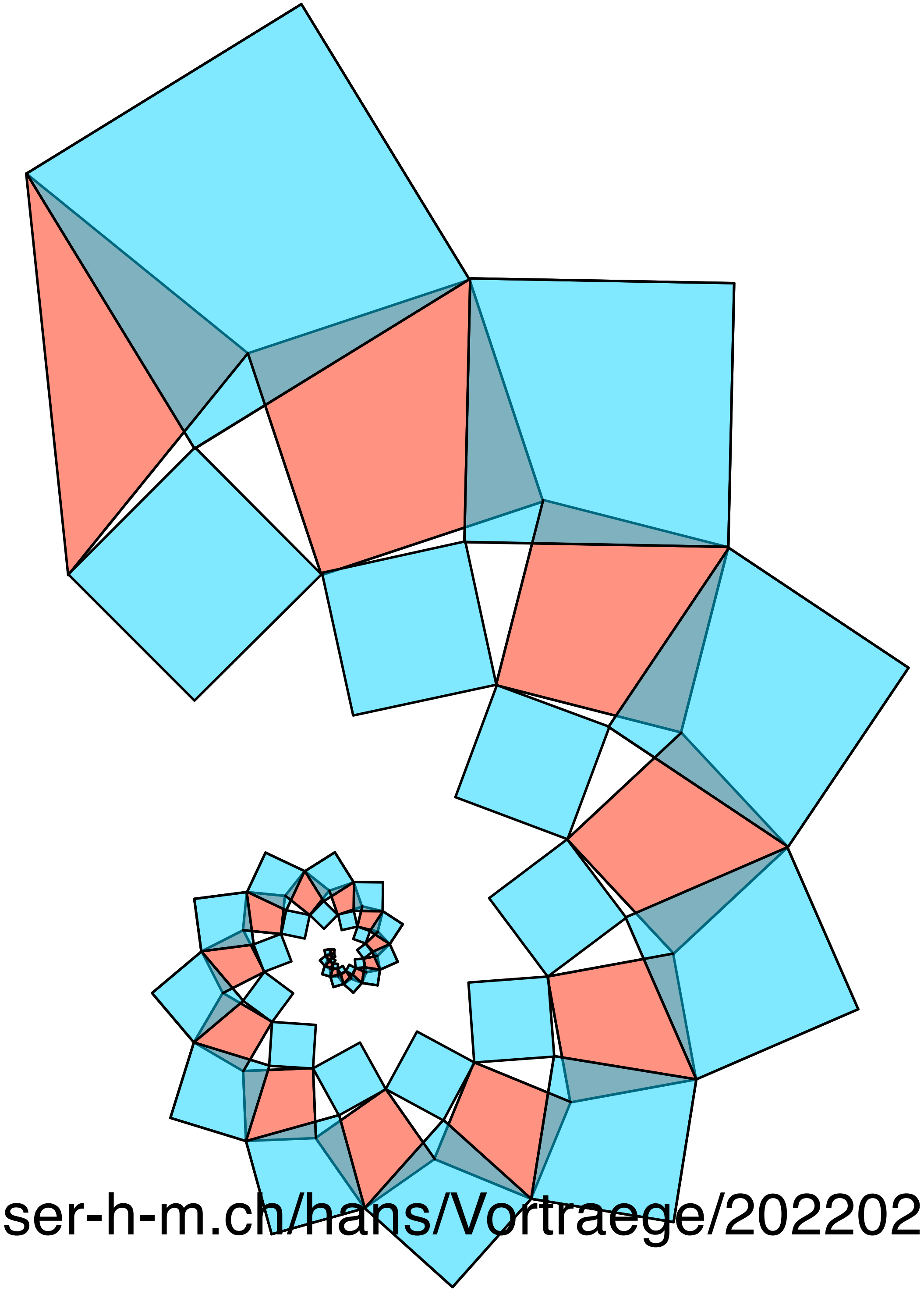


Papillonspirale

rot = ½ blau



Danke



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