Towards Optimal Distributed Edge Coloring with Fewer Colors

Manuel Jakob · Yannic Maus · **Florian Schager**

30. October 2025

TU Graz

FWF Österreichischer Wissenschaftsfonds

This research was funded in whole or in part by the Austrian Science Fund (FWF) [10.55776/P36280] & [10.55776/I6915].

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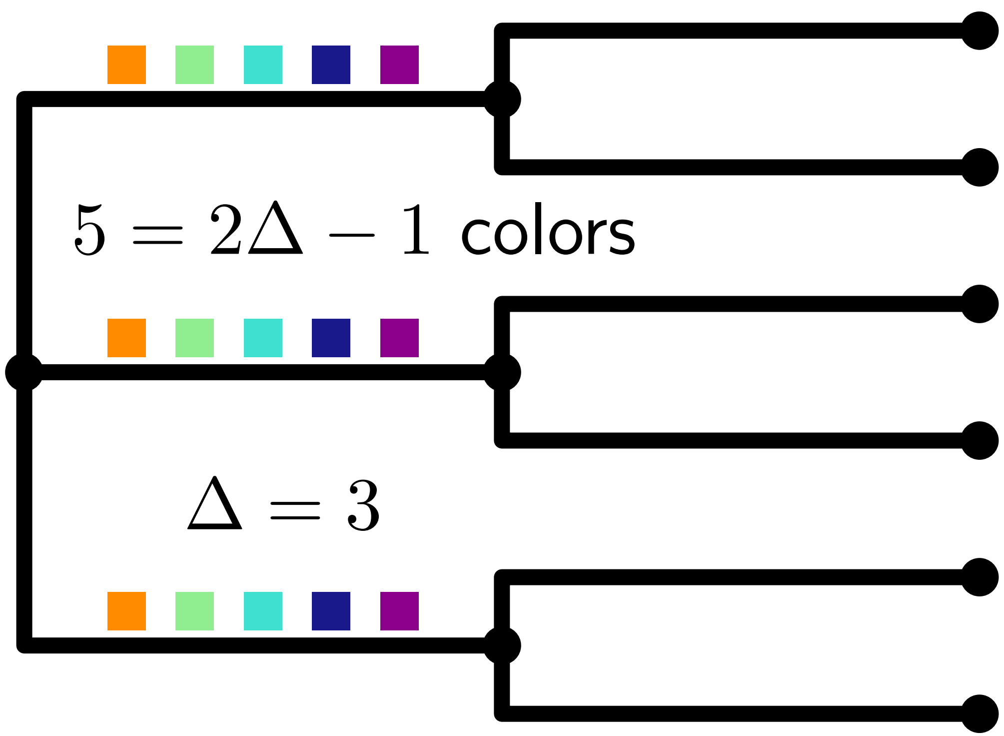

Goal: assign colors to edges; adjacent edges receive different colors

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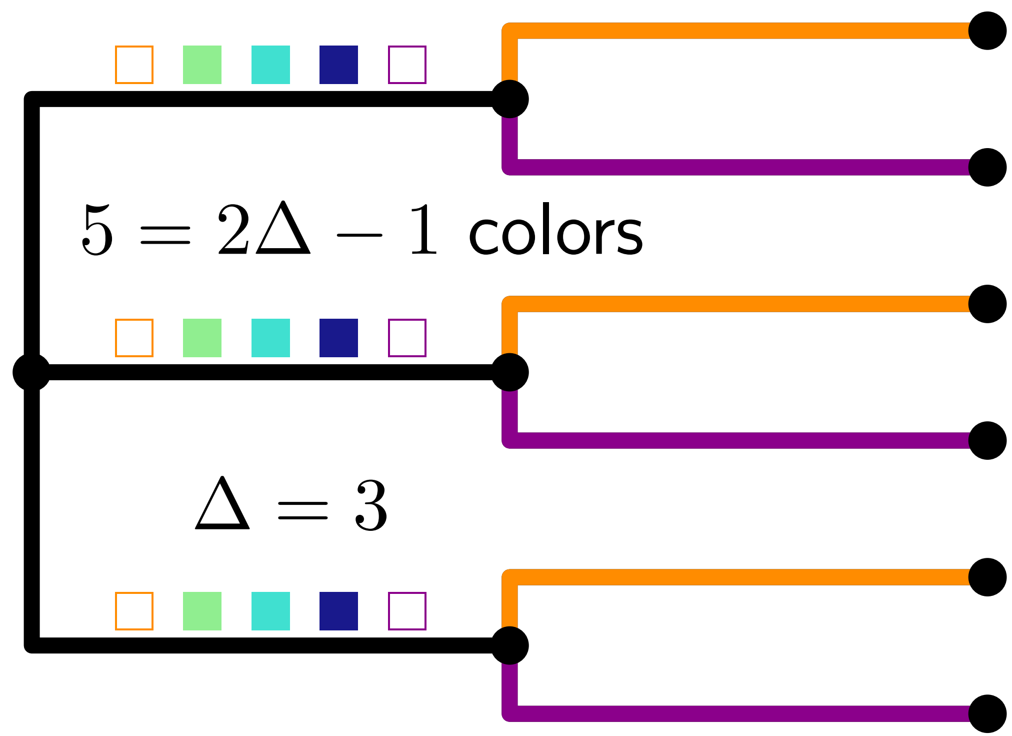

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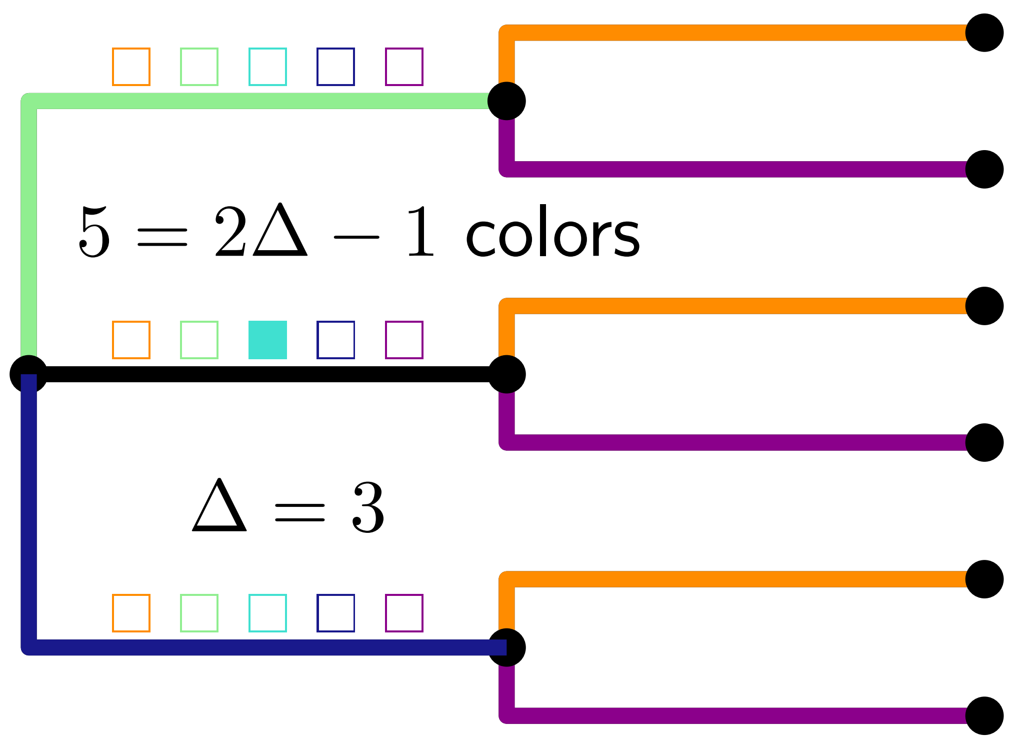

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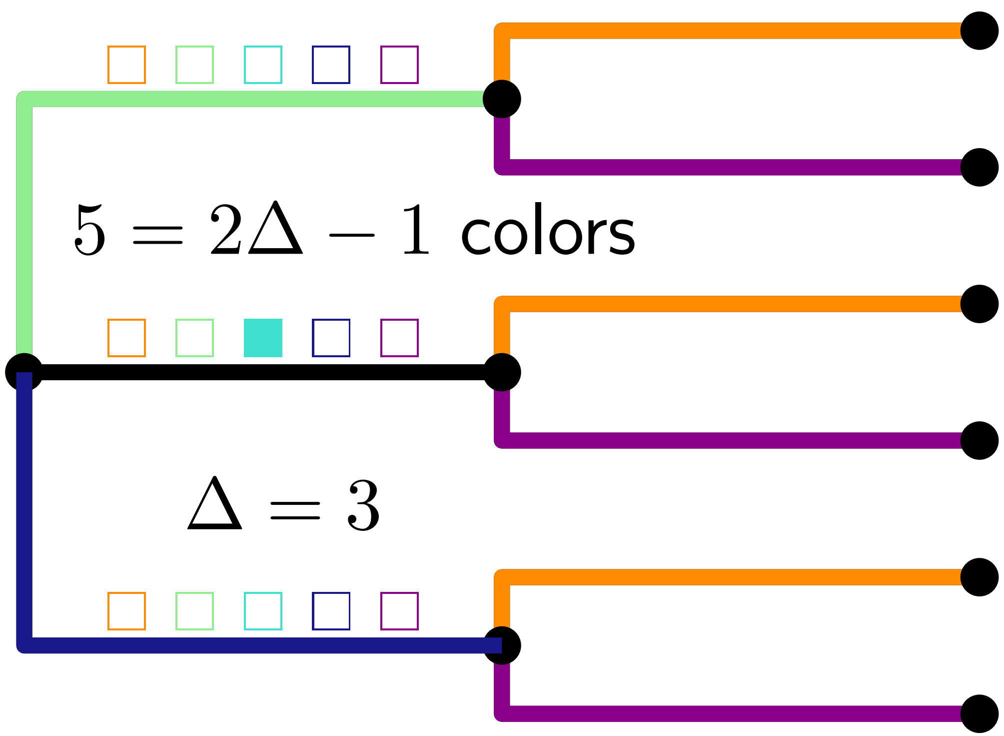
$2\Delta - 1$

EASY

$\mathcal{O}(\log^\star n)$



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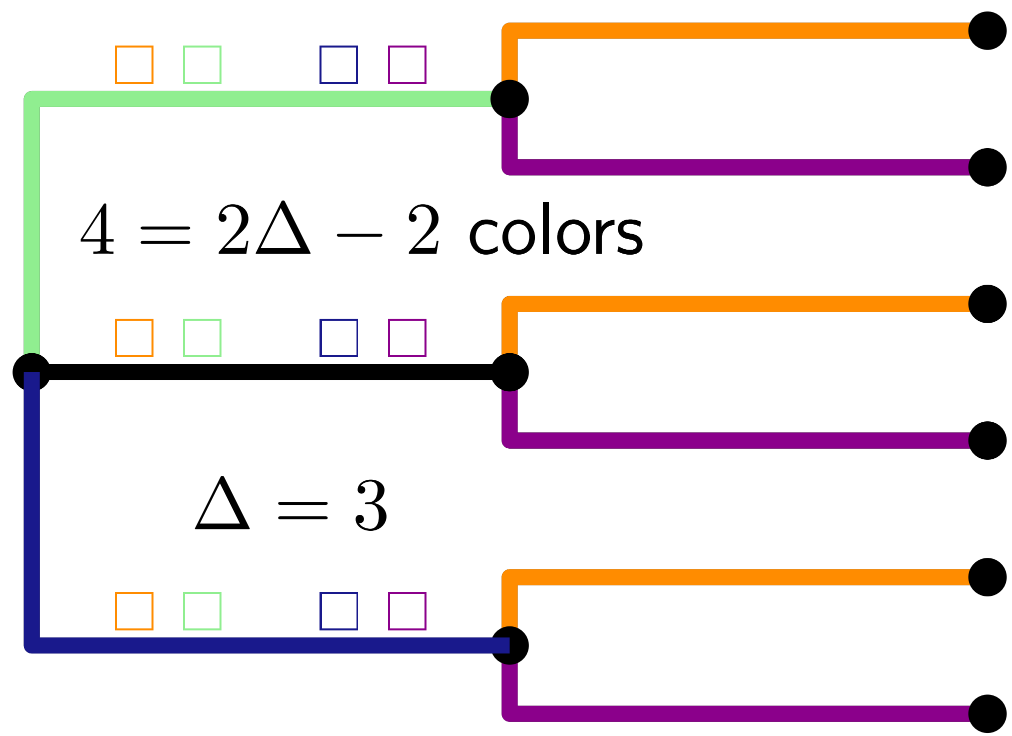

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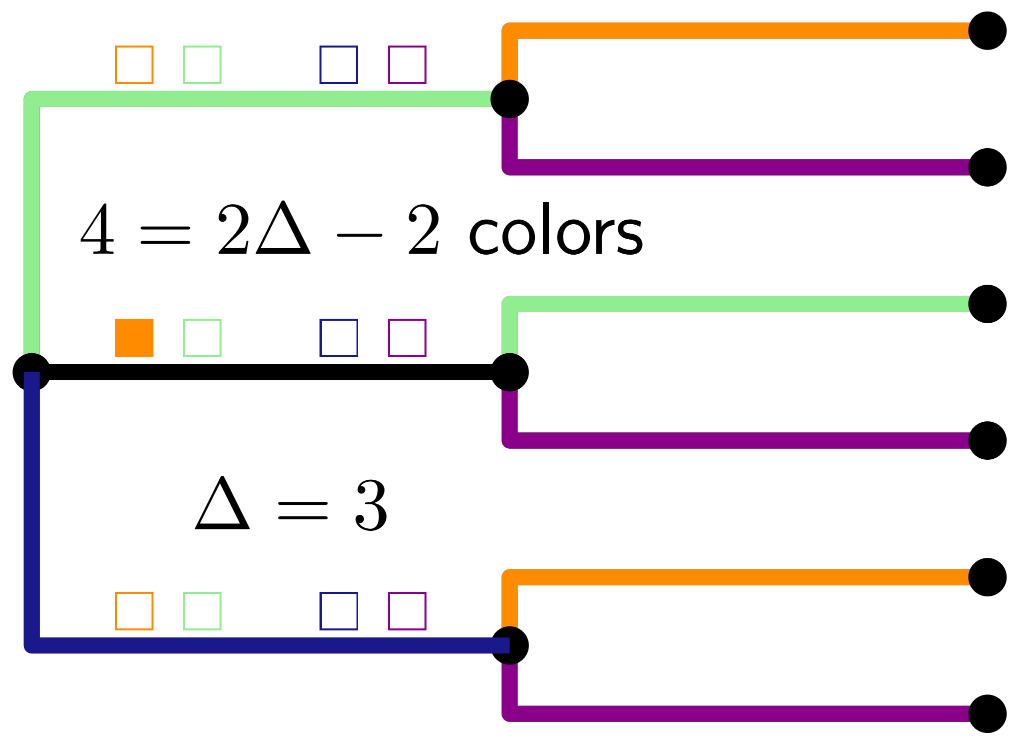

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Technique

Question: When can we extend partial $(2\Delta - 2)$-colorings?

extendable?

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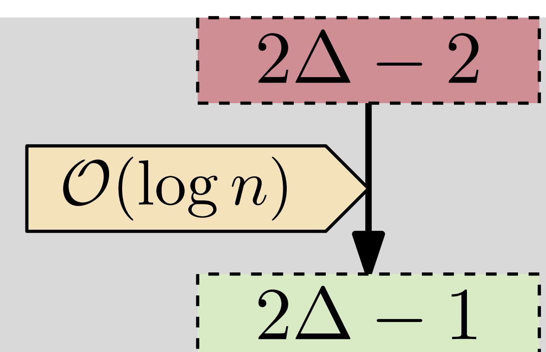
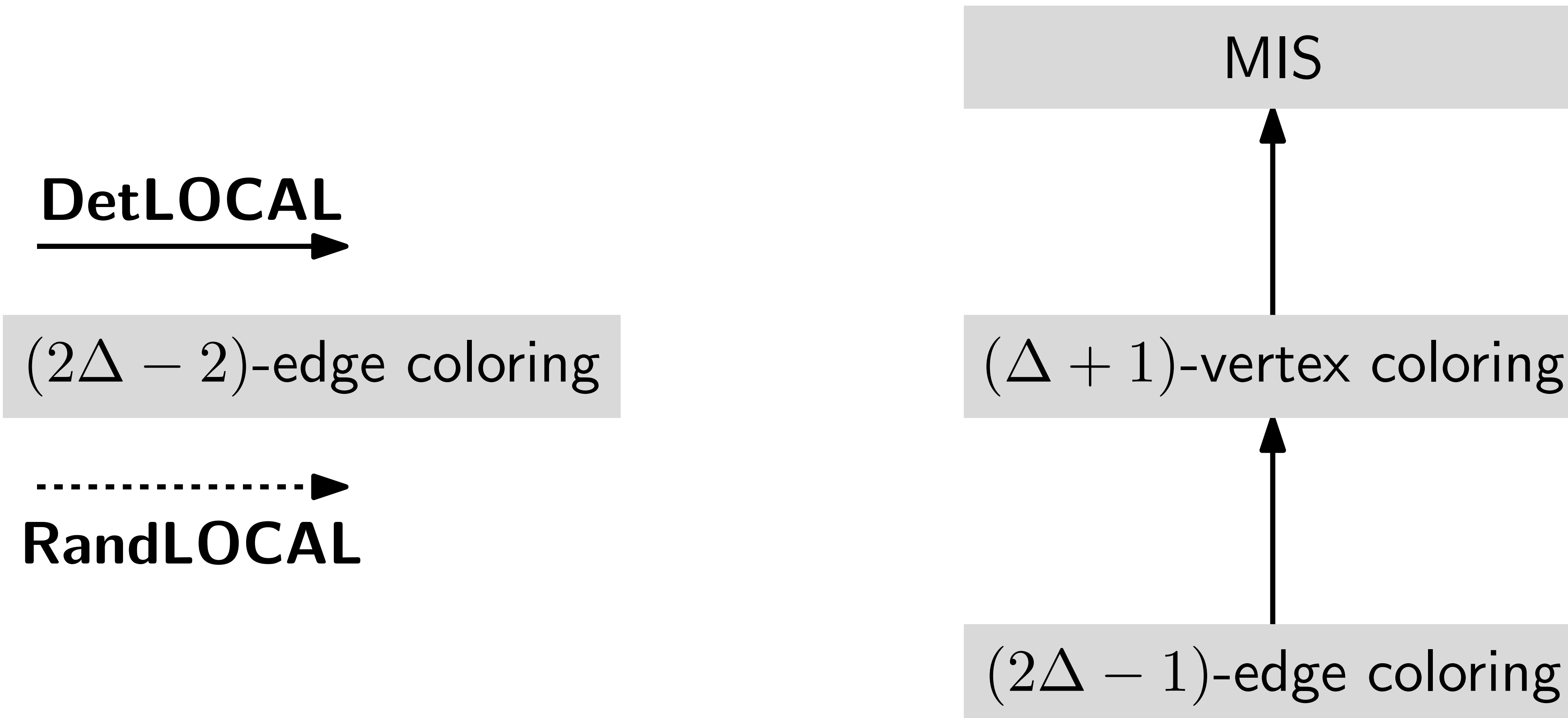
Result

optimal reduction from non-greedy edge coloring to greedy edge coloring



Reductions


MIS
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RandLOCAL
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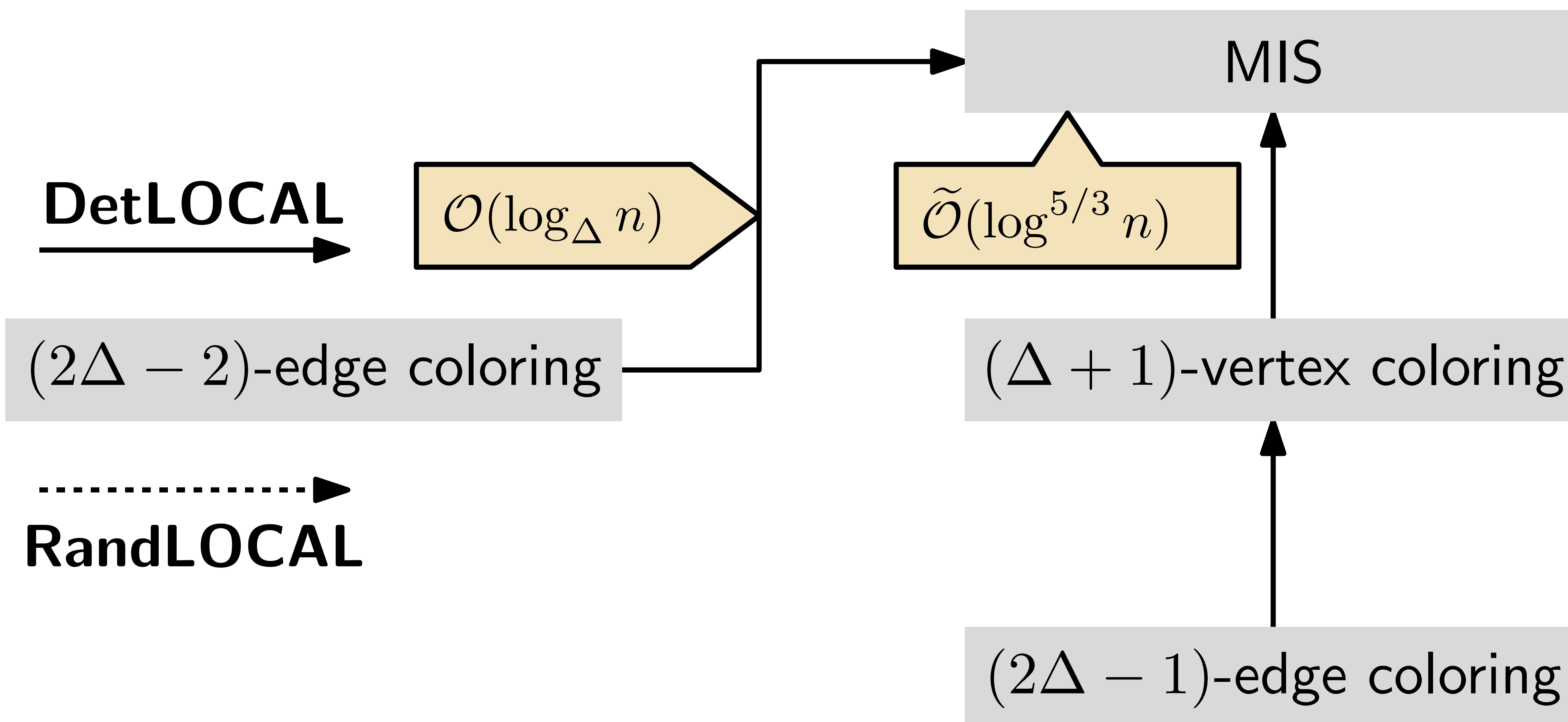
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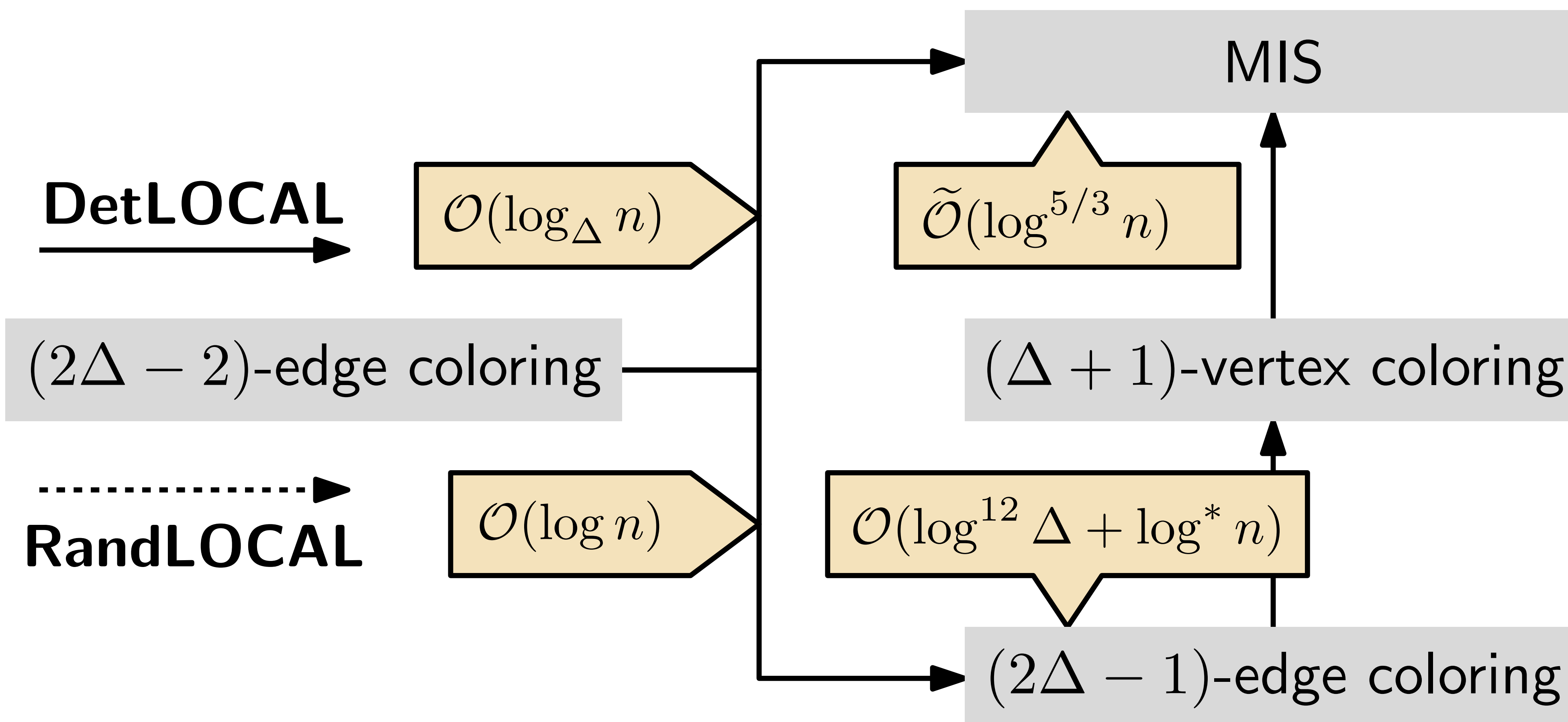
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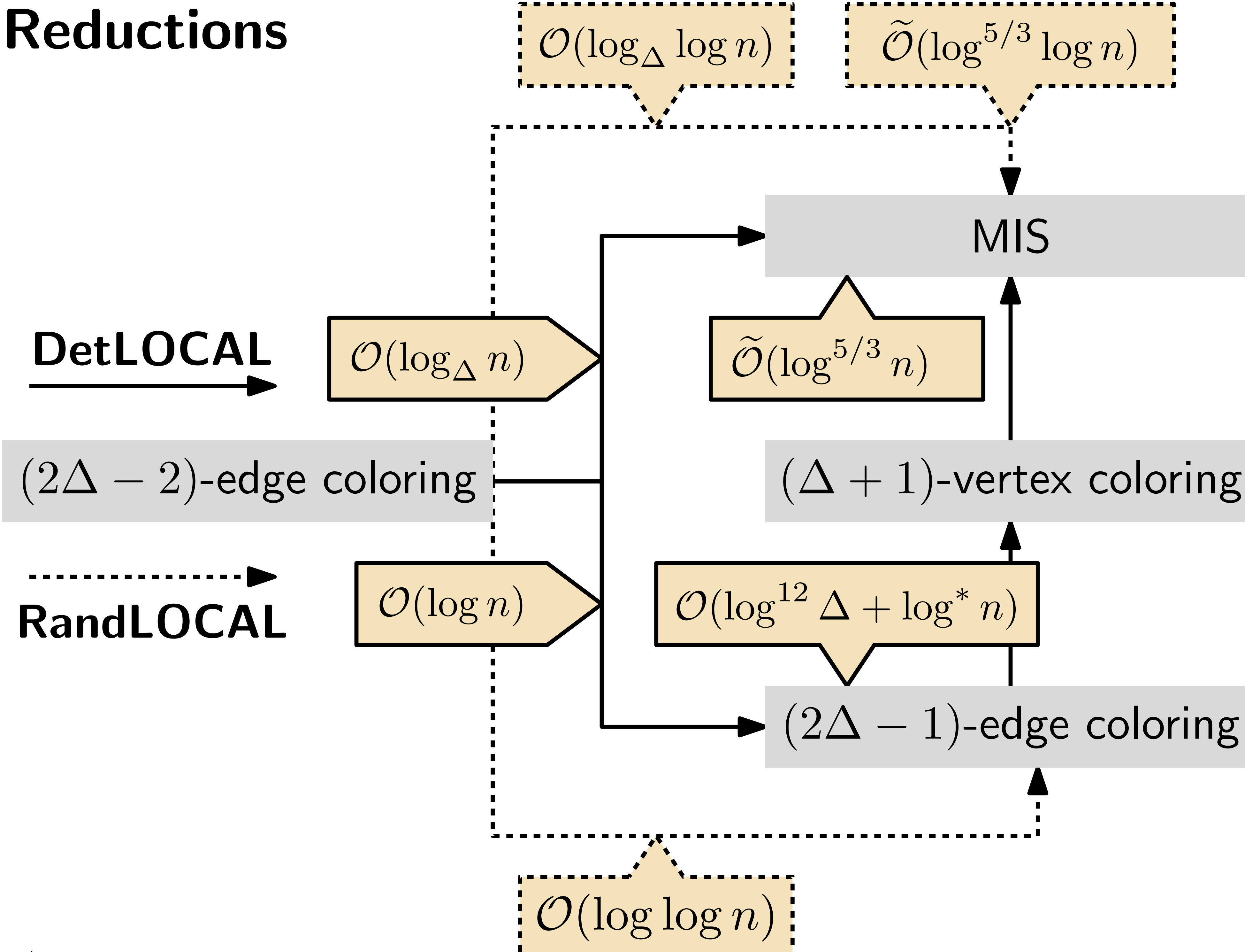

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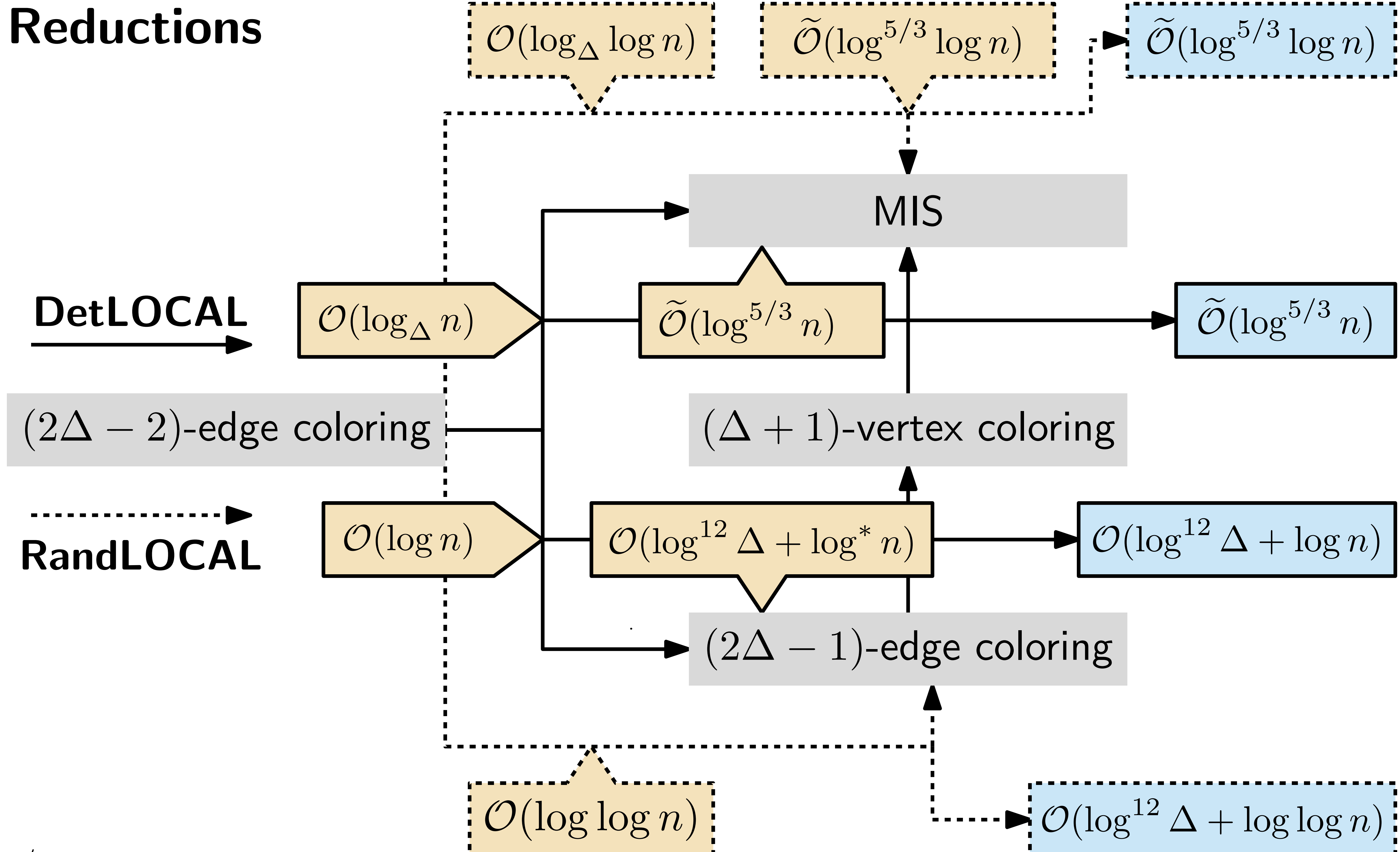

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Runtimes

deterministic | randomized

Δ-regime	NEW	OLD	Sources
general graphs			
$\Delta \leq 2^{\log^{1/12} n}$			
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1: [Brandt, Bourreau & Nolin, STOC'25]

2: [Ghaffari, Kuhn, FOCS'21]

3: [Ghaffari, Grunau, FOCS'24]

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After publication: [Brandt, Bourreau & Nolin, SODA'26]

$\mathcal{O}(\log_\Delta n)$ reduction from Δ-vertex coloring to MIS

Contributions

Motivation

huge difference between greedy and non-greedy problems

$2\Delta - 2$	$2\Delta - 1$
$\Omega(\log n)$	$\mathcal{O}(\log^\star n)$
hard	**easy**

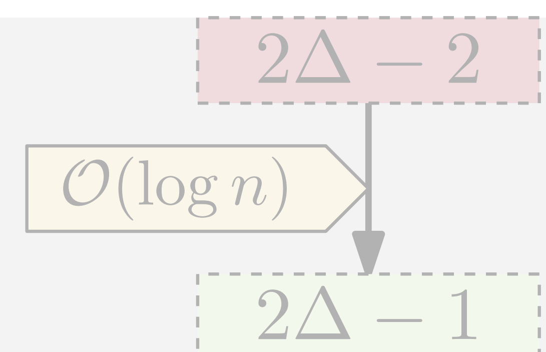
Technique

Question: When can we extend partial $(2\Delta - 2)$-colorings?

extendable?

Result

optimal reduction from greedy edge coloring to non-greedy edge coloring



When can we extend a partial coloring to an uncolored star?

Lemma 1 (reaching for the stars)

- a graph $G = (V, E)$ and a partial $(2\Delta - 2)$-edge coloring φ
- an uncolored star graph $S \subseteq G$

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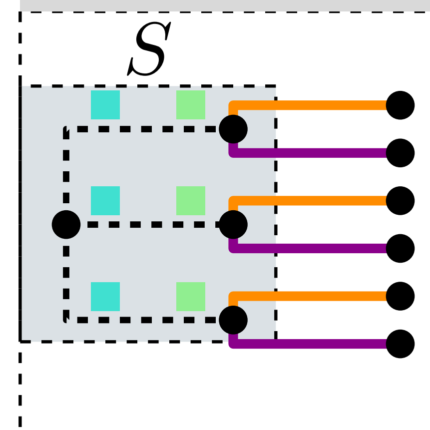

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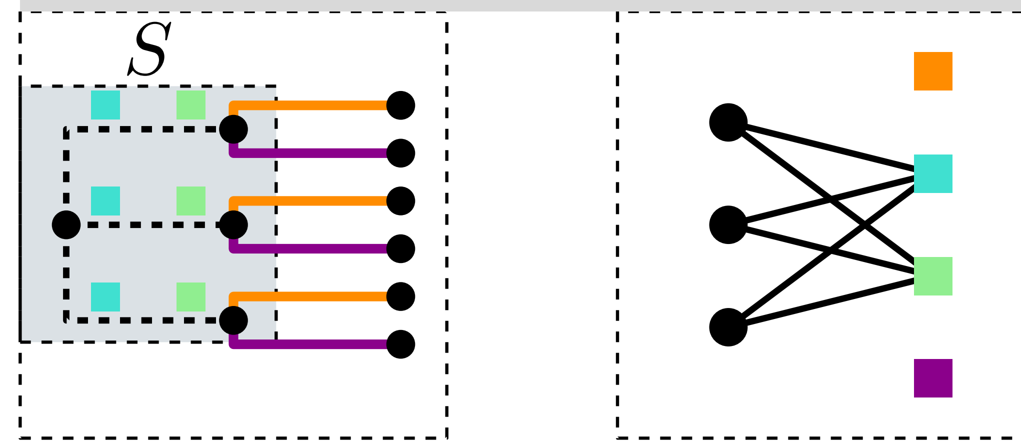

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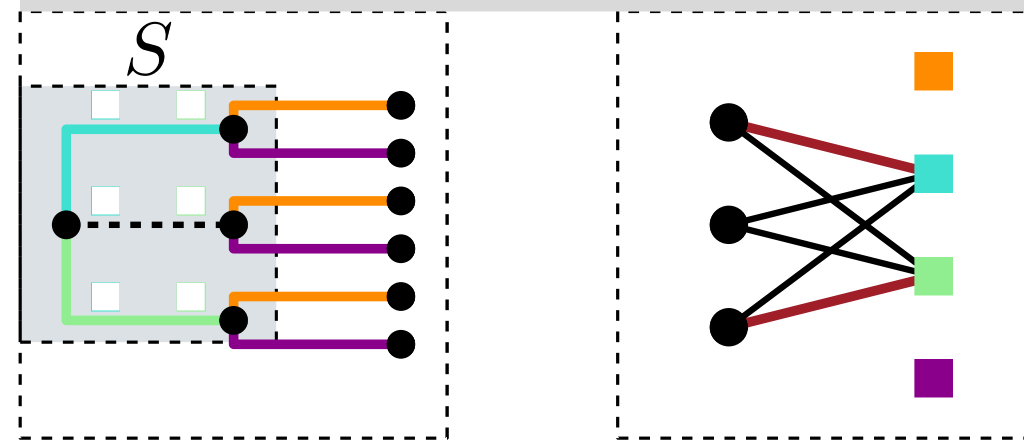

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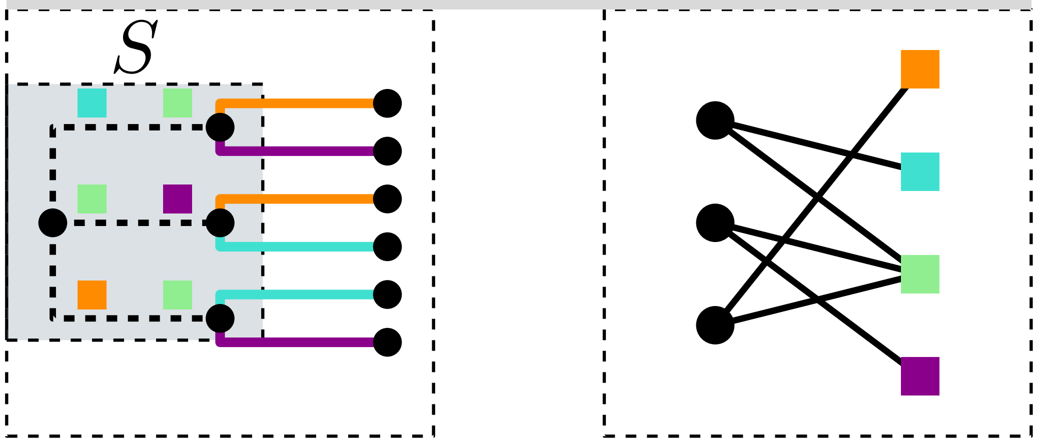

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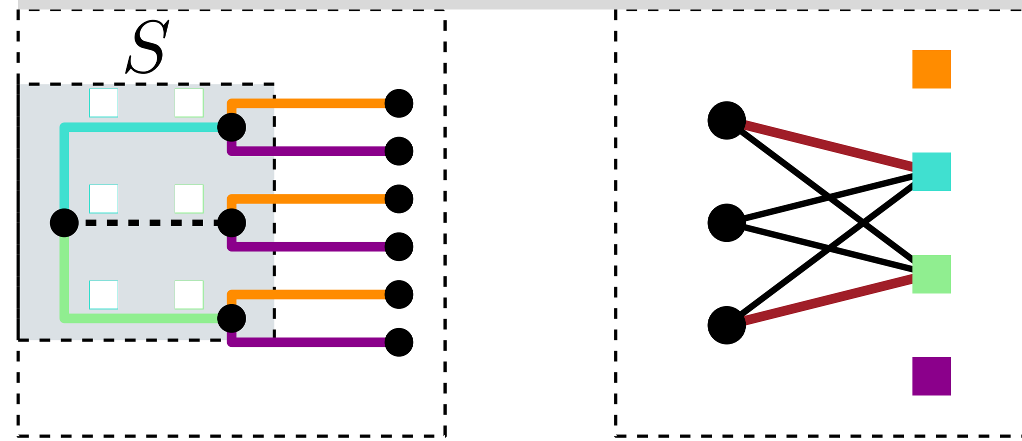

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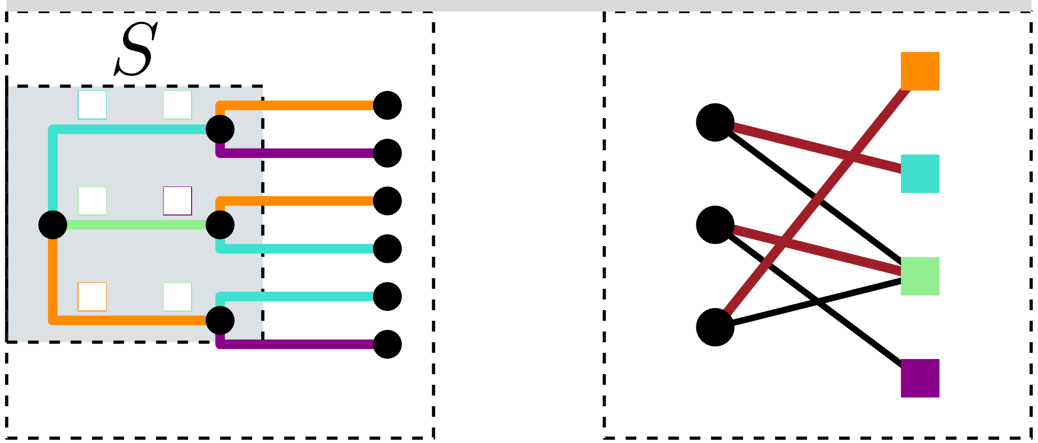

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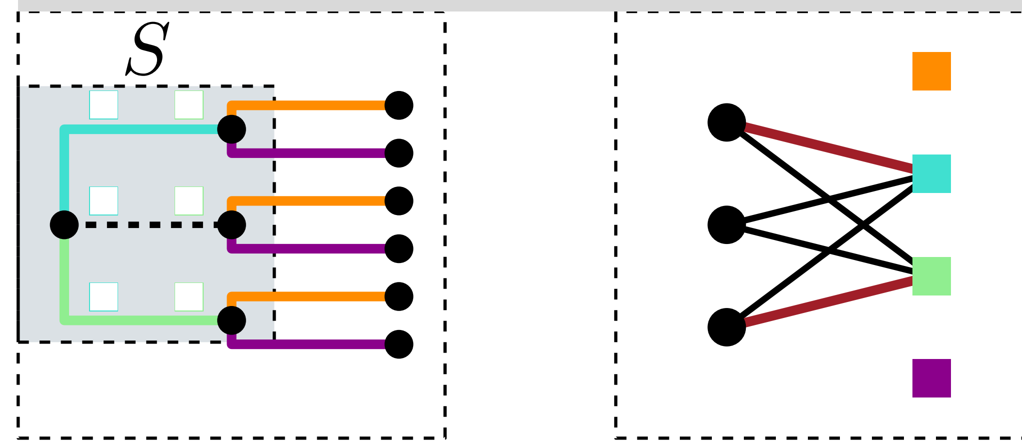

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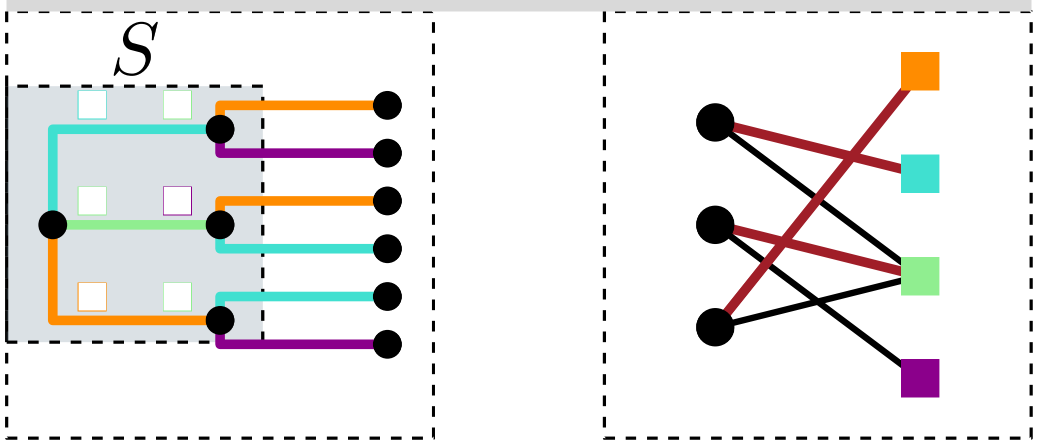

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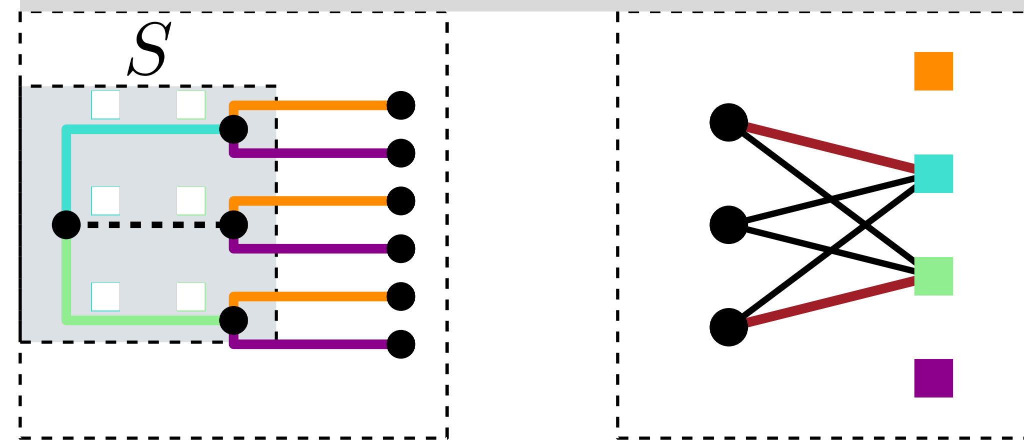

Hall's theorem: $\exists\, U$-saturating matching $\iff$ $|N(S)| \geq |S|$ for all $S \subseteq U$

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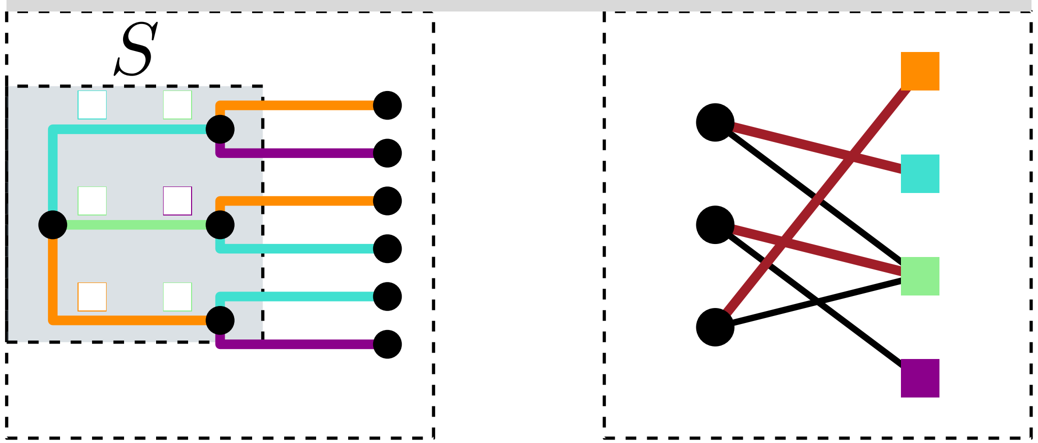

$\varphi(N_E(V_S)) = \{$ ■ , ■ , ■ $\}$

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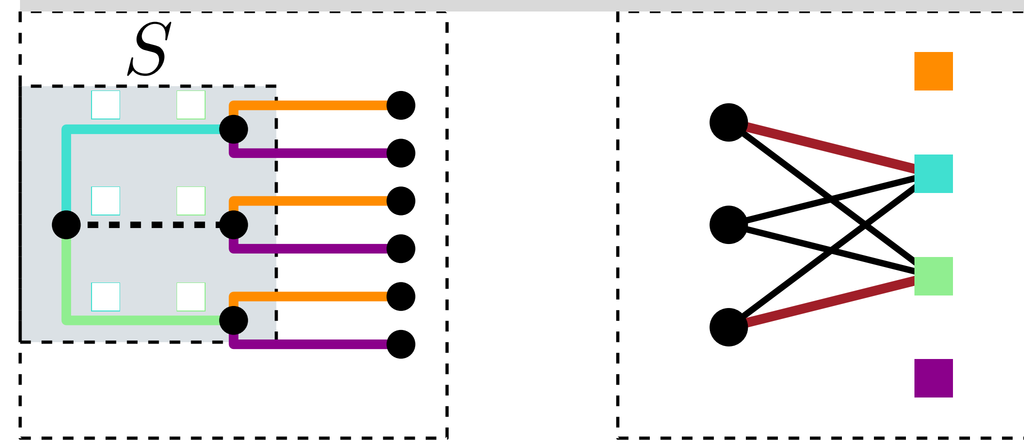

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Lemma 2 (colorful leaves make any tree happy)

- Δ-regular graph G, rooted tree $T \ni r$
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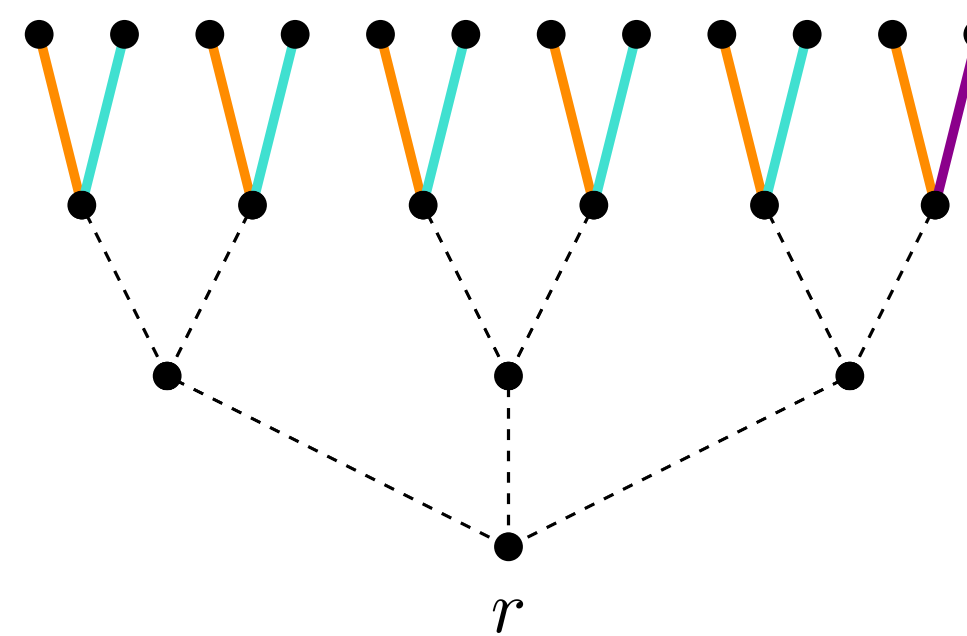

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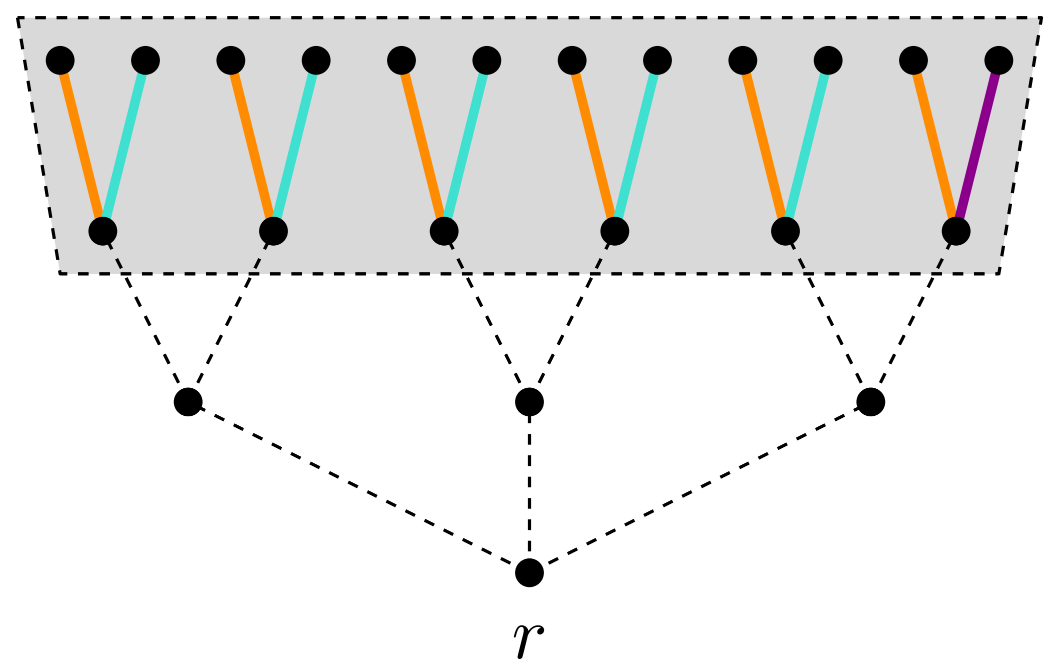
$|\varphi(N_E(V_2))| \geq 3$ ✓



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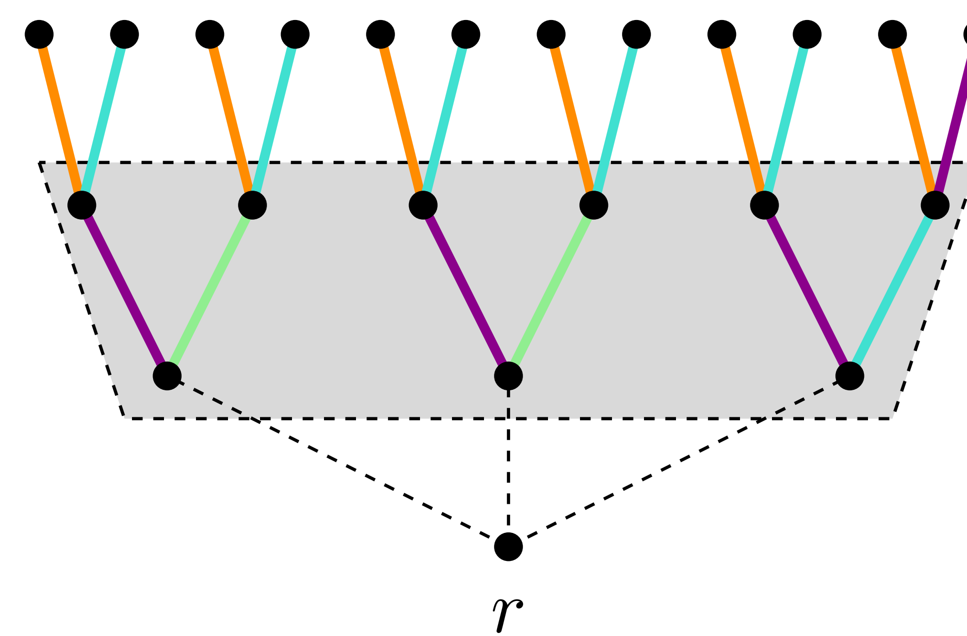

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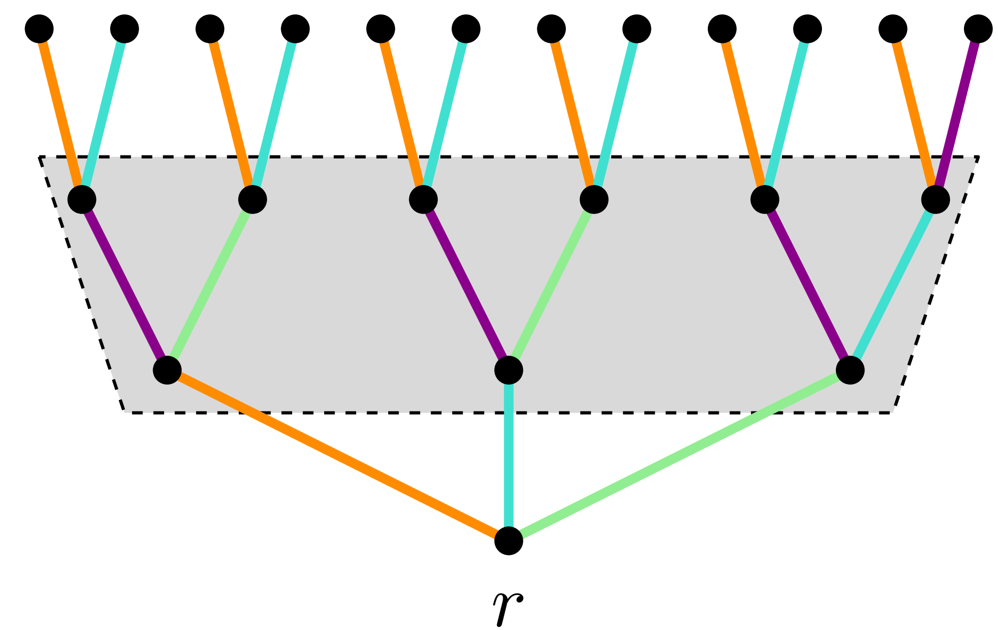
Lemma 1 ↓

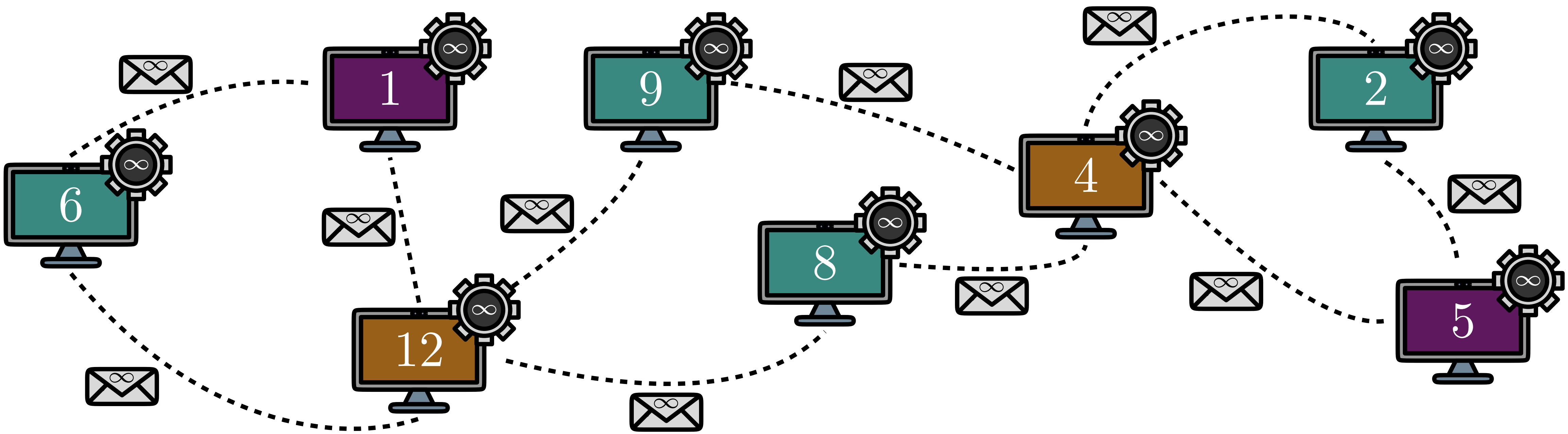
φ is extendable to T ✓



Model of computation

Definition (LOCAL model) **[Linial, 1992]**

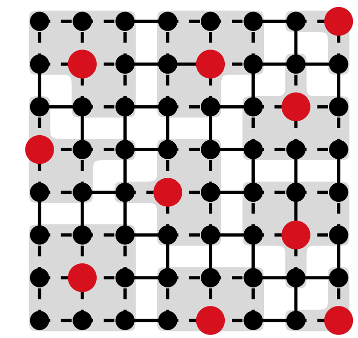
- communication network: **undirected graph**
- nodes have **unique IDs**
- communication happens in **synchronous rounds**
- message size and local computation is unlimited
- time complexity: **# of synchronous rounds**



High level overview of our algorithm

Phase 1: Partition vertices into clusters

- compute MIS on power graph G^k
- every vertex joins the cluster of its closest MIS-node
- compute $(2\Delta - 3)$-edge coloring of intercluster edges



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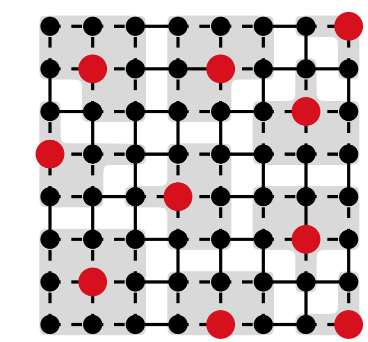
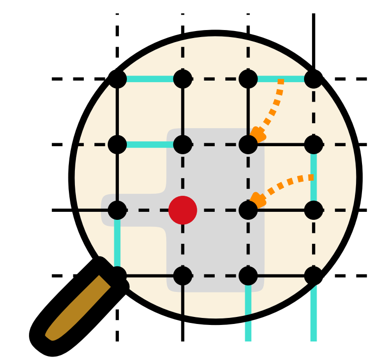


Phase 2: Assign two exclusive edges to each cluster

- compute maximal matching of intercluster edges
- modify the matching via hypergraph sinkless orientation



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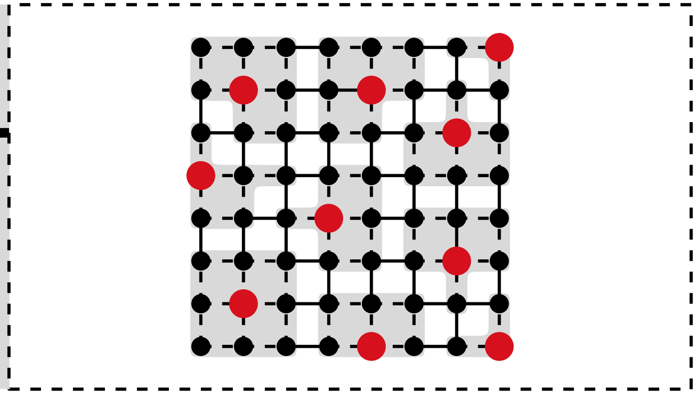

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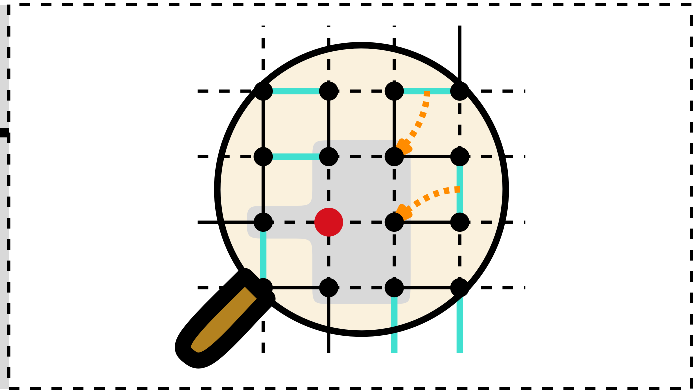
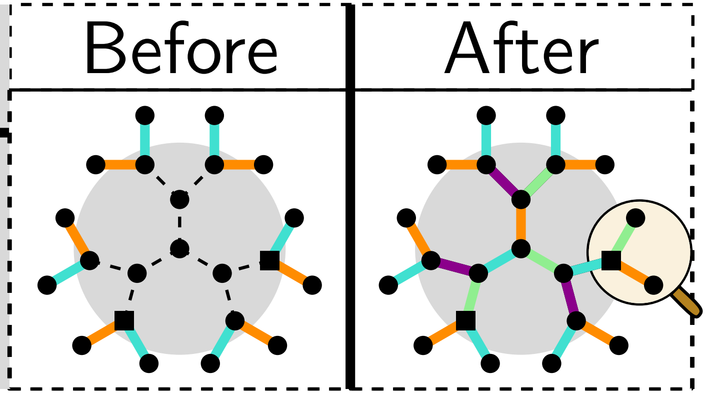


Phase 3: Switch colors and complete the coloring

- adapt colors of assigned edges in order to
- complete the coloring on the intracluster edges



Phase 1: Partition vertices into clusters

Goal: symmetry breaking + every cluster gets sufficiently many vertices

Technique:

- compute a maximal independent set $\mathcal{I}$ on G^8
- every vertex joins the cluster of its closest node in $\mathcal{I}$
- compute greedy edge coloring of intercluster edges

Result:

- every cluster $C \in \mathcal{C}$ has $\mathrm{diam}(C) \leq 8$
- for every $v \in \mathcal{I} : N^4(v) \subseteq C(v)$
- intercluster edges are colored with just $2\Delta - 3$ colors

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Phase 1: Partition vertices into clusters

Goal: symmetry breaking + every cluster gets sufficiently many vertices

Technique:

- compute a maximal independent set $\mathcal{I}$ on G^8
- every vertex joins the cluster of its closest node in $\mathcal{I}$
- compute greedy edge coloring of intercluster edges

Standard techniques

Result:

- every cluster $C \in \mathcal{C}$ has $\mathrm{diam}(C) \leq 8$
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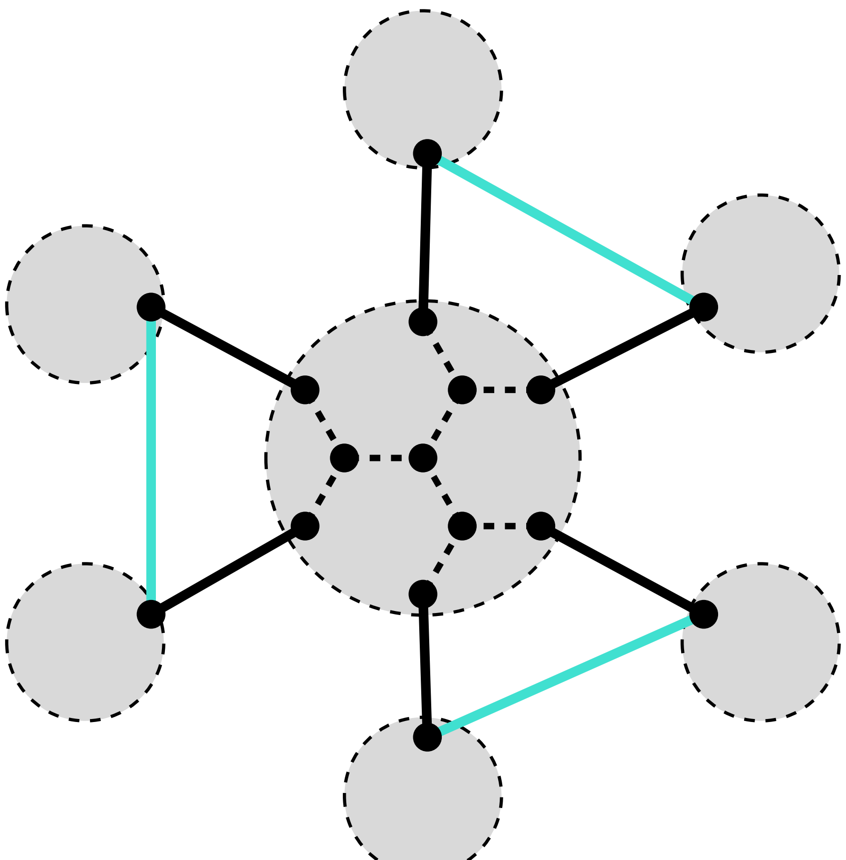

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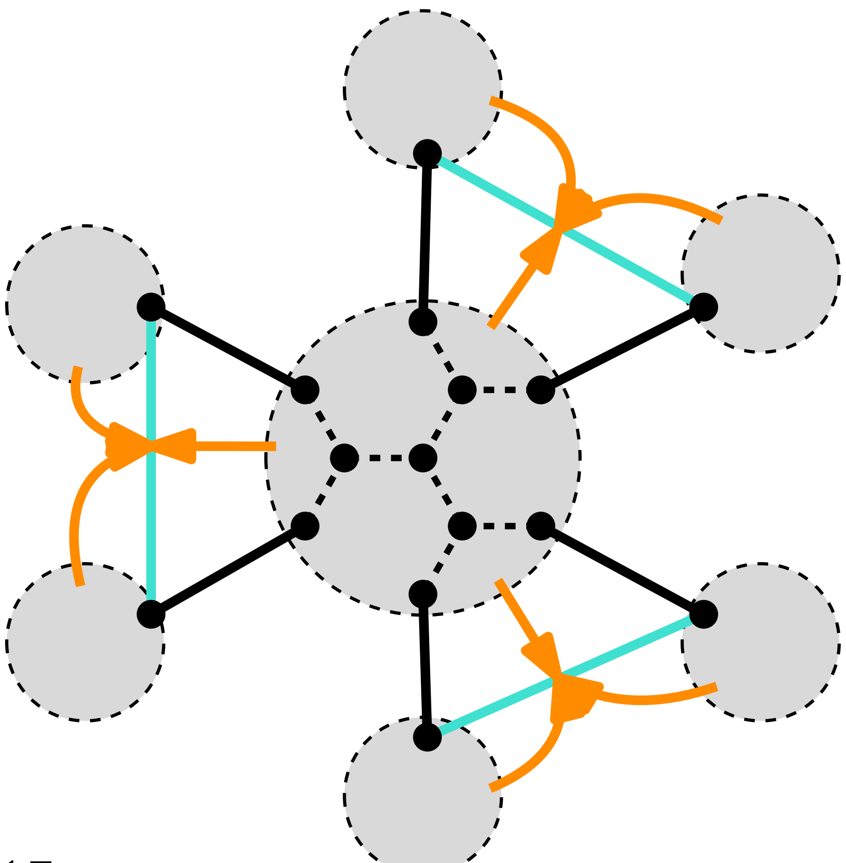

- each cluster sends requests to all matching edges in its 2-hop neighborhood
- each cluster sends at least $\delta := 2\Delta^2$ requests
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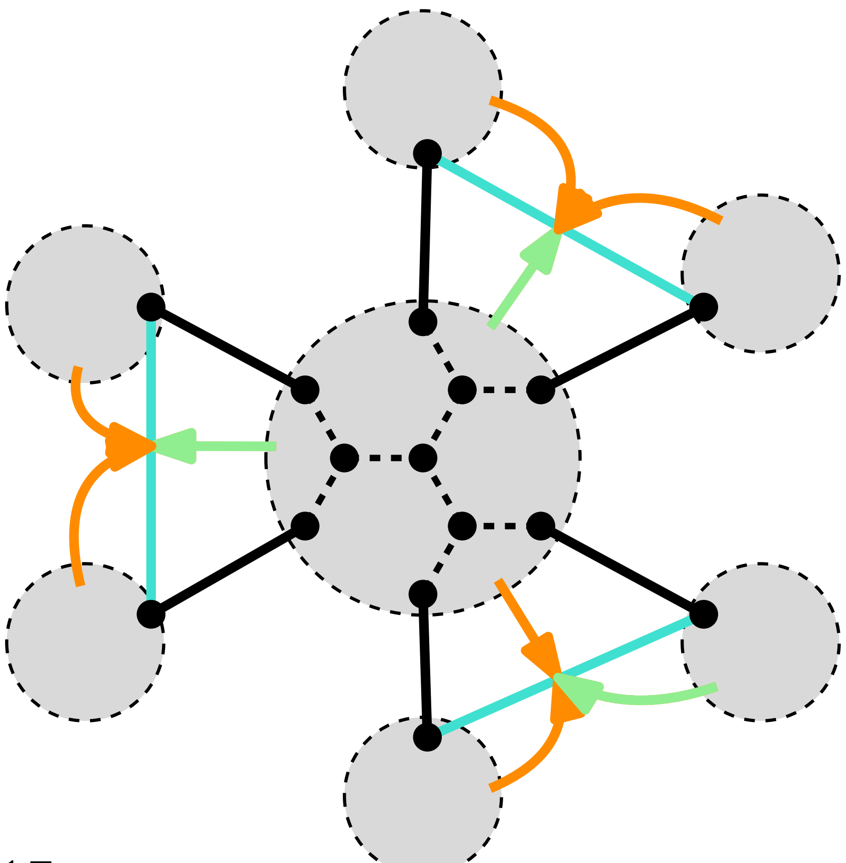

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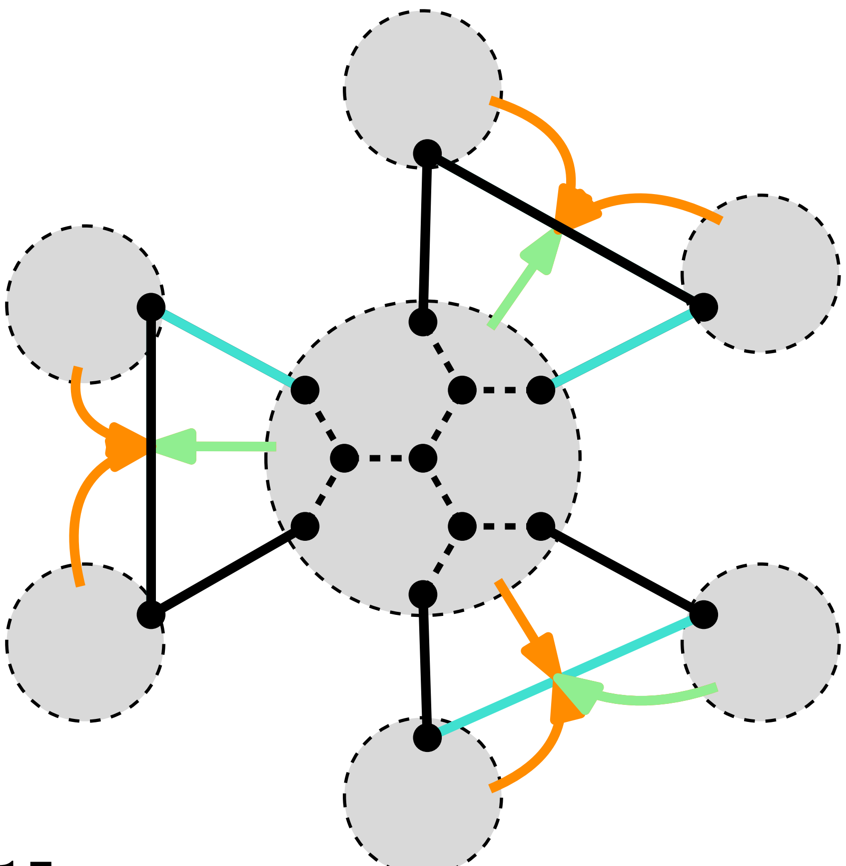

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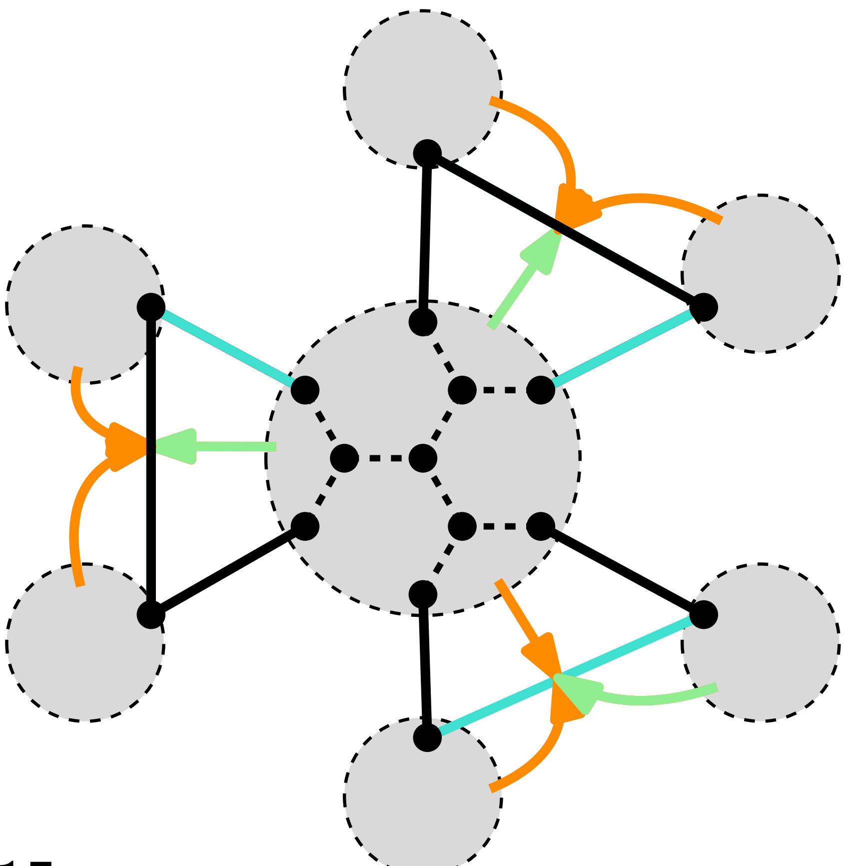

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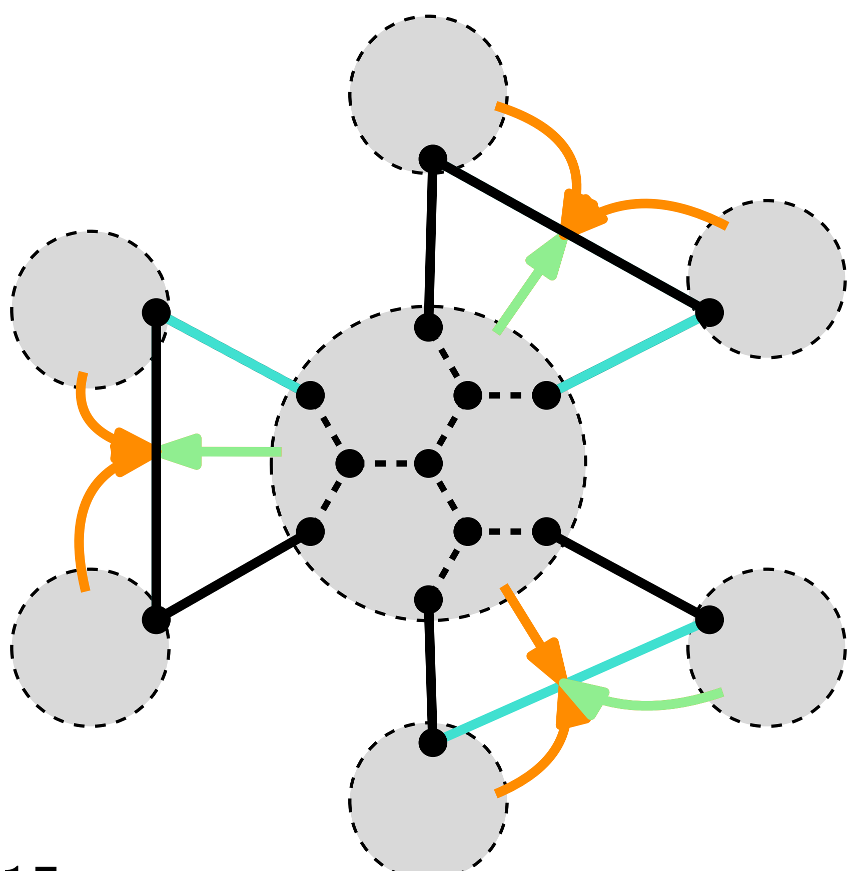

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Runtime:

$$T_{\mathrm{MM}}(n) + T_{\mathrm{HSO}}(n, 2\Delta^2, 2\Delta)$$
$$=$$
$$T_{\mathrm{MIS}}(\Delta \cdot n, 2\Delta - 2) + \mathcal{O}(\log_\Delta n)$$

Phase 3: Switch colors in order to complete the coloring

Goal:	extend the coloring inside the clusters

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- move assigned edges to the immediate neighborhood of the cluster
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Reduction from $2\Delta - 2$-edge coloring to MIS	$T_{\mathrm{MIS}}(n) + \mathcal{O}(\log_{\Delta} n)$

Reduction to greedy edge coloring

Phase 1: Partition vertices into clusters $\mathcal{O}(\log \Delta) + T_{2\Delta-1}(n)$

- ~~compute MIS on power graph G^k~~
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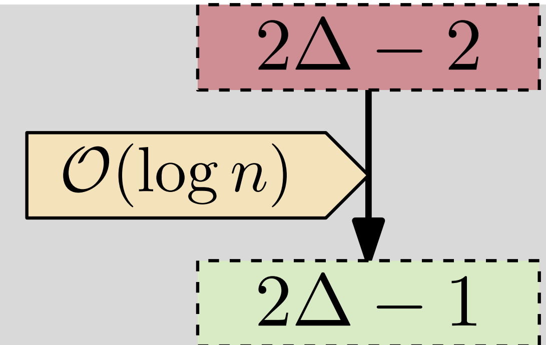
Reduction from $2\Delta - 2$ to $2\Delta - 1$-edge coloring	$T_{2\Delta-1}(n) + \mathcal{O}(\log n)$

Wrapping up

Result

optimal reduction from greedy edge coloring to non-greedy edge coloring



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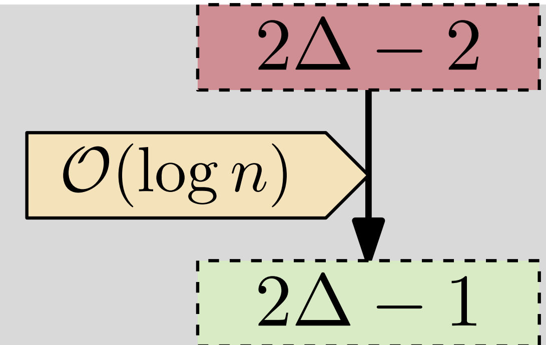
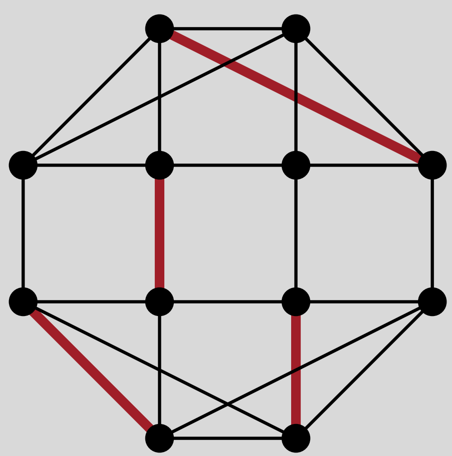


Open problem

Given

4-regular graph $G = (V, E)$



matching $M \subseteq E$

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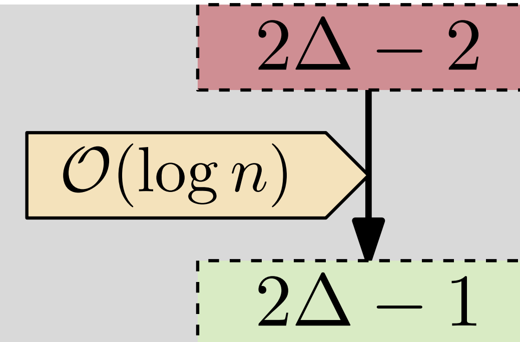
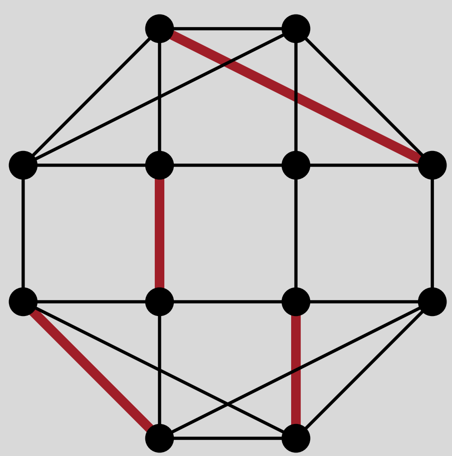


Open problem

Given	**Goal**
4-regular graph $G = (V, E)$	find small-diameter subgraph H



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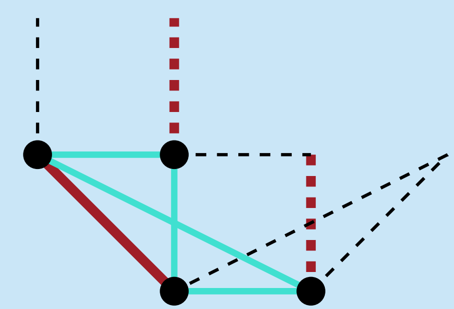

such that H can still be properly 5-colored at the end