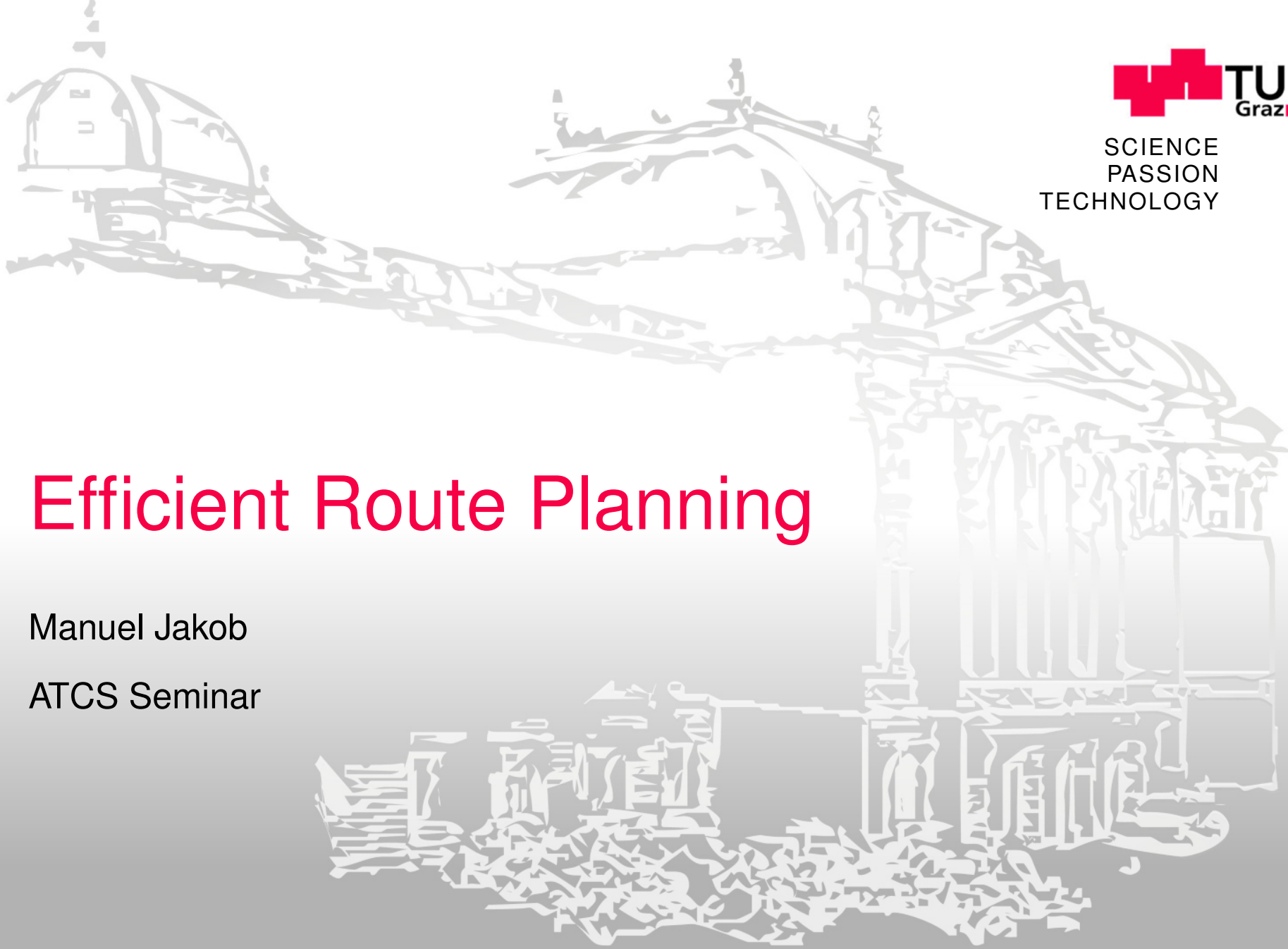


SCIENCE
PASSION
TECHNOLOGY

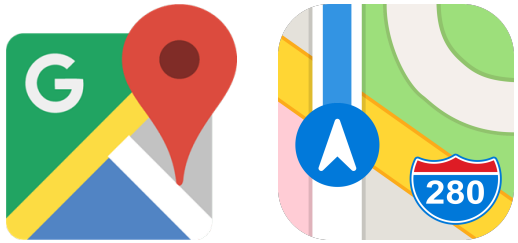
Efficient Route Planning

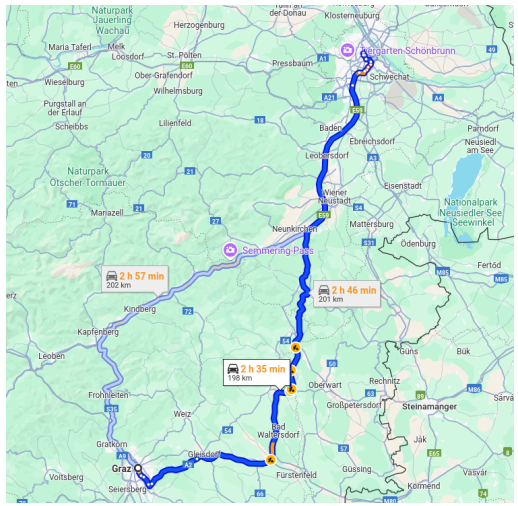
Manuel Jakob

ATCS Seminar

Motivation

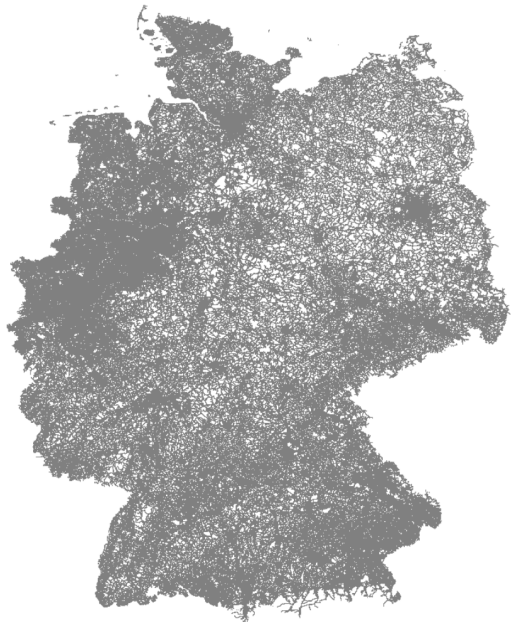
- real-time navigation
- big data
- new algorithms





Road Network

Graph $G = (V, E, \omega)$, $\omega : E \to R_+$
$|V| = n$, $|E| = m$



$|V| \approx 11.5M$
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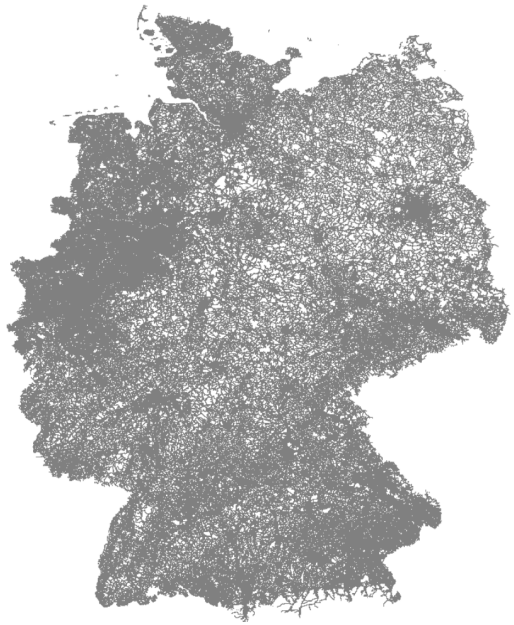
Characteristics:

- almost planar
- constant max. degree
- sparse
- strongly connected
- hierarchical structure
- static



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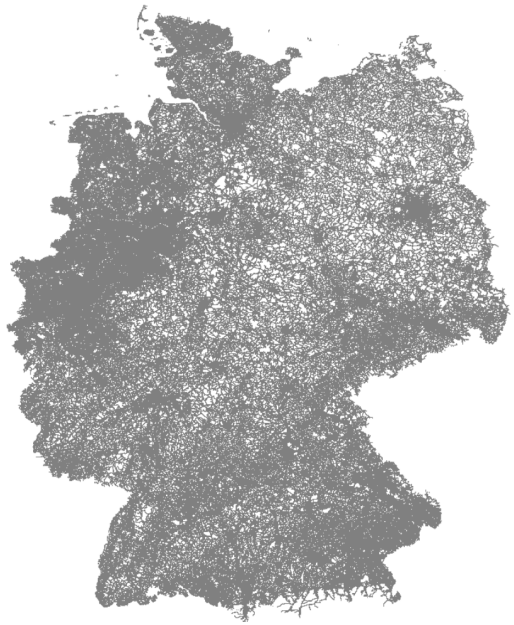


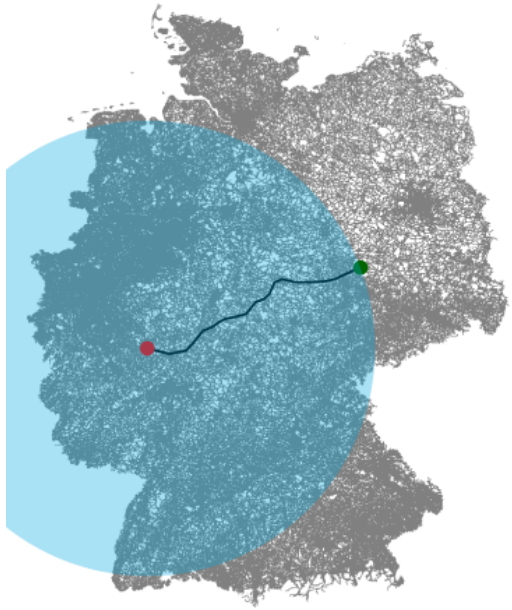
$|V| \approx 11.5M$
$|E| \approx 12.4M$

GOAL: find *shortest s-t* path (P2P)

Dijkstra's Algorithm [Dij59]

RUNTIME:

Search Space contains all vertices $v \in V$ with $\omega_s(v) < \omega_s(t)$



Dijkstra's Algorithm [Dij59]

RUNTIME: $\mathcal{O}(m + n \log n)$

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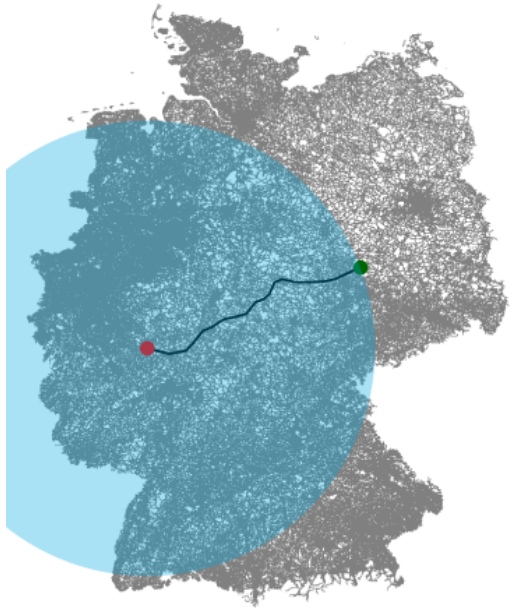

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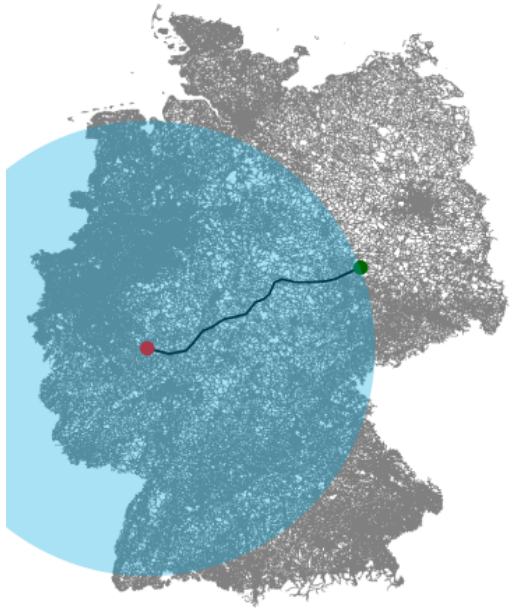


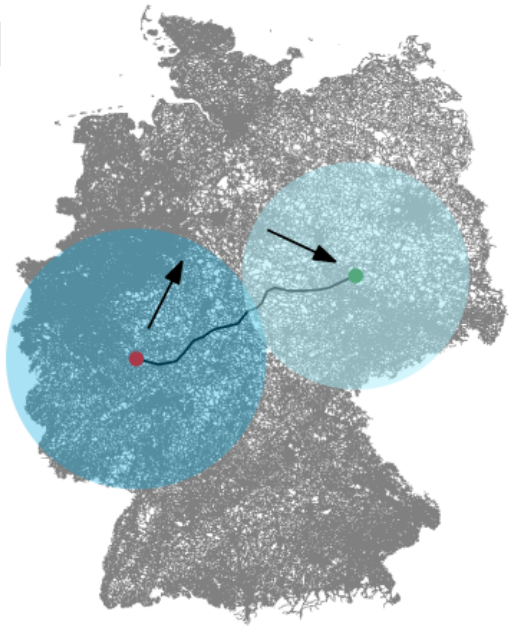
Running Dijkstra on country sized networks takes seconds!

Bidirectional Dijkstra's Algorithm

IDEA: perform the search from start and target simultaneously

Find $v \in V$ with

$$\min_{v \in V} \omega_s^{\rightarrow}(v) + \omega_t^{\leftarrow}(v)$$

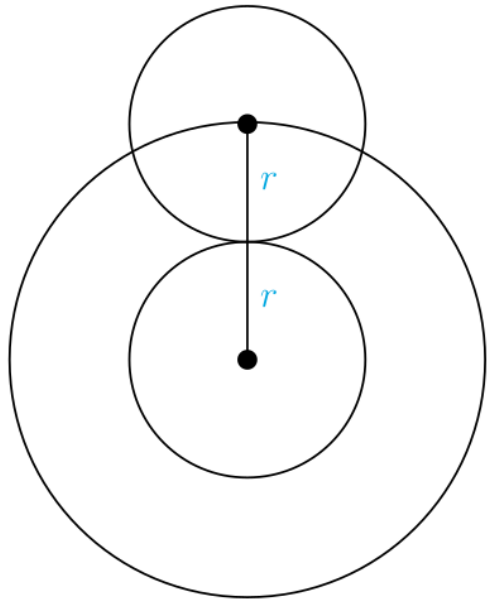


Bidirectional Dijkstra's Algorithm

Improvement:

Dijkstra: $\pi \cdot (2r)^2 = 4\pi r^2$
Bi-Dijkstra: $2 \cdot \pi r^2$

Decrease by a factor of 2



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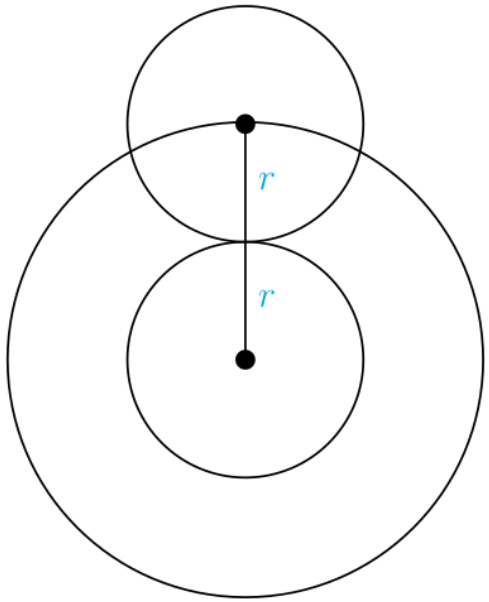


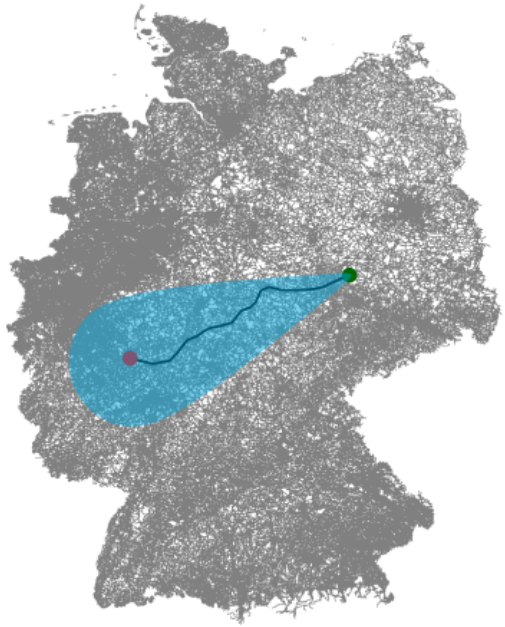
Search space still too large!

A* Algorithm

IDEA: guide search towards the target

Let $h(v)$ be an heuristic, estimates the costs from v to the target

Modify Dijkstra by setting
$\omega_s(v) + h(v)$ as key in PQ



A* Algorithm

A heuristic $h : E \rightarrow \mathcal{R}$ is...

A* Algorithm

A heuristic $h : E \to \mathcal{R}$ is...

admissible
$h(v)$ never overestimates

A* Algorithm

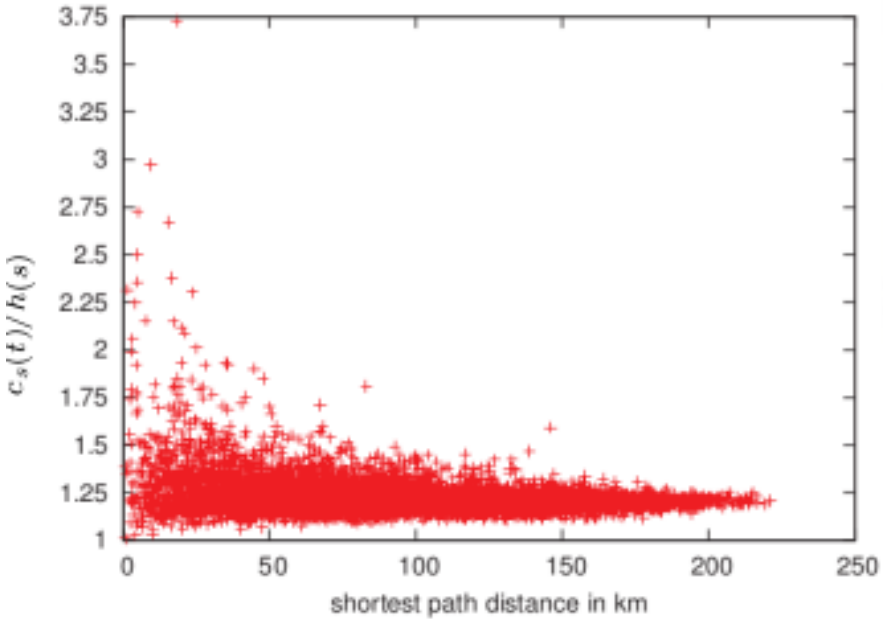
A heuristic $h : E \to \mathcal{R}$ is...

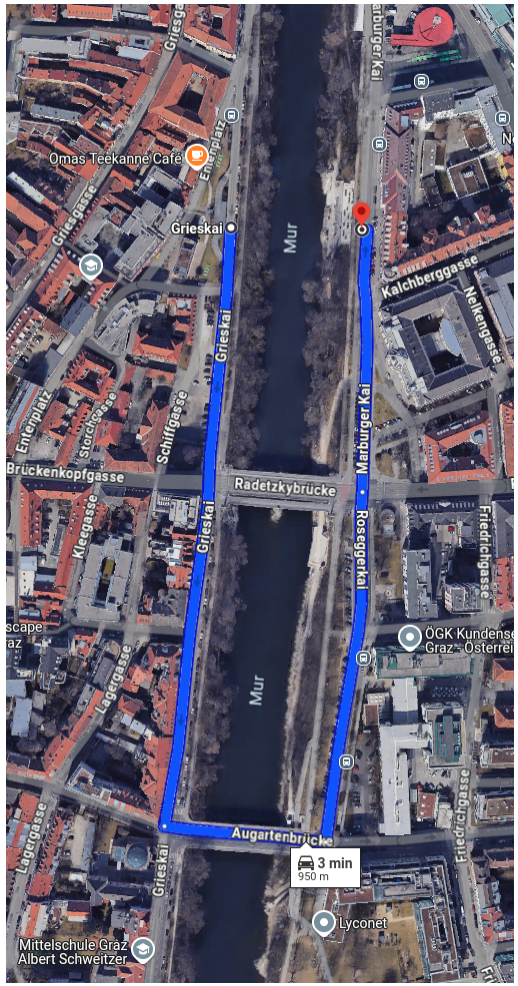
admissible
$h(v)$ never overestimates

consistent
$\forall e = (v, w) \in E : h(v) \leq \omega(e) + h(w)$

A* Algorithm

Example: bee line distance [FSS15]





A* Algorithm

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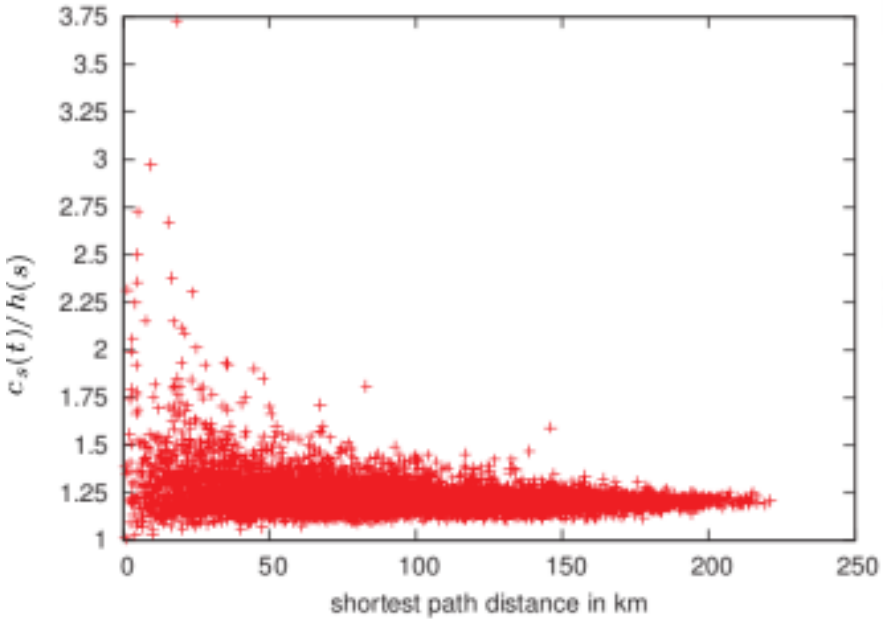

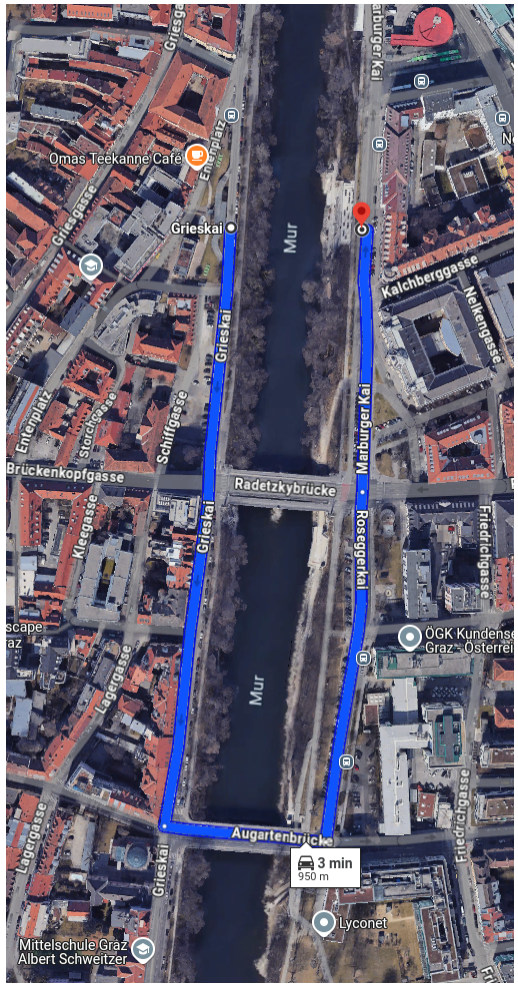

Search space still too large!

How to speed up computation?

PREPROCESSING

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1. Compute additional information in advance once

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2. Use data to reduce search space during query answering

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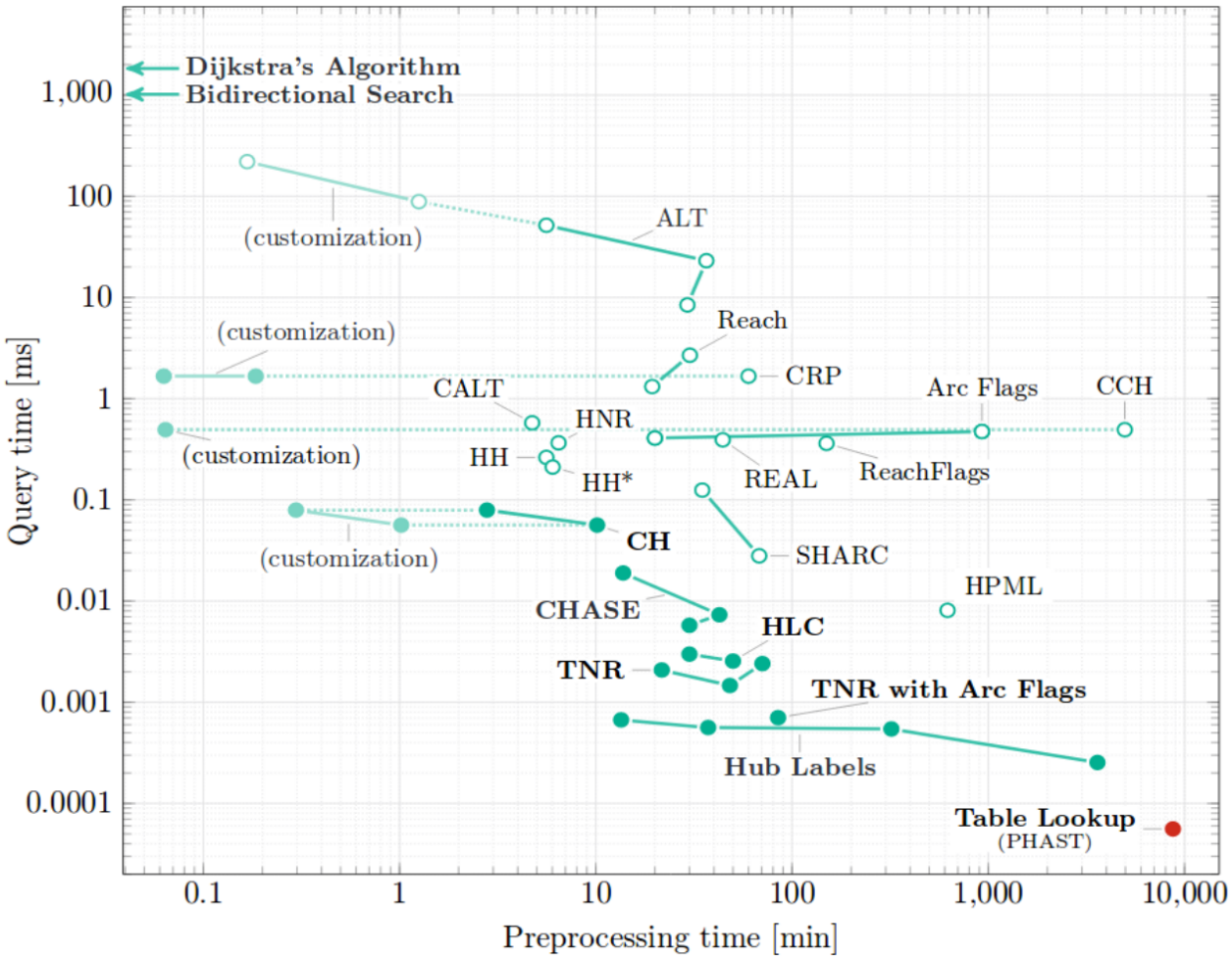
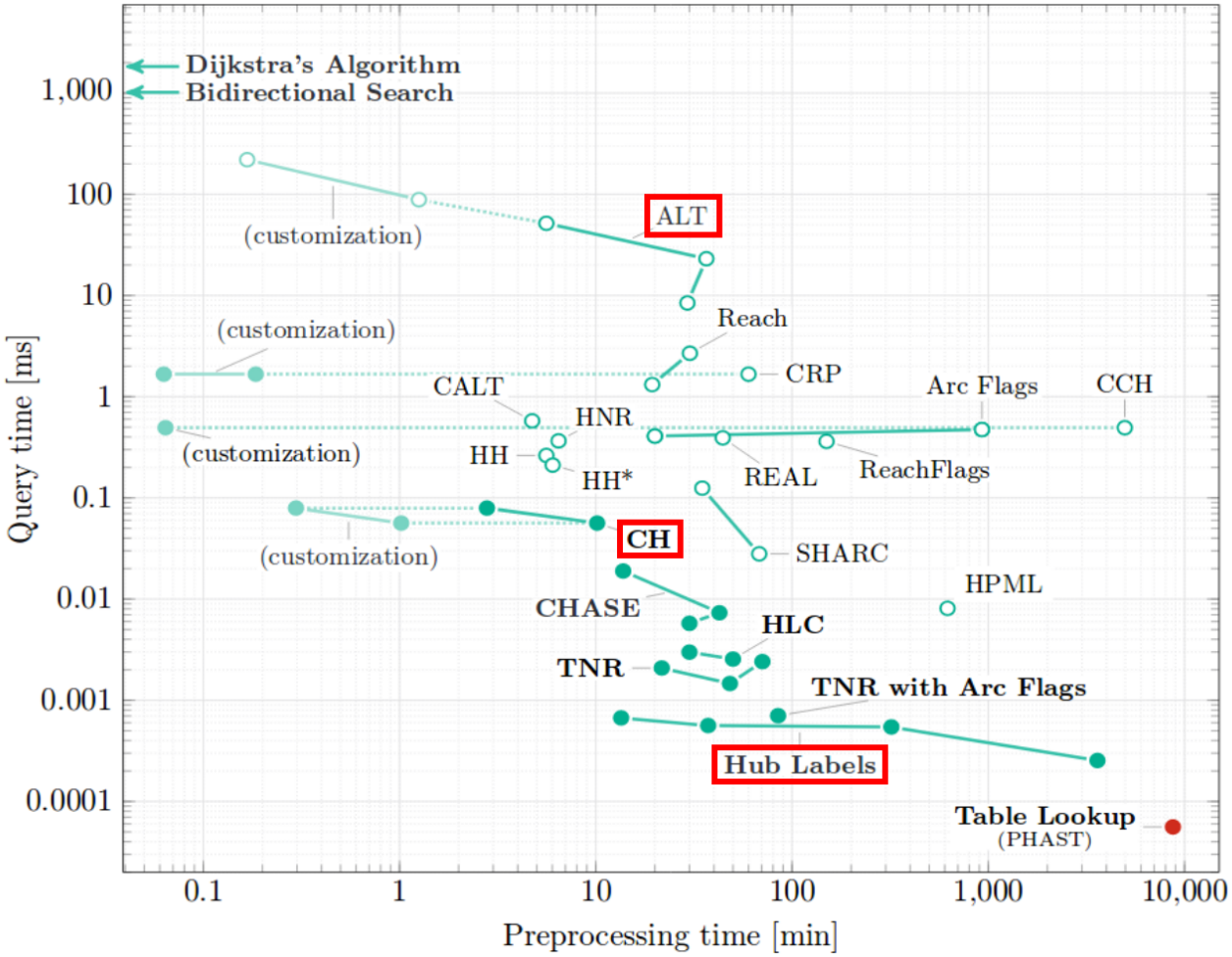
Caution: preprocessing takes time and memory!

Big Picture


Dijkstra's Algorithm
Bidirectional Search
1,000
100
10
1
0.1
0.01
0.001
0.0001
Query time [ms]
(customization)
ALT
Reach
(customization)
CRP
CALT
Arc Flags
CCH
HNR
(customization)
HH
HH*
REAL
ReachFlags
CH
(customization)
SHARC
HPML
CHASE
HLC
TNR
TNR with Arc Flags
Hub Labels
Table Lookup
(PHAST)
0.1
1
10
100
1,000
10,000
Preprocessing time [min]

Big Picture


Dijkstra's Algorithm
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(customization)
ALT
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CCH
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HH
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REAL
ReachFlags
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CH
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0.1
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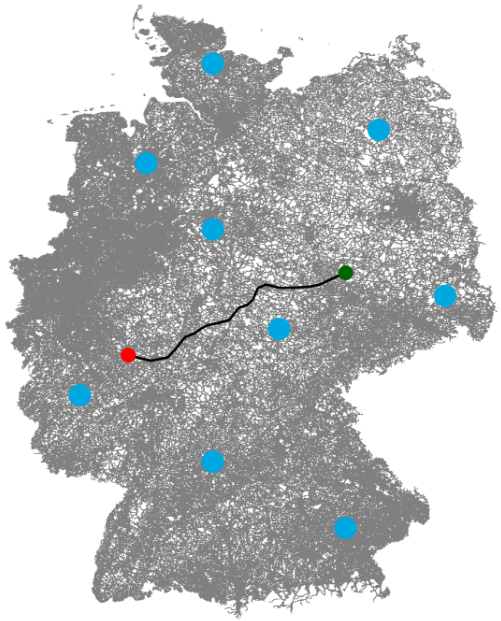
10
100
1,000
10,000
Preprocessing time [min]

A* + **L**andmarks + **T**riangle inequality [GH05]

Preprocessing:

1. select set $L \subseteq V$ of landmarks
2. $\forall l \in L$: run Dijkstra and store costs to all other vertices



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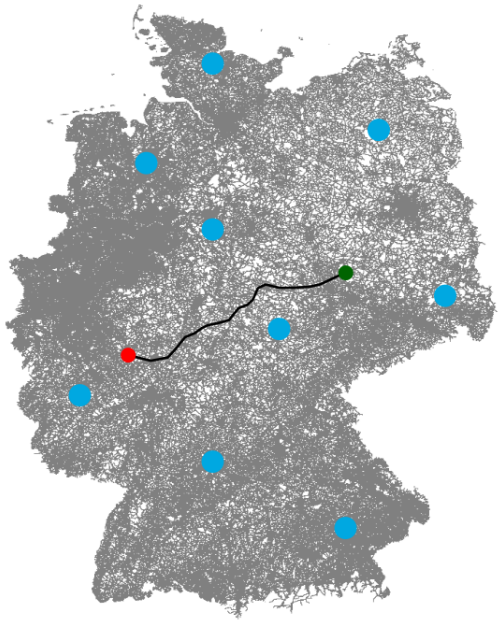
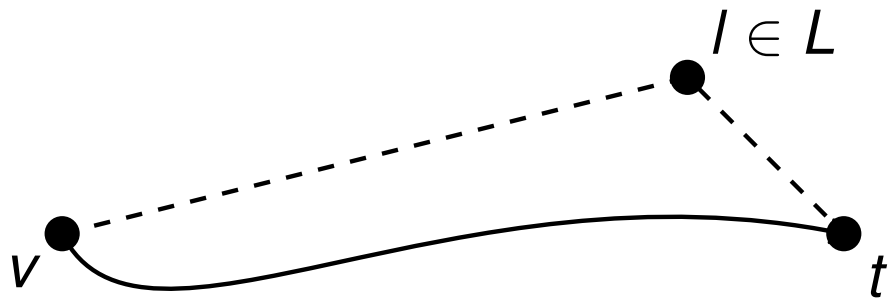


Heuristic:



$$\omega_v(l) \leq \omega_v(t) + \omega_t(l) \Leftrightarrow$$

$$\omega_v(l) - \omega_t(l) \leq \omega_v(t)$$

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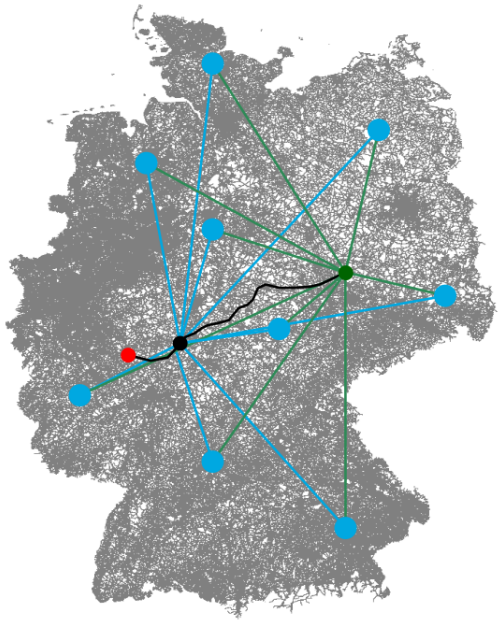

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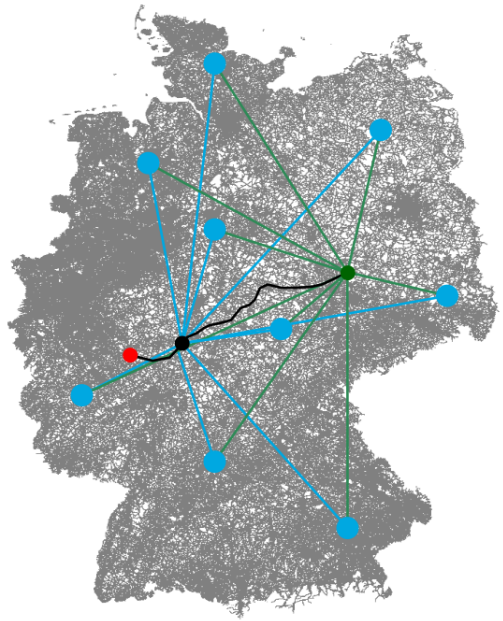
$h(v) = \max\{\omega_v(l) - \omega_t(l) \mid l \in L\}$



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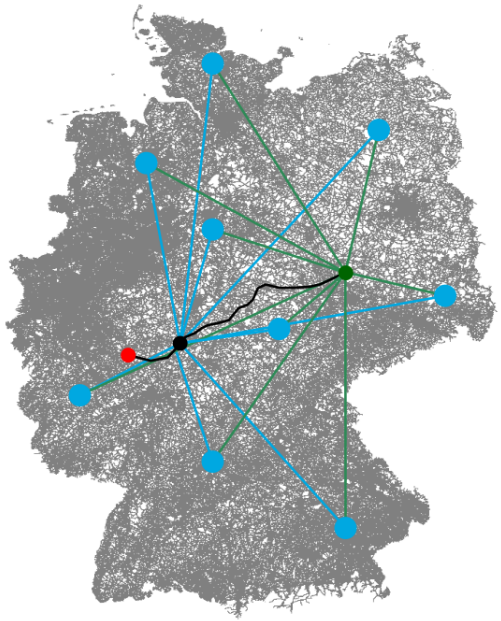

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$h(v) = \max\{\omega_v(l) - \omega_t(l) \mid l \in L\}$

Small amount of landmarks is sufficient!

A* + **L**andmarks + **T**riangle inequality

Preprocessing:

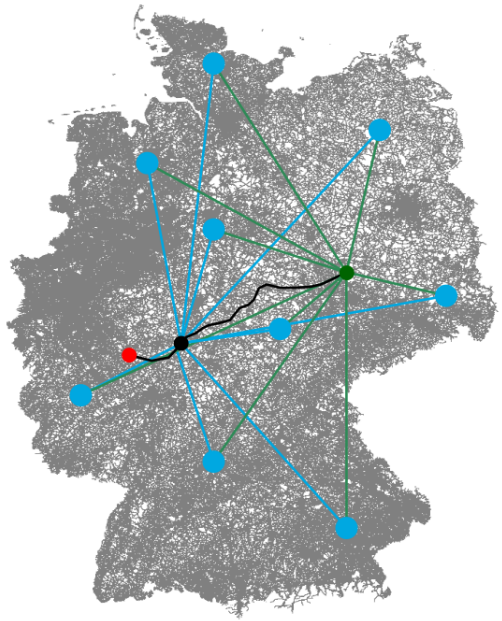
$$\mathcal{O}(|L| \cdot (n \log n + m)) + \text{select } L$$

Space Consumption:

$$\mathcal{O}(|L| \cdot n)$$

Query Time:

$$\mathcal{O}(n \log n + |L| \cdot m)$$



A* + Landmarks + Triangle inequality

Results: [GH05][Gol07]

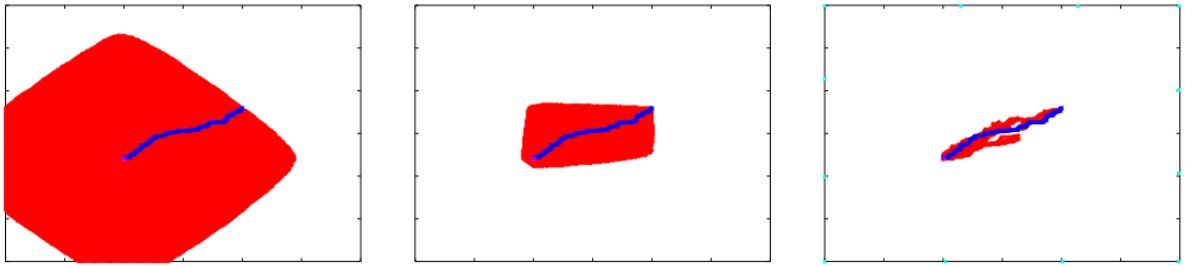


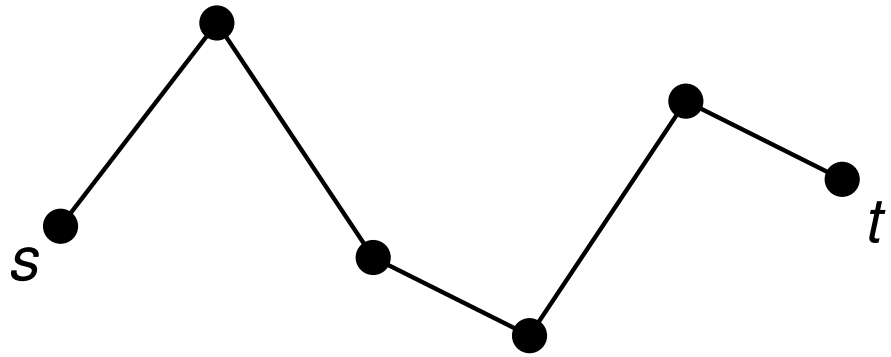
Figure 1: Vertices visited by Dijkstra's algorithm (left), A^* search with Manhattan lower bounds (middle), and an ALT algorithm (right) on the same input.

	preprocessing		query		
method	seconds	MB	avgscan	maxscan	ms
B	—	6.1	116 588	293 152	30.49
ALT	5.7	27.8	4 430	54 194	2.91
RE	45.4	12.3	668	1 697	0.55
REAL	51.1	34.0	172	982	0.28
RE (no shortcuts)	753.3	7.4	13 419	28 670	5.24
RE (no shortcuts, exact reaches)	8 629.9	7.4	11 140	24 565	4.41

Table 2. Data for the Bay Area road network.

Contraction Hierarchies [GSSD08]

IDEA: Add shortcuts to the graph.



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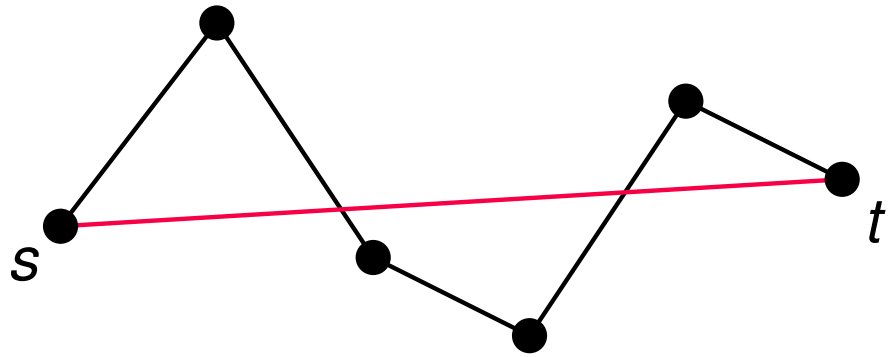
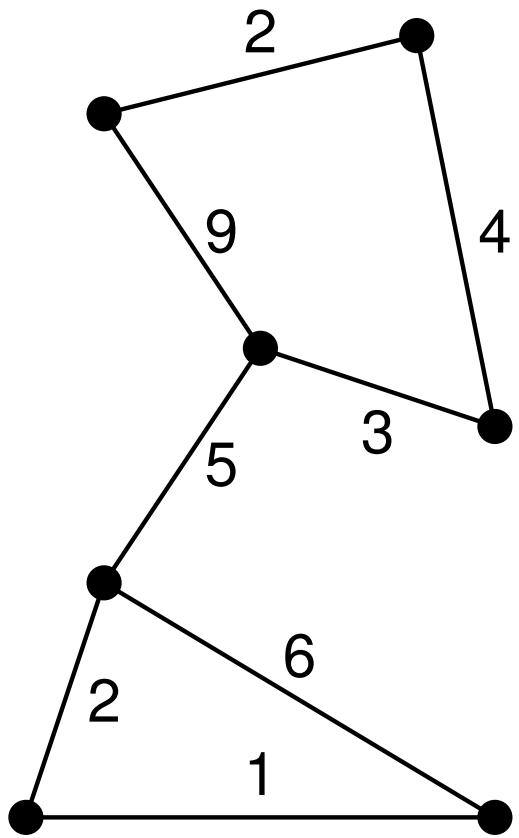


Contraction Hierarchies

Preprocessing:

1. Define rank $r : V \to \mathbb{N}$
2. Contract nodes based on their rank
   - remove node v and incident edges
   - add shortcut between neighbors if SP goes only through v
3. Add shortcuts E^+ to G remember contraction order!



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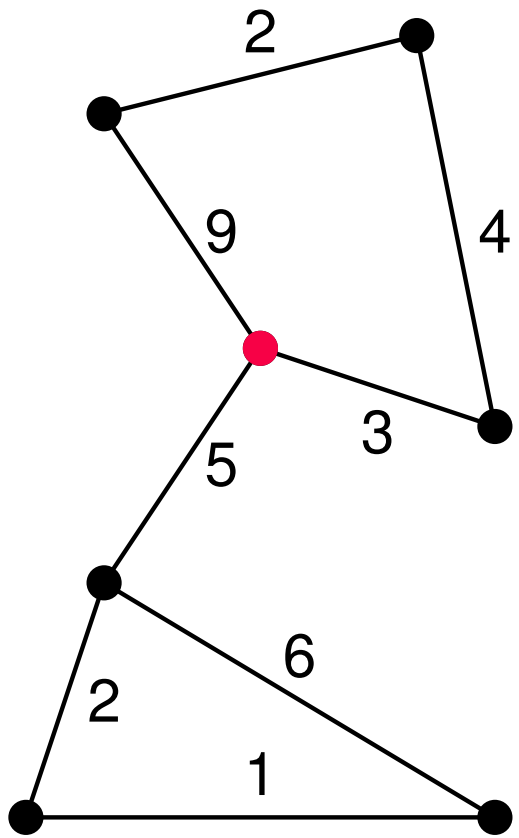

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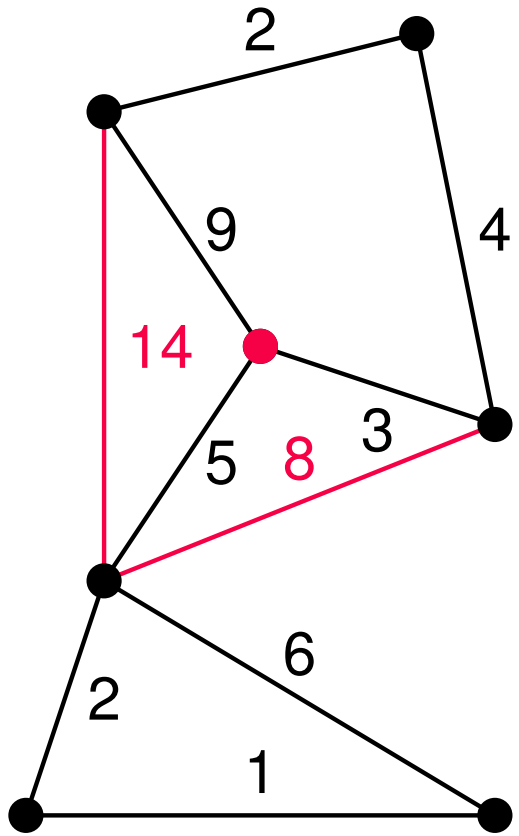

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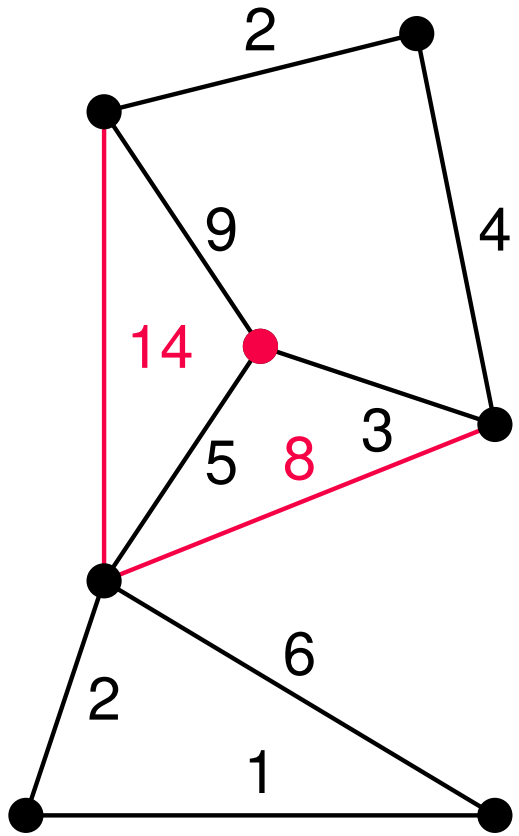

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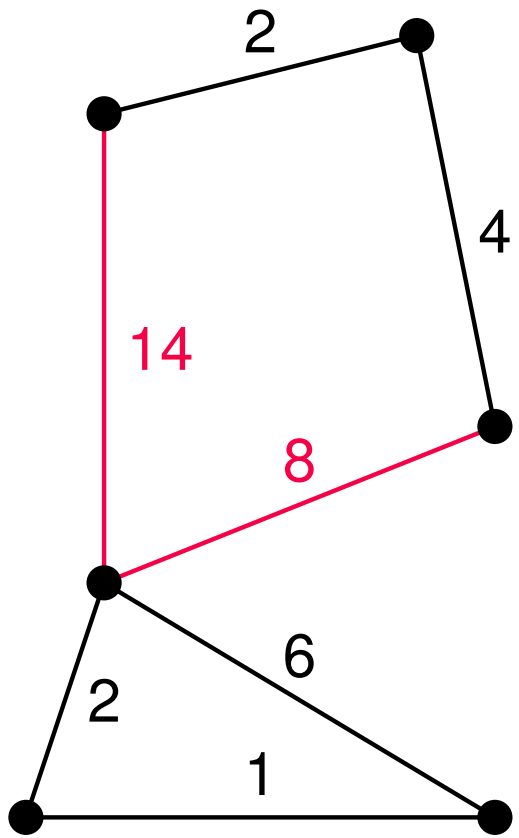

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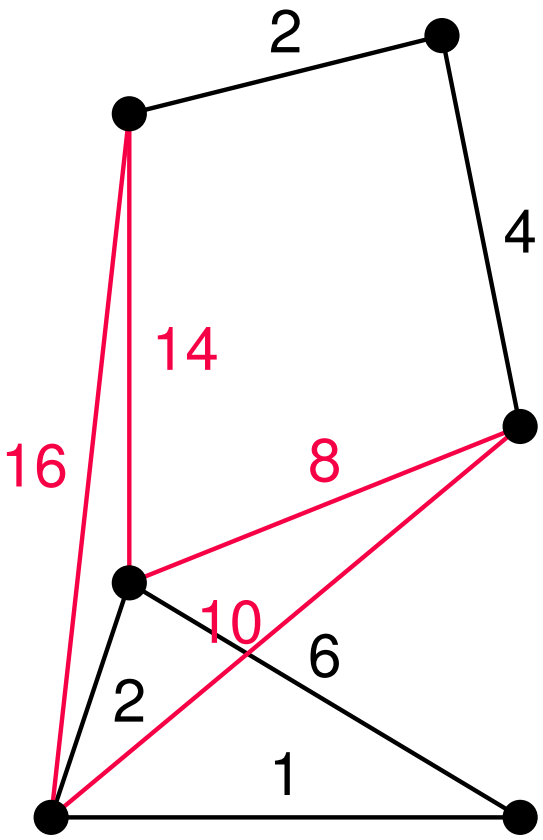

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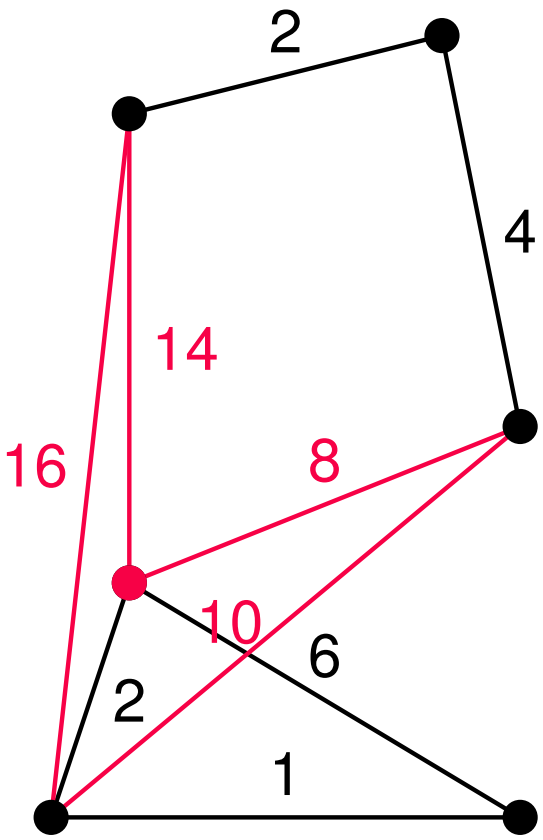

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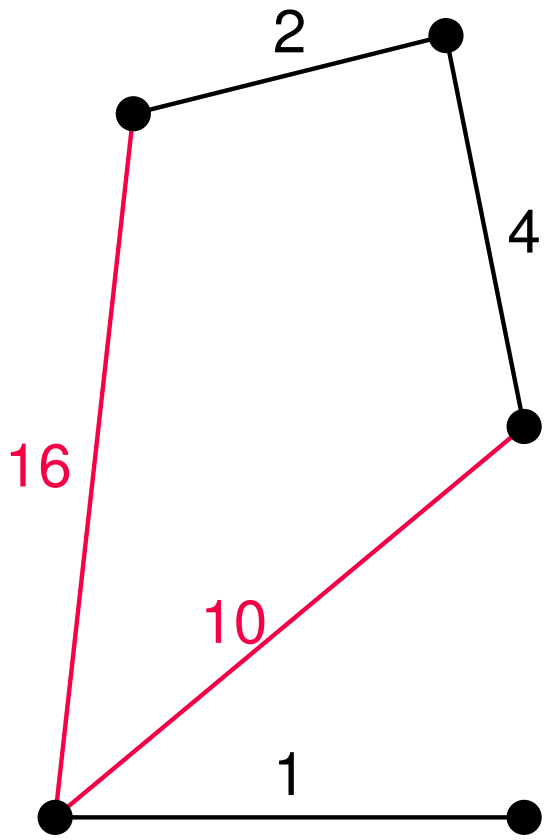

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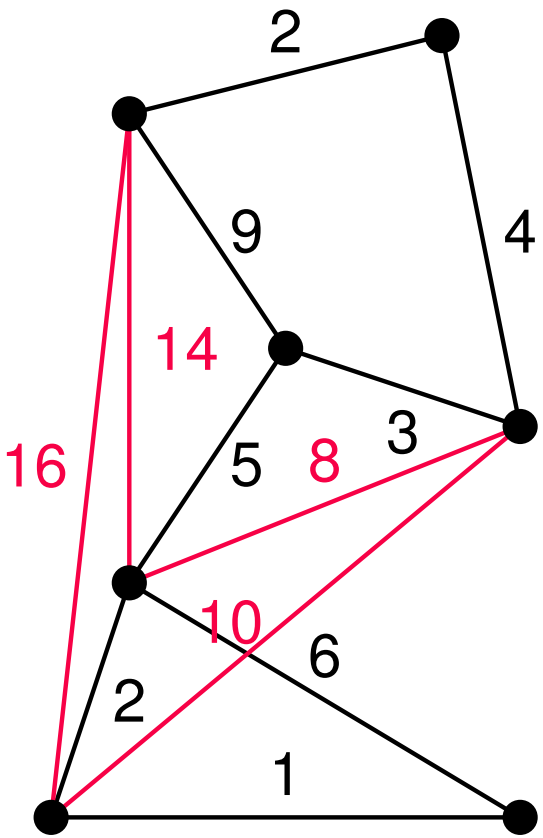
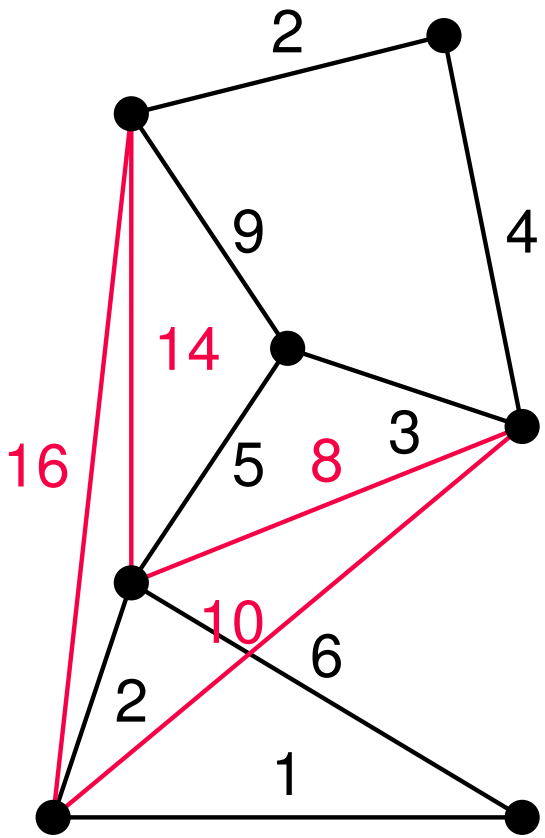


Contraction Hierarchies

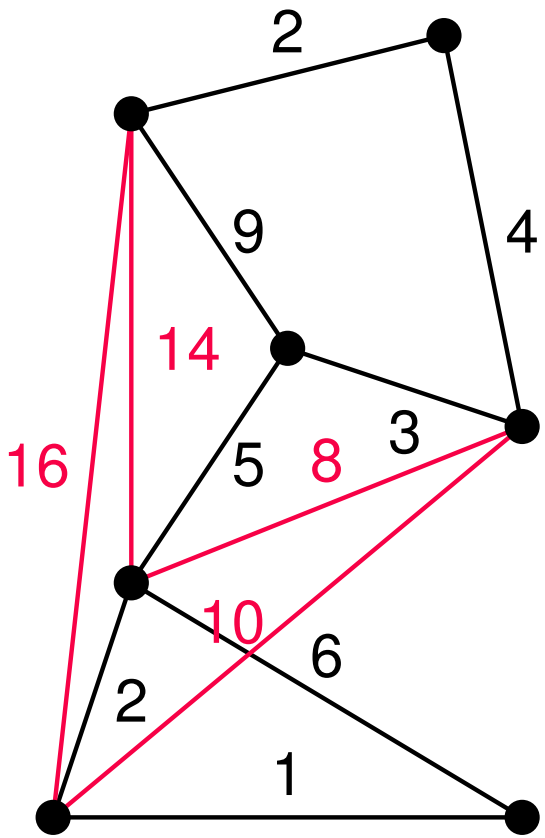
Preprocessing:
What is a good contraction order?



Contraction Hierarchies

Preprocessing:
What is a good contraction order?

Intuition: contract *important* nodes late



Contraction Hierarchies

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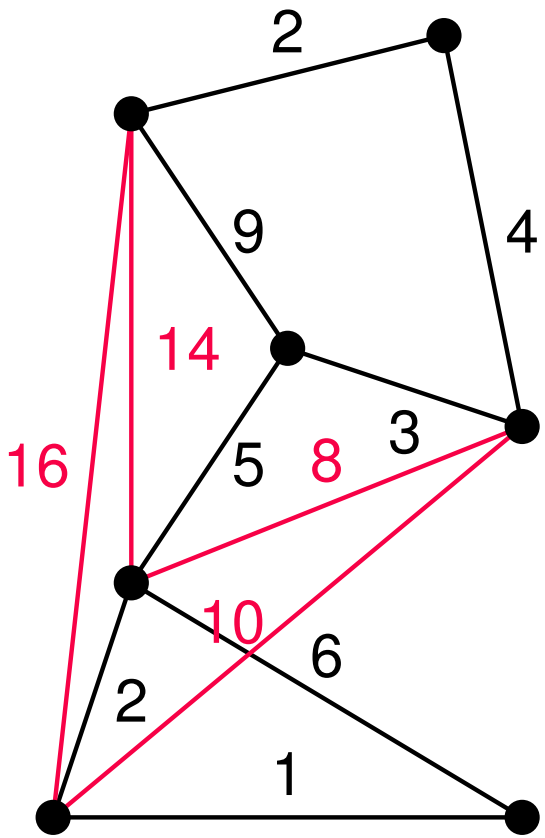
minimizing $|E^+|$ is NP-hard!



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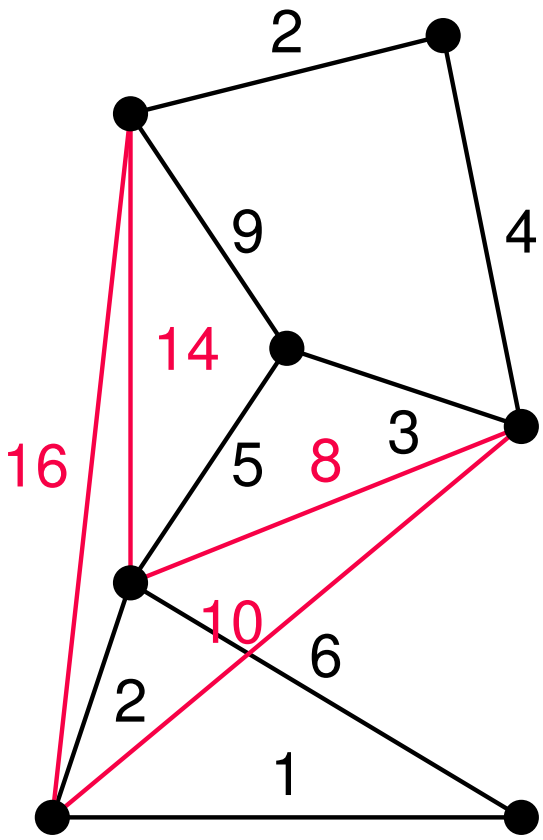
$ED(v) = -|N(v)| + \#\text{shortcuts of } v$

$$\Rightarrow |E^+| \approx |E|$$



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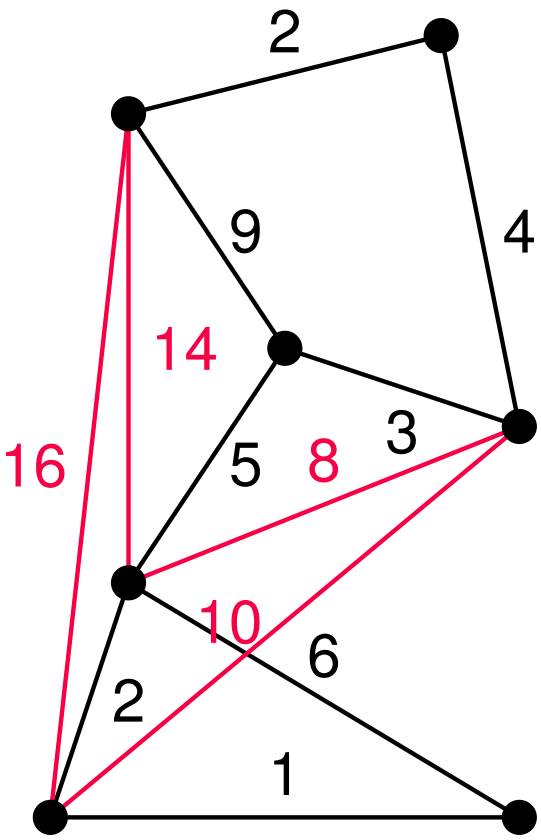
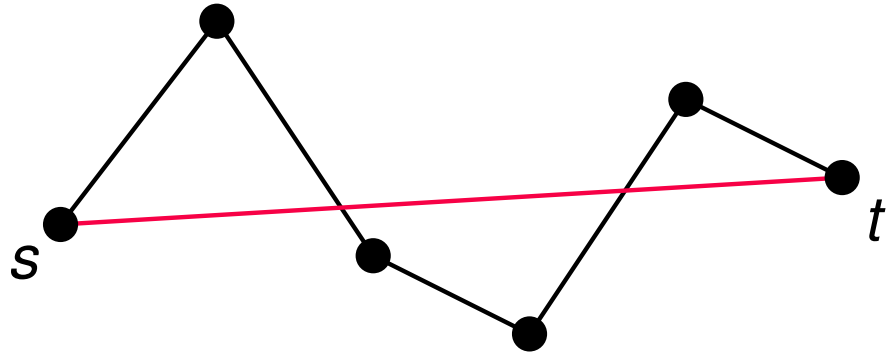


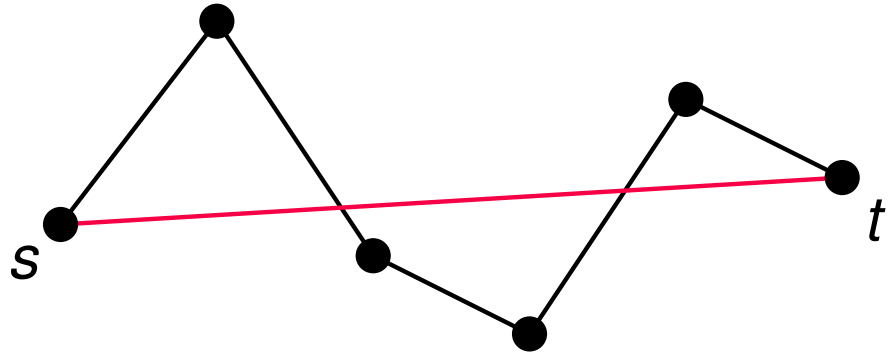
Preprocessing Time: $\mathcal{O}(n^2 \log n + nm)$ but only local search

Contraction Hierarchies



Query Answering: Convince Dijkstra to use shortcuts!

Contraction Hierarchies



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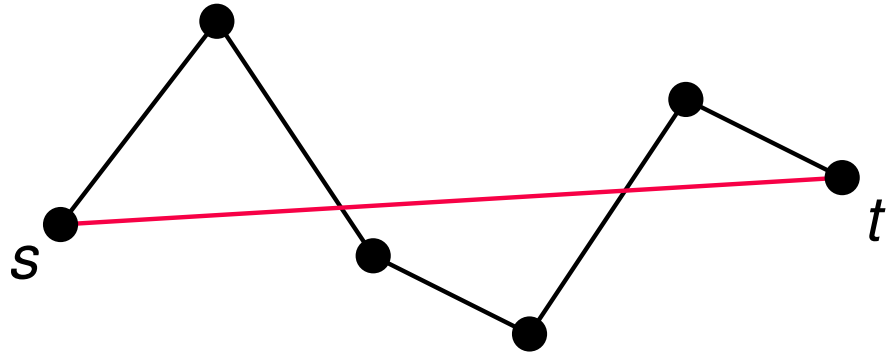

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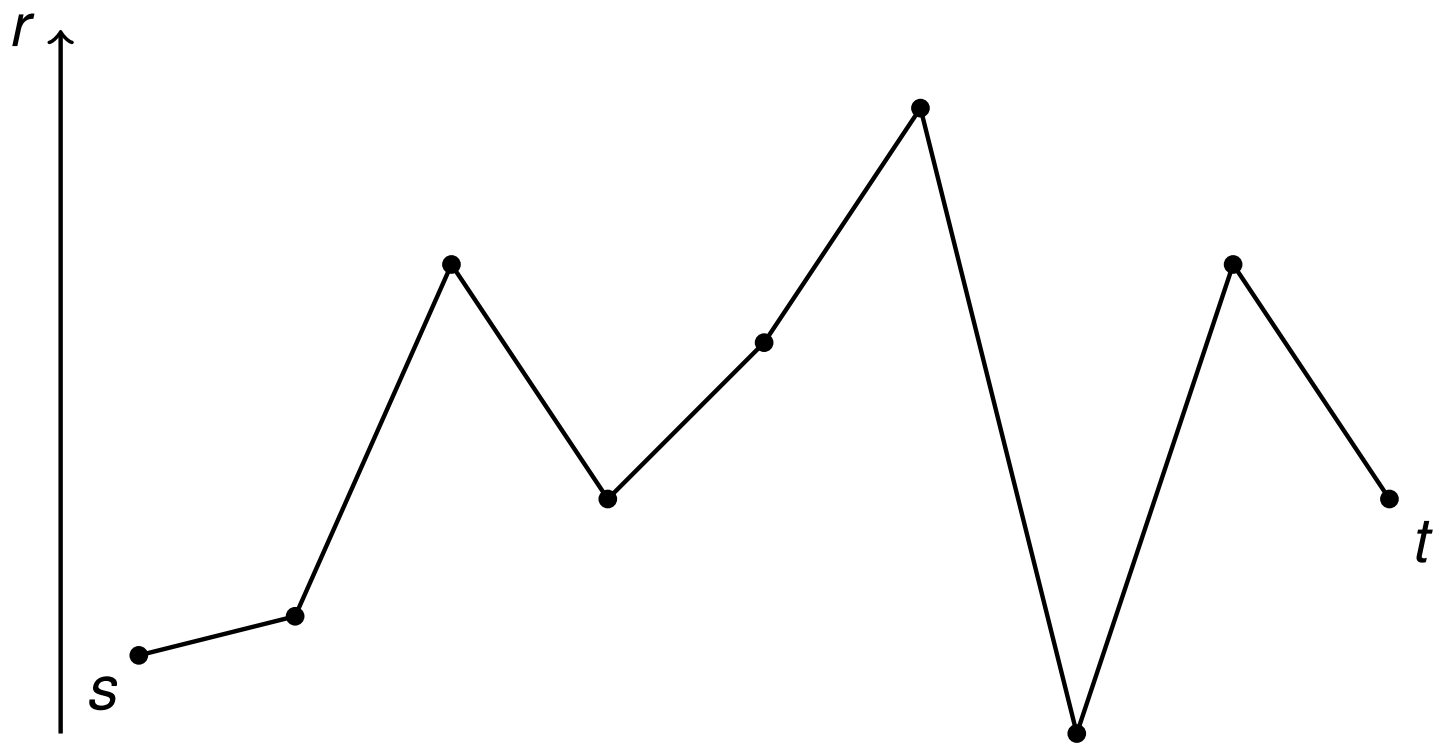
CH-Dijkstra: Bi-directional Dijkstra where forwards run is restricted to $G^{\uparrow}(s)$ and the backwards run is restricted to $G^{\downarrow}(t)$

Contraction Hierarchies

Lemma: CH-Dijkstra computes the shortest *s*-*t* path



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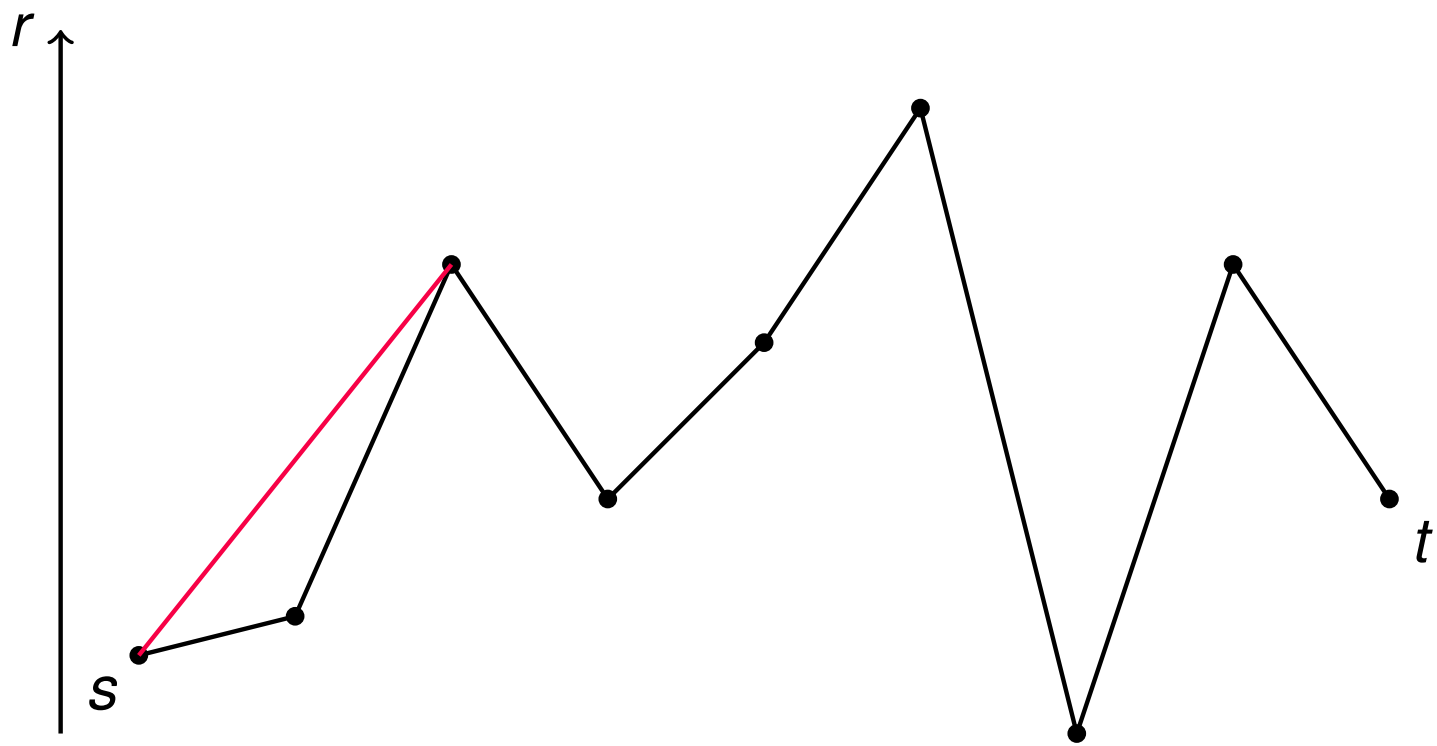

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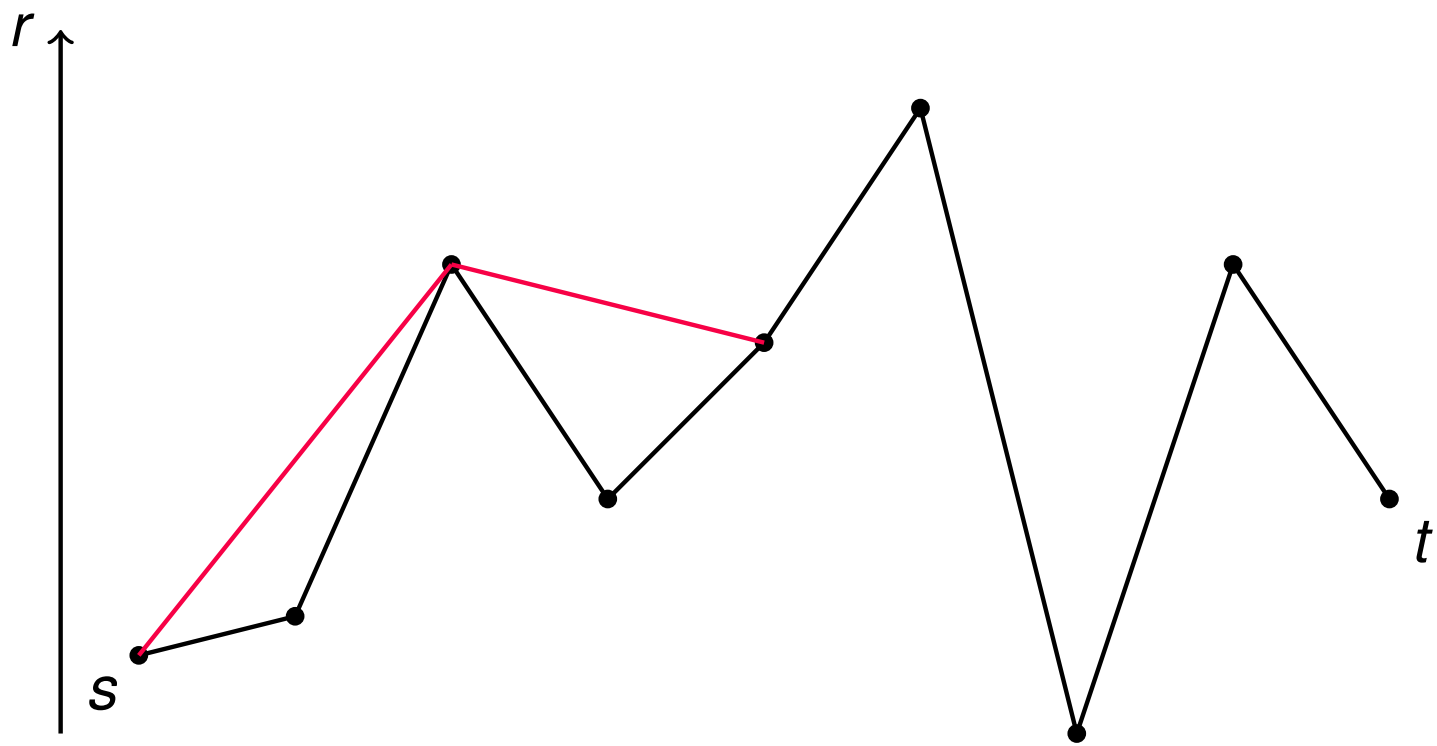

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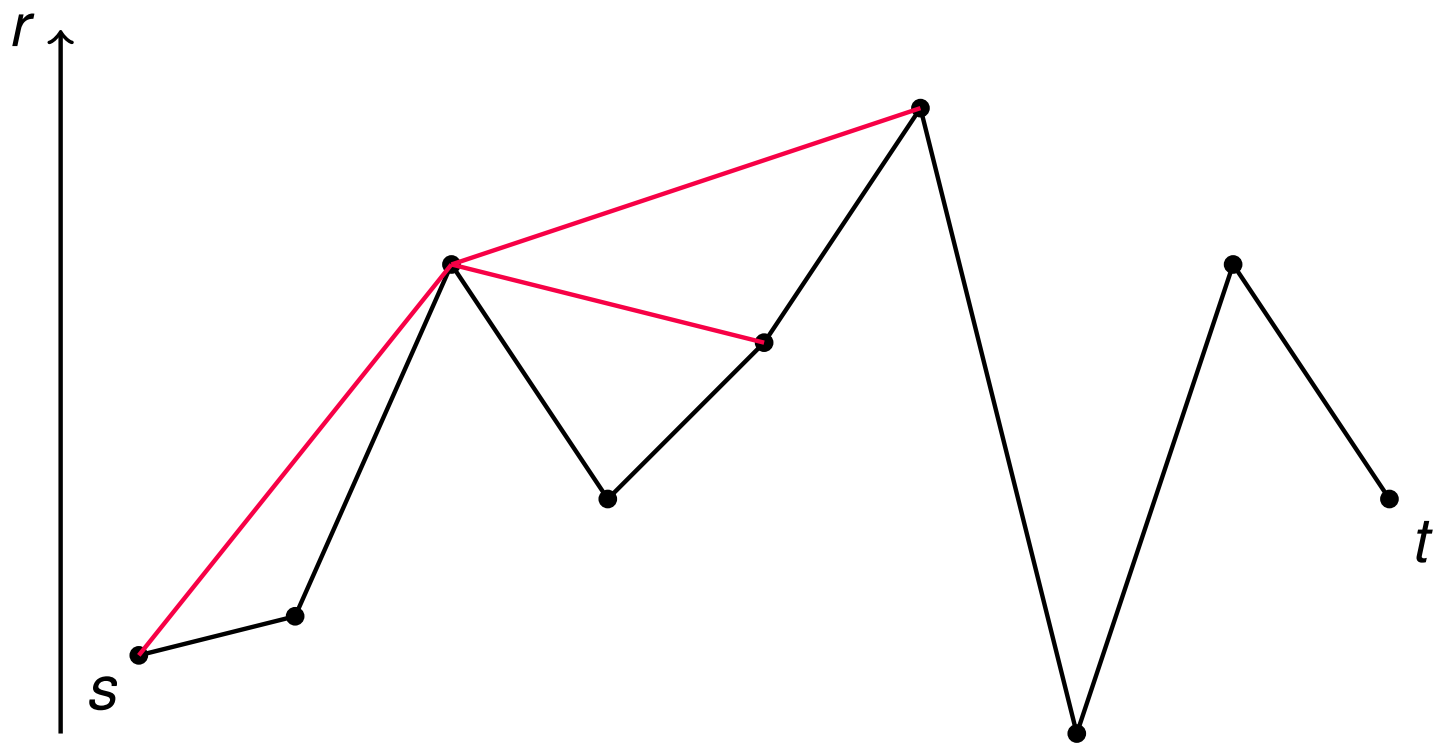

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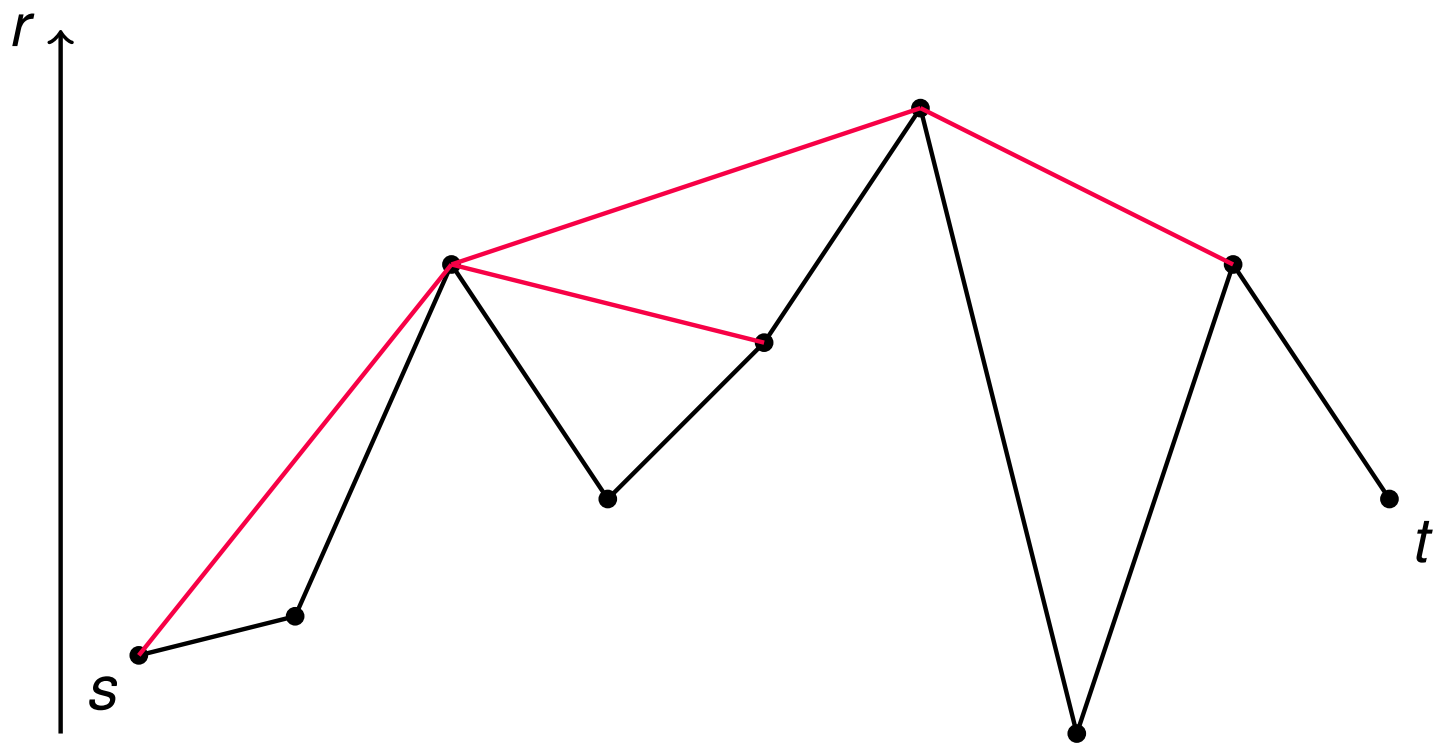

Contraction Hierarchies

Results: [GSSV12]

		prepro.		query			dom.
		time	overh.	settled	time	uses	by
method	data from	[min]	[B/n.]	nodes	[ms]	CH	CH
Dijkstra[a]	[Bauer et al. 2010b]	0	0	9.11 M	5 591		
bidir. Dijkstra[a]	[Bauer et al. 2010b]	0	0	4.76 M	2 713		
eco. CH	this paper	**8**	0.6	487	0.21	✓	
aggr. CH	this paper	27	-2.1	356	0.15	✓	
ALT-16[b]	[Goldberg et al. 2009]	13	70	82 348	120.1		✓
ALT-64[a]	[Delling and Wagner 2007]	68	512	25 234	19.6		✓
AF[c]	[Hilger et al. 2009]	2 156	25	1 593	1.1		✓
REAL[b]	[Goldberg et al. 2009]	103	36	610	0.91		✓
HH	[Schultes 2008]	13	48	709	0.61		✓
HNR	[Schultes 2008]	15	2.4	981	0.85		✓
SHARC[a]	[Bauer and Delling 2009]	81	14.5	654	0.29		✓
bidir. SHARC[a]	[Bauer and Delling 2009]	158	21.0	125	0.065		✓[d]
CALT[a]	[Bauer et al. 2010b]	11	15.4	1 394	1.34		✓
eco. CH+AF[a]	[Bauer et al. 2010b]	32	0.0	111	0.044	✓	
gen. CH+AF[a]	[Bauer et al. 2010b]	99	12	**45**	0.017	✓	
partial CH[a]	[Bauer et al. 2010b]	15	**-2.9**	965 k	53.63	✓	
TNR	this paper	46	193	N/A	0.0033	✓[e]	
TNR+AF	[Bauer et al. 2010b]	229	321	N/A	**0.0019**	✓[e]	

Europe: $|N| \approx 18M$, $|E| \approx 42M$

Hub Labels

IDEA: Store distances to *important* nodes (hubs).

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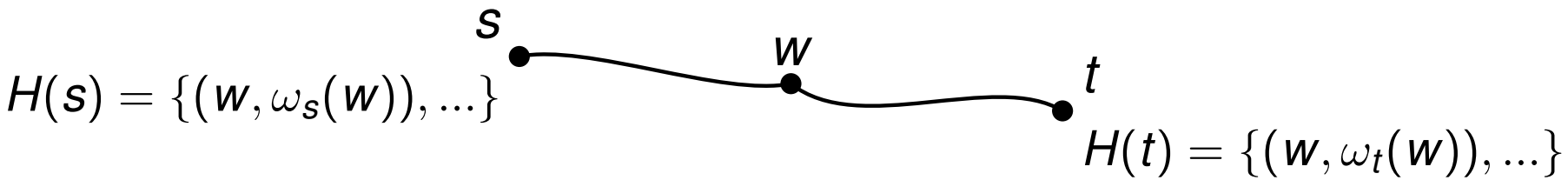
Def:

$H(v) = \{(w_1, \omega_s(w_1)), (w_2, \omega_s(w_2)), ...\}$ - hub labels of v



$$\omega_s(t) = \min\{\omega_s(w) + \omega_t(w) \mid w \in H(s) \cap H(t)\}$$

Cover Property:

$\forall s, t \in V : \exists w \in H(s) \cap H(t) : \omega_s(t) = \omega_s(w) + \omega_t(w)$

Query Time: $\mathcal{O}(|H(s)| + |H(t)|)$

Hub Labels

Space Consumption: $\mathcal{O}(\Sigma_{v \in V} |H(v)|)$

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Find minimal Hub label sets is NP-Hard!

Hub Labels

Approach 1: Set Cover Approximation

Def: Set Cover

U - Universe of size n

$\mathcal{S}$ - collection of subsets S of U with weights $w(s)$

Find $\mathcal{S}' \subseteq \mathcal{S}$ of min. weight s.t. $\bigcup_{S \in \mathcal{S}'} S = U$

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Reduction:

$U = \{(s,t) \in V^2 \mid \omega(s,t) < \infty\}$

$S(A, v, B) := \{(s,t) \mid s \in A, t \in B, v \in \pi(s,t)\}$,
with $w(S) = |A| + |B|$
$\mathcal{S}$ contains all possible $S(A, v, B)$

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$\Rightarrow$ Greedy algorithm gets a $\mathcal{O}(\log n)$ approximation

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Approach 2: CH-based Hub Labels

Computing CHs is very fast and memory efficient!

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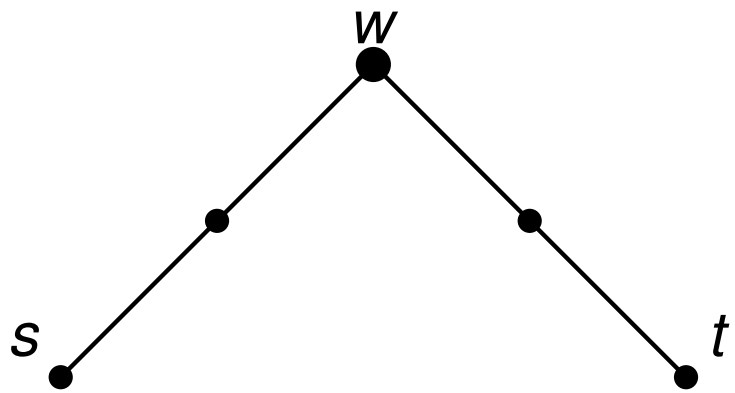
Observation: $H(v) = G^{\uparrow}(v)$ fulfills the cover property.



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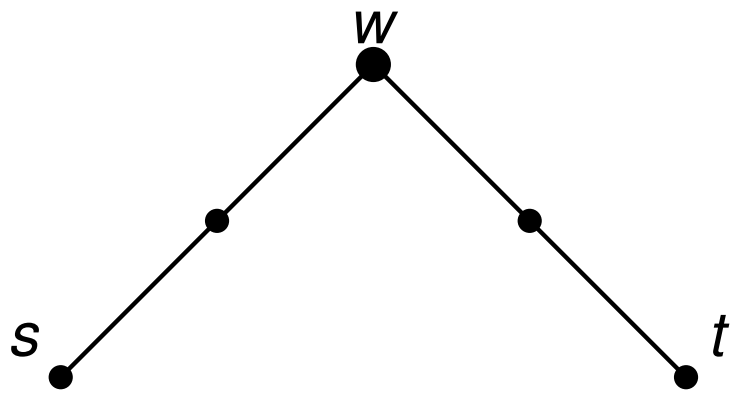


Note: not every in $G^{\uparrow}(v)$ is the peak node of some s-t path

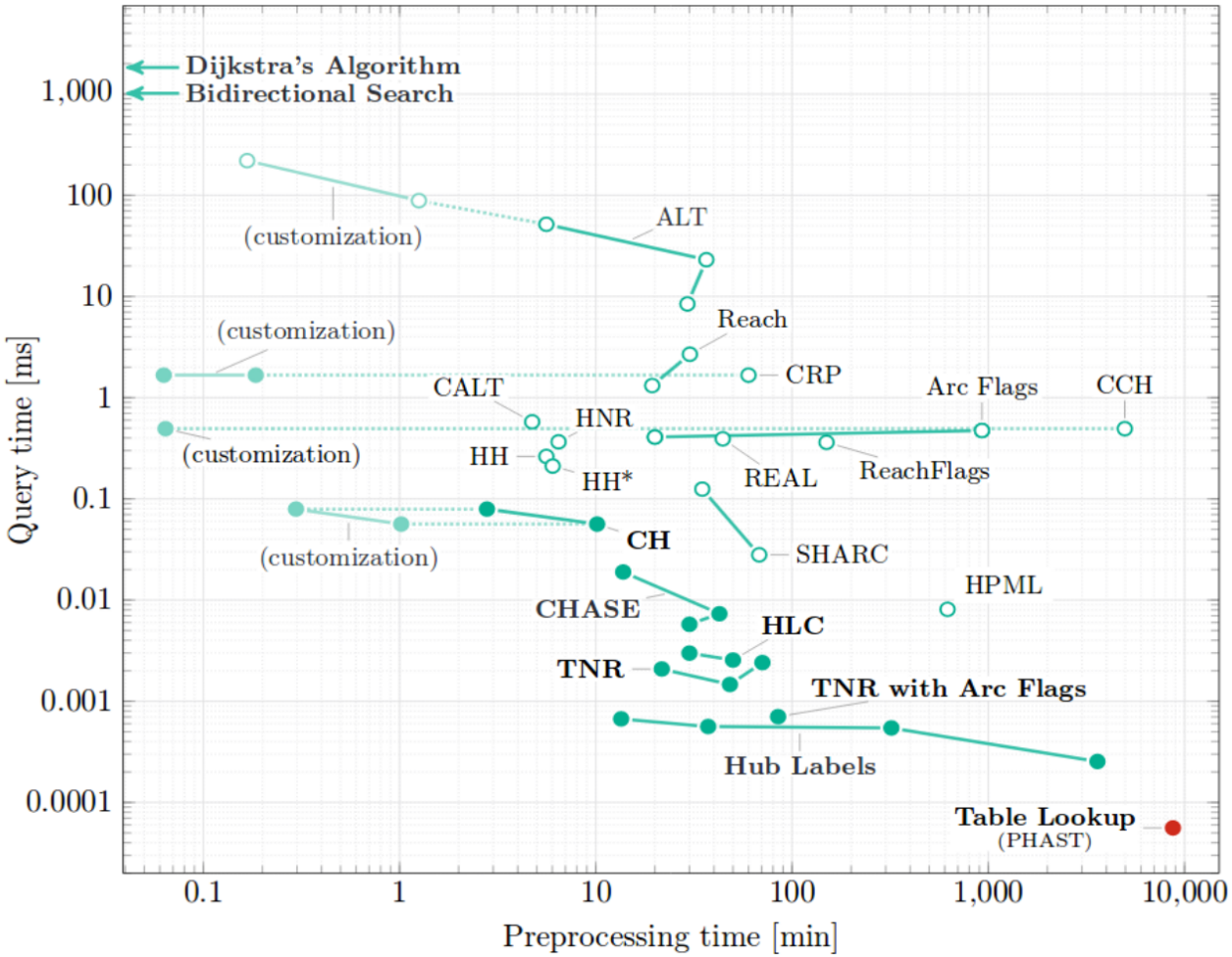
Hub Labels

Results: [ADGW11]

	Europe				USA			
	preprocessing		query		preprocessing		query	
	time	space	time [ns]		time	space	time [ns]	
method	[h:m]	[GB]	real	scaled	[h:m]	[GB]	real	scaled
CH	0:25	0.4	180000	93995	0:27	0.5	130000	67885
CHASE	1:39	0.6	17300	9034	3:48	0.7	19000	9922
HPML	≈ 24:00	3.0	18800	9817	≈ 24:00	5.1	19300	10078
TNR	1:52	3.7	3400	1775	1:30	5.4	3000	1566
TNR+AF	3:49	5.7	1900	992	2:37	6.3	1700	888
HL prefix	0:45	5.7	527	527	0:40	6.4	542	542
HL local	0:08	20.1	572	572	0:07	22.7	627	627
HL	0:14	21.3	276	276	0:18	25.4	266	266
Table Lookup	> 14:01	1 208 358.7	56	56	> 29:52	2 293 902.1	56	56

Related Topics

- One-to-Many, One-to-All Queries
- Edge cost functions
- Multicriteria route planning
- Multimodal route planning
- Public Transit Networks
- ...


Dijkstra's Algorithm
Bidirectional Search
(customization)
ALT
Reach
(customization)
CRP
CALT
Arc Flags
CCH
HNR
(customization)
HH
HH*
REAL
ReachFlags
(customization)
CH
SHARC
HPML
CHASE
HLC
TNR
TNR with Arc Flags
Hub Labels
Table Lookup
(PHAST)
Query time [ms]
Preprocessing time [min]
1,000
100
10
1
0.1
0.01
0.001
0.0001
0.1
1
10
100
1,000
10,000

Thank you!

References I

[ADGW11] Ittai Abraham, Daniel Delling, Andrew V Goldberg, and Renato F Werneck, *A hub-based labeling algorithm for shortest paths in road networks*, Experimental Algorithms: 10th International Symposium, SEA 2011, Kolimpari, Chania, Crete, Greece, May 5-7, 2011. Proceedings 10, Springer, 2011, pp. 230–241.

[Dij59] Edsger W Dijkstra, *A note on two problems in connexion with graphs*, Numerische mathematik **1** (1959), no. 1, 269–271.

References II

[FSS15] Stefan Funke, Robin Tibor Schirrmeister, and Sabine Storandt, *Automatic extrapolation of missing road network data in openstreetmap*, MUD@ICML, 2015.

[GH05] Andrew V Goldberg and Chris Harrelson, *Computing the shortest path: A search meets graph theory.*, SODA, vol. 5, 2005, pp. 156–165.

References III

[Gol07] Andrew V Goldberg, *Point-to-point shortest path algorithms with preprocessing*, International Conference on Current Trends in Theory and Practice of Computer Science, Springer, 2007, pp. 88–102.

References IV

[GSSD08] Robert Geisberger, Peter Sanders, Dominik Schultes, and Daniel Delling, *Contraction hierarchies: Faster and simpler hierarchical routing in road networks*, Experimental Algorithms: 7th International Workshop, WEA 2008 Provincetown, MA, USA, May 30-June 1, 2008 Proceedings 7, Springer, 2008, pp. 319–333.

[GSSV12] Robert Geisberger, Peter Sanders, Dominik Schultes, and Christian Vetter, *Exact routing in large road networks using contraction hierarchies*, Transportation Science **46** (2012), no. 3, 388–404.