Interference in the Internet of Things with Sparse Code Multiple Access

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***Abstract*—A massive number of connected devices each exchanging a small amount of data is an important characteristic of the Internet of Things. In order to support such a large number of devices, non-orthogonal multiple access strategies such as sparse code multiple access have been proposed. A key question is therefore how to characterize the interference statistics of this network. In this paper, we show that the interference for single user detection with devices governed by a Poisson point process is non-Gaussian, and in particular α-stable with non-trivial dependence structure between the interference on each frequency band. This raises a number of new questions for system design. For example, how should each user's signal be decoded, and what are the fundamental information theoretic limits? These questions are discussed and tractable interference models for the design of detection schemes are proposed.**

I. Introduction

A key feature of the Internet of Things (IoT) is the presence of a massive number of devices [1]. These devices are typically simple, which prevents coordination and therefore advanced interference mitigation strategies. Nevertheless, it is important to ensure that each device is able to eventually transmit small amounts of data to a base station for the purpose of, for example, data collection.

One approach to reduce interference on the uplink is to ensure that device transmissions are nearly orthogonal [2]. That is, transmissions are on different frequencies, at different times, in different spatial dimensions, or at different power levels. This approach falls under the class of nearly orthogonal multiple access (NOMA) transmission strategies.

A recently proposed NOMA strategy is sparse code multiple access (SCMA) [3] for OFDM systems, where users transmit on a sparse subset of all frequency bands. SCMA can be viewed as performing coding over frequency bands, analogous to CDMA where the coding is performed over time slots.

A key question is the impact on system performance of having a very large network of uncoordinated devices that transmit small amounts of data via SCMA codebooks. Unlike the scenario where there is a small number of users, it is not clear that the interference statistics are Gaussian.

In this paper, we study the interference statistics for devices in a large scale network of devices exploiting SCMA with locations forming a homogeneous Poisson point process. We show that the interference on each frequency band is in fact non-Gaussian, following an α-stable distribution. Moreover, the interference on each band is *non-linearly* dependent, with the dependence structure arising from the design of the SCMA codebook. This is due to the fact that α-stable random variables have infinite variance and as such the correlation is also infinite.

The presence of dependent α-stable interference on each band raises a number of new challenges. These include how to tractably model the dependence structure for the purpose of designing detection schemes, and the achievable rate limits in the presence of small amounts of data transmitted by each device. To this end, we introduce a copula-based framework to tractably model the statistics of the interference.

In Section II, we detail our model of a large scale IoT system exploiting SCMA. In Section III, we derive the statistics of the non-Gaussian interference for each user's transmission. In Section IV, we discuss the implications of the non-Gaussian interference on detection and fundamental limits of the system when users only transmit a small amount of data. In Section V, we conclude.

II. System Model

Consider an uplink single-cell network in which a massive number of devices transmit to a single base station. The device locations form a homogeneous Poisson point process (PPP) Φ with intensity λ. Without loss of generality, the base station is assumed to lie on the origin.

The users can transmit over a set of orthogonal frequency bands $\mathcal{B} = \{1, 2, \ldots, K\}$. In particular, each user transmits a sparse code multiple access (SCMA) codeword, which is a K-dimensional codeword in the set $\mathcal{C}_k = \{\mathbf{x}_{k,1}, \ldots, \mathbf{x}_{k,M}\} \subset \mathbb{R}^K$. Each codeword in $\mathcal{C}_k$ is m-sparse, which means there are m non-zero elements in each $\mathbf{x}_{k,n},\ n = 1, 2, \ldots, M$. Each element of $\mathbf{x}_{k,n}$ is the symbol transmitted by user k on band n, where band n is used only if the n-th element of $\mathbf{x}_{k,n}$ is non-zero.

As the devices are very simple and are capable of limited coordination, we assume that each device selects a random SCMA codebook. The random SCMA codebook of each device is constructed as follows. Let $\mathcal{S}$ be the set of all subsets of $\{1, 2, \ldots, K\}$ of size m, and define the random variable S as being any element in $\mathcal{S}$ with probability $1/\binom{K}{m}$. Further let $\mathbf{u}_{k,l},\ k = 1, 2, \ldots, N,\ l = 1, 2, \ldots, M$ be random vectors

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on $\mathbb{R}^m$ with law P_U, independent and identically distributed. Codebook $\mathcal{C}_k$ then consists of codewords $\mathbf{x}_{k,l}$, $l = 1, 2, \ldots, M$ with each codeword containing the same non-zero elements, determined by the random variable S on $\mathcal{S}$, and values $\mathbf{u}_{k,l}$. That is, the i-th element of codeword $\mathbf{x}_{k,l}$ is given by

$$\mathbf{x}_{k,l} = \mathbf{u}_{k,l} \circ \mathbf{s}, \tag{1}$$

where $\mathbf{s} \in \{0,1\}^K$ with $s_i = 1$ if $i \in S$ and $s_i = 0$ if $i \notin S$ and $\circ$ is the Hadamard product.

Let the transmitted codeword of user k be $\mathbf{X}_k$, with the signal on band n denoted by $X_{k,n}$. The received signal from device k on band n is then given by

$$y_{k,n} = h_{k,n} r_k^{-\eta} X_{k,n} + \sum_{l \in \Phi \setminus \{k\}} h_{l,n} r_l^{-\eta} X_{l,n}, \tag{2}$$

where $h_{k,n}$ is the fading coefficient of device k on band n, r_k is the distance from device k to the base station, and η is the path-loss exponent.

III. Interference Characterization for Single User Detection

In this section, we study the interference statistics for each user in the IoT system in Section II, where the interference of the other users is treated as noise. In particular, we show that the interference is isotropic α-stable with non-trivial dependence between the interference on each band arising from the design of the SCMA codebook.

A. Preliminaries

The α-stable random variables are an important class of random variables with heavy-tailed probability density functions, which have been widely used to model impulsive signals. The probability density function of an α-stable random variable is parameterized by four parameters: the exponent $0 < \alpha \leq 2$; the scale parameter $\gamma \in \mathbb{R}_+$; the skew parameter $\beta \in [-1, 1]$; and the shift parameter $\delta \in \mathbb{R}$. As such, a common notation for an α-stable distributed random variable is $X \sim S_\alpha(\gamma, \beta, \delta)$. In the case, $\beta = \delta = 0$, the random variable X is said to be symmetric α-stable.

In general, α-stable random variables do not have closed-form probability density functions. Instead, they are usually represented by their characteristic function, given by

$$\mathbb{E}[e^{i\theta X}] \\ = \begin{cases} \exp\left\{-\gamma^\alpha |\theta|^\alpha (1 - i\beta(\mathrm{sign}\theta)\tan\frac{\pi\alpha}{2}) + i\delta\theta\right\}, & \alpha \neq 1 \\ \exp\left\{-\gamma|\theta|(1 + i\beta\frac{2}{\pi}(\mathrm{sign}\theta)\log|\theta|) + i\delta\theta\right\}, & \alpha = 1 \end{cases} \tag{3}$$

As baseband signals are typically complex, we require an extension of α-stable random variables to the complex case. An important class of complex random variables follow the isotropic α-stable distribution, which generalizes the symmetric α-stable distribution. Let N_1, N_2 be two symmetric α-stable random variables. An isotropic α-stable random variable $N = N_1 + iN_2$ then satisfies the following two conditions:

C1: The random vector $\mathbf{N} = (N_1, N_2)^T$ is symmetric in $\mathbb{R}^2$; i.e., $\Pr(-\mathbf{N} \in A) = \Pr(\mathbf{N} \in A)$ for all Borel set $A \in \mathbb{R}^2$.

C2: $e^{i\phi} N \overset{(d)}{=} N$ for any $\phi \in [0, 2\pi)$.

B. Interference Statistics

We now characterize the statistics of the interference arising from all other user transmissions for user k on band n in the IoT model in Section II.

Theorem 1. *Suppose for each l, n, the complex random variable $h_{l,n} X_{l,n}$ is isotropic and $\mathbb{E}[(\mathrm{Re}(h_{l,n} X_{l,n}))^{\frac{\eta}{4}}] < \infty$. Then, the interference random variable*

$$I_{k,n} = \sum_{l \in \Phi \setminus \{k\}} h_{l,n} r_l^{-\eta} X_{l,n} \tag{4}$$

converges almost surely to an isotropic $\frac{4}{\eta}$-stable random variable.

Proof. (*Sketch*) Using the mapping theorem for PPPs, it follows that $\{r_l\}_l$ is a one-dimensional Poisson point process with intensity $\lambda\pi$ [4, Theorem 1]. By the LePage series representation of symmetric α-stable random variables [5, Theorem 1.4.2], it then follows that the interference has real and imaginary parts that are almost surely symmetric $\frac{4}{\eta}$-stable random variables. By the isotropy condition on $h_{l,n} x_{l,n}$, condition **C2** for isotropic α-stable random variables is also satisfied, as required. □

A consequence of Theorem 1 is that each $I_{k,n}$ is isotropic α-stable and hence the interference random vector for user k, $\mathbf{I}_k = [\mathrm{Re}(I_{k,1}), \mathrm{Im}(I_{k,1}), \ldots, \mathrm{Re}(I_{k,n}), \mathrm{Im}(I_{k,n})]^T$ has α-stable marginal distributions. If all linear combinations of elements of $\mathbf{I}_k$ form α-stable random variables, $\mathbf{I}_k$ will be jointly α-stable. Moreover, the elements of $\mathbf{I}_k$ will not in general be independent due to the fact that the same users can be interferers on multiple bands utilized by user k. In fact, this dependence is non-linear since the correlation between two α-stable random variables ($\alpha < 2$) is infinite. As such, the dependence structure is more commonly studied in the form of the covariation [5].

Remark 1. *Note that the PPP assumption on device locations is crucial for the application of the LePage series representation of α-stable random variables. As such, when guard zones or a finite network area are required, the interference will have different statistics [6]. Despite this, α-stable interference provides a tractable model that is accurate for sufficiently large networks.*

IV. Discussion

In the IoT system in Section II, the signal $\mathbf{y} \in \mathbb{C}^K$ at the receiver is given by

$$\mathbf{y} = \sum_{k \in \Phi} \mathrm{diag}(\mathbf{h}_k) \mathbf{X}_k + \mathbf{n}, \tag{5}$$

where $\mathbf{h}_l \in \mathbb{C}^K$ is the vector of fading coefficients for user k, $\mathbf{x}_l$ is the SCMA codeword of user k and $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}_{K\times 1}, \sigma^2 \mathbf{I}_{K\times K})$ is additive white Gaussian noise. We have shown that for a given user k, the interference

$$\mathbf{I}_k = \sum_{l \in \Phi \setminus \{k\}} \mathrm{diag}(\mathbf{h}_l) r_l^{-\eta} \mathbf{X}_l \tag{6}$$

is a random vector with α-stable marginal distributions and non-trivial dependence structure.

The presence of non-Gaussian interference raises a number of important questions, which we pose in the following sections.

A. Detection

In the presence of non-Gaussian interference, how should detection be performed, where only one user's signal must be detected? One approach to resolving this problem is to exploit a copula representation of the dependence structure, which was considered in [7].

In particular, let $F_{\mathbf{I}_k}$ be the joint distribution of the random vector $\mathbf{I}_k$. We seek a simple and general characterization of the joint distribution. To this end, let C_k be a copula, which is a function $[0,1]^K \to [0,1]$. Since the elements of $\mathbf{I}_k$ are α-stable random variables, it follows that the cumulative distribution function of each element is continuous. As a consequence, there exists a unique copula C_k such that the joint distribution of $\mathbf{I}_k$ can be written as

$$F_{\mathbf{I}_k}(z_1,\ldots,z_{2K}) = C_k(F_{\mathrm{Re}(I_{k,1})}(z_1),\ldots,F_{\mathrm{Im}(I_{k,K})}(z_{2K})) \tag{7}$$

by Sklar's theorem [8].

As such, the joint distribution of $\mathbf{I}_k$ can characterized by the isotropic α-stable distributions of each $I_{k,n}$ and the copula C_k. This provides a simple means of optimizing receiver structures and other system components by selecting copulas C_k with a small number of parameters. For example, let $\phi : [0,1] \to [0,1]$ be a convex and continuous function such that $\phi(1) = 0$. Then, C_k is called an Archimedean copula if

$$C_k(u_1,\ldots,u_{2K}) = \phi^{-1}\left(\sum_{i=1}^{2N}\phi(u_i)\right). \tag{8}$$

Such a simple representation of the dependence structure for the interference forms a basis for maximum likelihood decoding schemes at the receiver, which are otherwise intractable.

A similar approach also is applicable for multi-user detection in non-Gaussian noise. The most general approach is to jointly perform detection for each user's data [9]; however, due to the large number of users this is an extremely high-dimensional problem. A suboptimal approach is therefore to only perform detection for D users and treat the remaining users' signals as interference. If $D = 1$, then we have shown that the interference is α-stable.

In the case $D > 1$, then the interference will only be approximately α-stable. In particular, suppose that a sequential detection approach is adopted where user signals are decoded one at a time. In the case that the signals of users $k_1,\ldots,k_L$ have been decoded, it then follows that the interference for the $L+1$-th user on band n is given by

$$I_{k_{L+1},n,L} = \sum_{l\in\Phi\setminus\{k_1,\ldots,k_L\}} h_{l,n} r_l^{-\eta} X_{l,n}. \tag{9}$$

It remains an open question how to tractably characterize the distribution of $I_{k_{L+1},n,L}$ due to the fact that the LePage series representation does not hold.

B. Achievable Rate Limits

Another key question is what are the fundamental limits of the IoT system? In IoT systems, devices often transmit a very small number of symbols. As such, consider the case that each user k sends a very short packet, $\mathbf{X}_k$ consisting of only K symbols, corresponding to a codeword in the SCMA codebook. That is, each user only transmits in a single time slot, but over multiple bands simultaneously.

Formally, the system is a multiple access channel in the finite blocklength regime [10]. The challenge is therefore to establish the maximum achievable rates subject to an average error probability. If all users are decoded, then the system is a Gaussian multiple access channel. However as in the detection problem, due to the massive number of users it will be typical to only decode D users and treat the contributions of the other users as noise, which will be non-Gaussian.

In particular, for the case $D = 1$ the channel reduces to a finite blocklength point-to-point channel with additive non-Gaussian noise, where each band corresponds to a channel use. However, the interference $I_{k,n}$ for each band is dependent and the noise on the n-th band corresponds to the sum of $I_{k,n}$ and Gaussian noise w_n, which is non-Gaussian. As such, standard results for the Gaussian point-to-point channel in the finite blocklength regime [11] and additive α-stable noise channels in the infinite blocklength regime [12–14] are not applicable. As a further consequence, maximum achievable rates for the multiple access channel subject to an error probability constraint in the case $D > 1$ is also an open problem.

V. Conclusion

We have considered a large-scale Internet of Things network where each user transmits a small amount of data and exploits SCMA. We have shown that the interference statistics for each user are non-Gaussian. In fact, for single user detection schemes the interference is isotropic α-stable, with a non-trivial dependence structure between the interference on each band. This raises new questions for detection and characterization of the fundamental limits of the system.

To this end, we have proposed a simple and general representation of the dependence structure via the copula framework. We have also discussed the impact of multiuser detection, and the impact of transmissions of only a small amount of data. In particular, we have shown that the problem of characterizing maximum achievable rates lies in the finite blocklength regime with additive non-Gaussian noise and non-trivial dependence structure.

The presence of non-Gaussian interference with non-trivial dependence raises a number of new challenges both for the design of receiver structures and establishing fundamental information theoretic limits. Future work is therefore to develop receivers for noise with copula dependence structure and to establish achievable rates in the finite blocklength regime.

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