

Bayesian Multipath Channel Estimation Considering Dense Multipath

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Abstract—In this extended abstract we present a Bayesian estimation method applicable on single-input multiple-output radio channels. In addition to specular multipath components (MPC) also the parameters of a stochastic process are estimated that comprises non-resolvable dense multipath. Exploiting the hierarchical tree structure of a Bayesian graphical model, the parametric channel estimator is able to keep the number of unwanted MPC artifacts to a minimum and is jointly determining the model order. A delay-sum beamformer is used to consider the delay differences of MPCs at the array antenna elements which makes the estimator also applicable on wideband and ultra-wideband channels. An in-depth analysis of the methods is given on the basis of synthetically generated channel impulse responses. Further, a first glimpse is presented on how well the method performs on real data.

I. INTRODUCTION

Future 5G wireless communication technologies and the Internet of Things (IoT) paradigm will be characterized by supporting a variety of services with high quality requirements, addressing performance metrics such as reliability, latency, data throughput, and resource-efficient use of the infrastructure [1]–[3]. Spatial location information is expected to become an indispensable feature of these emerging wireless networks, considering that the user devices will have the capability of estimating accurately their locations and predicting relevant radio channel quality measures [4]–[6] that are of great importance, e.g. for spatial beamforming [7].

The robustness of location information strongly depends on the surrounding environment and hence multipath propagation. It has been shown that the actual use of multipath components (MPCs)—which can be associated to geometric features—has the potential to increase the robustness and accuracy of positioning systems [8], [9]. To be able to exploit as much position-related information from the acquired channel impulse responses [10], [11] as possible, high performance channel estimators are needed. Such channel estimators are a well investigated subject with a long history of publications, demonstrating the trend towards high accuracy. Expectation-maximization (EM) methods [12] were suggested first to estimate parameters of superimposed signals. Next, space-alternating EM methods [13] evolved and were applied on wideband radio channels. In recent years estimators that are based on more elaborate channel models which include dense/diffuse multipath (DM) (MPC components that can not be resolved by the measurement aperture) [14] have been applied. All these estimators do not include the model order in the estimation problem. Hence, MPC-artifacts that are induced by the estimators are a pitfall in the estimation procedure.

The next step in this evolution was to settle to sparse Bayesian variational methods that inherently are able to estimate a relevance metric and therefore the model order of MPCs by introducing a hierarchical graphical model structure [15]–[17]. With these methods it is possible to keep the number of unwanted estimated MPC-artifacts to a minimum, since they include the uncertainty of the parameters (gathered from the measurements themselves) in the estimation procedure.

In this work we present an extension of the sparse Bayesian variational methods of [18], [19] to a more general model, considering dense multipath for SIMO channel models. In addition the new method is using a delay beamformer [20] to cope with the propagation delay differences of the MPCs at the individual array elements making the approach suitable for wideband and ultra-wideband (UWB) channels.

II. PROBLEM FORMULATION AND SOLUTION OUTLINE

We are interested in estimating dispersion and noise parameters of a multipath channel according to the following signal model.

A. Signal Model

A baseband radio signal $s(t)$ is transmitted from an anchor to an agent equipped with an antenna array with J elements. The received signal at antenna element j reads

$$r^{(j)}(t) = \sum_{k=1}^K \alpha_k s(t - f(\tau_k, \varphi_k, \mathbf{p}^{(j)})) + s(t) * \nu^{(j)}(t) + w(t). \quad (1)$$

The first term describes K specular multipath components (MPC), each characterized by a complex amplitude α_k and a time delay $f(\tau_k, \varphi_k, \mathbf{p}^{(j)})$ where τ_k captures the delay to the center of gravity of the array, φ_k models the angle of arrival and $\mathbf{p}^{(j)}$ are the individual positions of the array elements. This first part includes the line-of-sight (LOS) and reflected specular MPCs which all hold position related information [10]. The second part in (1) denotes the convolution of $s(t)$ with the dense multipath (DM) $\nu^{(j)}(t)$, which is modeled as a non-stationary zero-mean Gaussian random process capturing all non-resolvable multipath propagation [14], [21]. Considering uncorrelated scattering along the delay axis τ the random process can be modeled by its auto-correlation function as $\mathbb{E}_{\nu} \{ \nu^{(j)}(\tau) \nu^{(j)*}(u) \} = S_{\nu}(\tau) \delta(\tau - u)$ where $S_{\nu}(\tau)$ represents the power delay profile of DM. The DM is assumed to be quasi-stationary in the spatial domain, hence

$S_\nu(\tau)$ does not change in a small vicinity around the receiver. Note that the DM presents an interference to the specular MPCs. The third term in (1) describes additive white Gaussian noise with double sided power spectral density of $N_0/2$.

By sampling the received signal $\mathbf{r}^{(j)} \in \mathbb{C}^{N \times 1}$ and stacking the received signals for the J array elements in a vector, the received signals read

$$\mathbf{r} = \mathbf{S}(\Psi)\boldsymbol{\alpha} + \mathbf{w}_c + \mathbf{w} \in \mathbb{C}^{NJ \times 1}, \quad (2)$$

where $\mathbf{r} = [(\mathbf{r}^{(1)})^T, \dots, (\mathbf{r}^{(J)})^T]^T$, $\boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_K]^T$, and $\mathbf{S}(\Psi) = [\mathbf{s}^T(\psi_1), \dots, \mathbf{s}^T(\psi_K)]^T$ with dispersion parameters $\Psi = [\psi_1^T, \dots, \psi_K^T]^T$ and $\psi_k = [\tau_k, \varphi_k]^T$. $\mathbf{w}_c + \mathbf{w}$ are sampled and stacked versions of the DM and AWGN realizations for the J array elements. According to the Gaussian assumptions the covariance matrix of the stacked random processes is $\mathbf{C}(\boldsymbol{\eta}) = \text{diag}([\tilde{\mathbf{C}}(\boldsymbol{\eta}), \tilde{\mathbf{C}}(\boldsymbol{\eta}), \dots, \tilde{\mathbf{C}}(\boldsymbol{\eta})])$ as the realizations of the DM are assumed to be uncorrelated at the array elements, $\tilde{\mathbf{C}}(\boldsymbol{\eta}) = \tilde{\mathbf{S}}\text{diag}(S_\nu(\boldsymbol{\Gamma}, iT_s)T_s)\tilde{\mathbf{S}} + N_0/T_s\mathbf{I}$, $\tilde{\mathbf{S}} = [\mathbf{s}(iT_s), \mathbf{s}(iT_s - 1), \dots, \mathbf{s}(iT_s - N)]^T$ and $f_s = 1/T_s$ as the sampling frequency. The covariance matrix depends on $\boldsymbol{\eta} = [N_0, \boldsymbol{\Gamma}]^T$, where $\boldsymbol{\Gamma}$ holds the parameters of the power delay profile we want to estimate, e.g. an exponential or double-exponential power delay profile [21].

B. Outline of the Estimation Solution

The likelihood function of the received signal model is a complex Gaussian random process modeled as

$$f(\mathbf{r}|\Psi, \boldsymbol{\eta}, \boldsymbol{\alpha}) = \frac{1}{\pi^{NJ}|\mathbf{C}|} \exp\{-(\mathbf{r} - \mathbf{S}\boldsymbol{\alpha})^H \mathbf{C}^{-1}(\mathbf{r} - \mathbf{S}\boldsymbol{\alpha})\},$$

where we have dropped the explicit dependence of $\mathbf{C}(\boldsymbol{\eta})$ and $\mathbf{S}(\Psi)$ on $\boldsymbol{\eta}$ and Ψ respectively for easier readability.

The aim is to infer the parameter posterior distribution of $f(\Psi, \boldsymbol{\eta}, \boldsymbol{\alpha}|\mathbf{r}) \propto f(\mathbf{r}|\Psi, \boldsymbol{\eta}, \boldsymbol{\alpha})f(\Psi)f(\boldsymbol{\eta})f(\boldsymbol{\alpha})$. If the number of multipath components K is known, a numerical maximization of the posterior distribution enables the estimation of the dispersion and noise parameters. Unfortunately, the number of MPCs is usually not known. Thus, to be able to automatically determine the number of MPCs and introduce sparsity, a set of hyperpriors $\boldsymbol{\kappa}$ is introduced for the complex amplitudes $\boldsymbol{\alpha}$, used to prune the model [17]. Therefore the parameter posterior distribution is $f(\Psi, \boldsymbol{\eta}, \boldsymbol{\alpha}, \boldsymbol{\kappa}|\mathbf{r}) \propto f(\mathbf{r}|\Psi, \boldsymbol{\eta}, \boldsymbol{\alpha})f(\Psi)f(\boldsymbol{\eta})f(\boldsymbol{\alpha}|\boldsymbol{\kappa})f(\boldsymbol{\kappa})$ [22].

We employ a uniform prior for the dispersion parameters Ψ and the noise parameters $\boldsymbol{\eta}$ as no prior knowledge is available. A classical sparse Bayesian learning approach assumes a factorable hierarchical prior $f(\boldsymbol{\alpha}|\boldsymbol{\kappa})f(\boldsymbol{\kappa}) = \prod_k f(\alpha_k|\kappa_k)f(\kappa_k) = \prod_k \mathcal{CN}(\alpha_k|0, \kappa_k^{-1})\text{Ga}(\kappa_k|a_k, b_k)$, with $\mathcal{CN}(\alpha_k|0, \kappa_k^{-1})$ as zero-mean complex normal distribution with variance κ_k^{-1} , and $\text{Ga}(\kappa_k|a_k, b_k)$ as gamma distribution with shape and rate parameter a_k and b_k respectively.

By employing a structured mean-field approximation [22] the parameter posterior can be approximated by

$$f(\Psi, \boldsymbol{\eta}, \boldsymbol{\alpha}, \boldsymbol{\kappa}|\mathbf{r}) \approx q(\Psi, \boldsymbol{\eta}, \boldsymbol{\alpha}) = q(\boldsymbol{\eta})q(\boldsymbol{\alpha}) \prod_{k=1}^K q(\psi_k)q(\kappa_k). \quad (3)$$

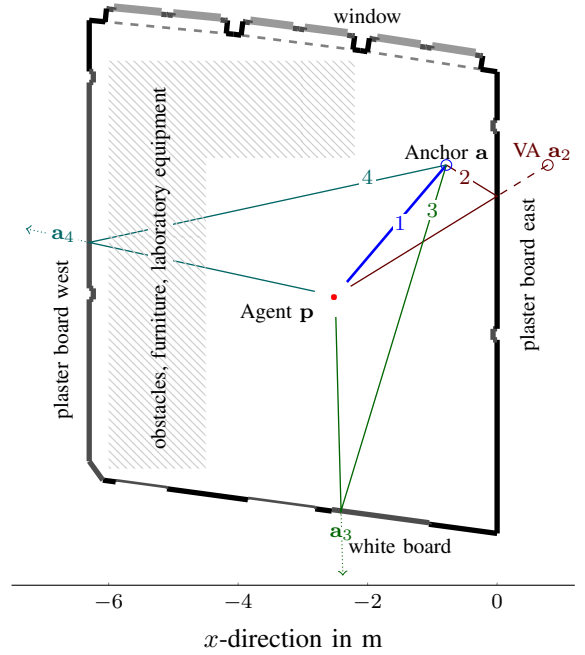


Fig. 1. Floorplan of the evaluation scenario and specular MPCs between agent and anchor located at $\mathbf{p}^{(j)}$ and \mathbf{a} are depicted. The line-of-sight ($k = 1$) and MPCs reflected at plaster board east ($k = 2$) and west (4), and white board (3) as well as their corresponding VA positions $\{\mathbf{a}_k\}_{k \in \mathcal{K}}$ are shown.

Variational Bayes, an iterative method, is used to find the approximations $q(\cdot)$ for the individual posteriors. By choosing point estimates for the dispersion and noise parameters, hence $q(\psi_k) = \delta(\psi_k - \hat{\psi}_k)$ and $q(\boldsymbol{\eta}) = \delta(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}})$, the unconstrained factor updates for the noise parameters are given by

$$\begin{aligned} \ln(q^*(\boldsymbol{\eta})) \propto & \text{const} - \ln(|\mathbf{C}(\boldsymbol{\eta})|) \\ & - (\mathbf{r} - \mathbf{S}(\hat{\Psi})\hat{\boldsymbol{\mu}}_\alpha)^H \mathbf{C}(\boldsymbol{\eta})^{-1}(\mathbf{r} - \mathbf{S}(\hat{\Psi})\hat{\boldsymbol{\mu}}_\alpha) \\ & - \text{tr}\{\mathbf{S}(\hat{\Psi})^H \mathbf{C}(\boldsymbol{\eta})^{-1} \mathbf{S}(\hat{\Psi})\hat{\mathbf{C}}_\alpha\}, \end{aligned} \quad (4)$$

where $\hat{\boldsymbol{\mu}}_\alpha$ and $\hat{\mathbf{C}}_\alpha$ are the estimated expected value and covariance matrix of the MPC amplitudes $\boldsymbol{\alpha}$. The factor update for the dispersion parameters looks comparable to (4), while analytical results are available for the updates of $q(\boldsymbol{\alpha})$ and $q(\kappa_k)$ (see also [15]–[17]).

C. Model order selection

By computing $\hat{\kappa}_k = \mathbb{E}_{q(\kappa_k)}\{\kappa_k\}$ and using a Jeffreys prior ($a_k = b_k = 0$) for the hyperparameter [17] the pruning condition can be analyzed. It is found that the stationary point for the hyperparameter is

$$\hat{\kappa}_k^{[\infty]} = \begin{cases} (|\rho_k|^2 - \zeta_k)^{-1} & \text{if } |\rho_k|^2 > \zeta_k \\ \infty & \text{else} \end{cases}, \quad (5)$$

if the other parameters are kept constant. $|\rho_k|^2$ and ζ_k are estimated values of the expectation of the mean and variance of the complex amplitudes. This means that a component is kept if $|\rho_k|^2 > \zeta_k$. Otherwise the MPC is removed from the model.

TABLE I

SIMULATED AND ESTIMATED (SHOWN AS $\hat{\cdot}$) PARAMETERS FOR THE SPECULAR MPCs. THE DISTANCE d_k IS LINKED TO THE DELAY VIA THE SPEED OF LIGHT c , THE EFFECTIVE SINR IS A RELIABILITY MEASURE REGARDING RANGING AND IS RELATED TO THE RANGING ERROR BOUND $\mathcal{R}(d)$ [23].

k	d_k	mean (\hat{d}_k)	φ_k	mean ($\hat{\varphi}_k$)	$ \alpha_k ^2$	mean ($ \hat{\alpha}_k ^2$)	SINR	mean ($\widehat{\text{SINR}}$)	$\mathcal{R}(d)$	std (\hat{d})
	m	m	°	°	dB	dB	dB	dB	m	m
1	2.883	2.883	45.06	45.04	-1.285	-1.279	21.16	21.33	0.0019	0.0020
2	4.141	4.141	29.52	29.38	-7.431	-7.465	10.28	10.25	0.0065	0.0068
3	8.900	8.901	273.61	273.79	-14.077	-14.052	7.82	7.87	0.0087	0.0089
4	9.225	9.225	167.22	167.17	-14.388	-14.376	7.82	7.88	0.0087	0.0081

TABLE II

PARAMETERS FOR THE GENERATION OF THE DM AND AWGN. Ω_1 IS THE TOTAL POWER OF THE DM, γ_1 , γ_{rise} , χ ARE SHAPE PARAMETERS, AND τ_{DM} IS THE DELAY AT WHICH THE ROOM IS EXCITED FOR THE FIRST TIME.

N_0	-30 dB	mean (\hat{N}_0)	-29.97 dB	std (\hat{N}_0)	-44.60 dB
Ω_1	-1.23 dB	mean ($\hat{\Omega}_1$)	-1.30 dB	std ($\hat{\Omega}_1$)	-15.31 dB
γ_1	11 ns	mean ($\hat{\gamma}_1$)	10.62 ns	std ($\hat{\gamma}_1$)	1.10 ns
γ_{rise}	5 ns	mean ($\hat{\gamma}_{\text{rise}}$)	18.71 ns	std ($\hat{\gamma}_{\text{rise}}$)	41.12 ns
τ_{DM}	9.617 ns	mean ($\hat{\tau}_{\text{DM}}$)	9.68 ns	std ($\hat{\tau}_{\text{DM}}$)	0.23 ns
χ	0.5	mean ($\hat{\chi}$)	0.61	std ($\hat{\chi}$)	0.16

III. RESULTS

A. Synthetic Data

To validate the algorithm, synthetic data is generated according to the model in Section II-A. The floorplan, depicted in Fig. 1, and a geometric ray tracer are used to generate the dispersion parameters of the specular MPCs. For an initial validation we restrict the set of specular components to 4 MPCs including the line-of-sight. The anchor is equipped with a single antenna, while the receiver is simulated with a 5×5 antenna array with a spacing of 1 cm. As baseband pulse $s(t)$ a root-raised-cosine pulse with a pulse duration of 1 ns and a roll-off factor of 0.6 is used which is transmitted at a center frequency of 7 GHz. The ray tracer considers only the room geometry and assumes a 3 dB loss per reflection. The total energy of the specular components E_{det} is normalized to 0 dB and a signal to noise ratio of $E_{\text{det}}/N_0 = 30$ dB is used. The parameters of the specular components are shown in Table I. To model the DM a double exponential power delay profile [21] is used with the parameters given in Table II. The Rician K-factor for the LOS component is set to 0 dB which is a representative value for the analyzed room [24].

To assess the performance of the estimator we compare the results of 100 realizations of the ranging estimator to the ranging error bound (REB, $\mathcal{R}(d)$) which is the square root of the Cramér Rao lower bound for the time-of-flight estimator [23]. As can be seen in Table I the estimated values for the distance $d_k = \tau_k c$, the angle-of-arrival φ_k , and the amplitude α_k are unbiased. The effective SINR which is a measure for the reliability of the MPC with respect to its ranging performance can also be estimated correctly by the algorithm. Moreover, the standard deviation of the estimated distance achieves the REB at this bandwidth.

In Table II the performance of the DM and AWGN parameter estimation can be seen. The AWGN-PSD N_0 , the power of the DM Ω_1 , the start time τ_{DM} , the fall time γ_1 and the

TABLE III

ESTIMATED PARAMETERS OF THE SPECULAR MPCs FOR REAL DATA.

k	\hat{d}_k	$\hat{\varphi}_k$	SINR
	m	°	dB
1	2.888	44.957	25.68
2	3.135	45.241	9.87
3	4.173	31.573	18.48
4	4.696	161.821	4.44
5	5.991	160.390	4.08
6	6.064	66.147	7.65
7	6.270	63.980	8.85
8	7.184	67.161	3.31
9	8.855	271.622	8.39
10	9.588	166.848	4.71
11	9.639	282.370	9.50
12	11.566	147.004	3.78

shape parameter χ are estimated with a reasonable accuracy. The rise time γ_{rise} on the other hand shows a large bias as well as a high standard deviation due to some large outliers.

B. Experimental Data

To validate the algorithm with real data, we performed measurements using an m-sequence channel sounder in the room depicted in Fig. 1¹. The channel sounder measures a bandwidth of 7 GHz centered around a center frequency of 7 GHz. To have a comparable setup as in Section III-A we reduced the overall bandwidth by filtering with a root-raised-cosine pulse with the same parameters as above. The algorithm detects twelve MPCs in the first 15 m of the signal which are listed in Table III. The detected MPCs $\{1, 3, 9\}$ match well with the MPCs used for simulation and presented in Table I. The differences can be explained by the uncertainty in the floorplan and that only a geometric ray tracer was used for generating the specular MPCs. MPC component 4 which is seen in the simulation (reflection at plaster board west) is not detected by the algorithm in the real data, as metal furniture was present in the western part of the room which is also depicted in Fig. 1.

IV. CONCLUSIONS

In this extended abstract we presented a variational Bayesian framework capable of estimating the dispersion parameters of specular multipath components as well as the power delay profile of interfering dense multipath. The algorithm is capable of inferring the number of multipath components and enforces a sparse solution to keep the number of unwanted multipath component artifacts at a minimum.

¹More details about the measurement setup can be found in [24]

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