Efficient Iterative Mobile Terminal Localization Based on Bayesian Time-of-Flight Estimations

Mathieu Van Eeckhaute[1], Thomas Van der Vorst[1], André Bourdoux[2], François Quitin[1], Philippe De Doncker[1], François Horlin[1]

[1]Université Libre de Bruxelles (ULB)
Av. Roosevelt 50, 1050 Brussels, Belgium
Emails: {mveeckha, tvdvorst, fquitin, pdedonck,fhorlin}@ulb.ac.be
[2]Inter-university Micro-Electronics Center (IMEC)
Kapeldreef 75, 3008 Leuven, Belgium
Email: bourdoux@imec.be

***Abstract*—Iterative localization has recently arisen as a promising solution to determine the position of a Mobile Station (MS) in a cellular network. We recently showed that iterating between the conventional delay estimation and multi-lateration steps allows to approach the performance of the direct localization. In this paper we present a new formulation of our iterative localization method that drastically reduces the computational effort compared to our original implementation. Simulation results prove that the proposed low complexity iterative algorithm performs close to the direct localization scheme while presenting a limited complexity increase compared to the conventional two-step approach.**

***Index Terms*—Iterative localization, Bayes framework, weighted linear least squares.**

I. Introduction

Time-of-Arrival (ToA) based localization methods rely on the estimation of the absolute signal Time-of-Flight (ToF) between the source and the receiver. The fourth generation of cellular systems (4G) includes a specific Positioning Reference Signal (PRS) in its protocol to finely estimate the signal ToF. This PRS is defined as an Orthogonal Frequency Division Multiplexing (OFDM) signal spread in time and frequency [1]. The signal ToF estimation constitutes the first step of the conventional two-step localization approach. The position of the Mobile Station (MS) is then determined in a multi-lateration step where the non-linear system of equations formed by the ToF estimates is solved. A lot of efficient algorithms have been developed in literature to perform this multi-lateration. Most of them work on a linearized version of the equation system, like the Weighted Linear Least Square (WLLS) solution presented in [2].

Another methodology to estimate the user position is the Direct Position Estimation (DPE) that directly estimates the position coordinates from the digitized received signal [3]. Paper [4] shows by simulations that DPE provides an important performance gain compared to the two-step approach, especially for lower Signal-to-Noise Ratio's (SNR's). DPE methods proposed in literature rely on the optimization of a multi-variate non-convex function, like the Maximum Likelihood (ML) estimator proposed in [3]. Although outperforming the two-step approach, they suffer from a significant complexity increase.

We recently demonstrated that the performance of the DPE can be approached by iterating between the two conventional steps [5]. However, our original iterative algorithm is not computationally attractive compared to DPE. It indeed relies on Bayesian delay and position estimation steps requiring the numerical assessment of posterior Probability Density Functions (PDF's) on a fine search grid. This paper therefore proposes a low complexity implementation of our iterative scheme and shows the achievable performance by simulations. We focus on an emerging cellular system scenario composed of small cells.

II. System Model

We consider a cellular system operating in Cyclic-Prefix OFDM (CP-OFDM). The MS is simultaneously connected and strictly time synchronized to K neighbouring base stations (BS's). We assume a single path channel introducing a delay $\tau_k(x, y)$ between the MS and BS k. This delay corresponds to the absolute ToF of the signal and is therefore equal to $\tau_k(x, y) = d_k(x, y)/c$ where c is the speed of light and $d_k(x, y) = \sqrt{(x - x_k)^2 + (y - y_k)^2}$. Coordinates $\{x, y\}$ and $\{x_k, y_k\}$ respectively denote the position of the MS and of base station k. As long as the delay $\tau_k(x, y)$ is shorter than the CP duration, sub-carriers remain orthogonal and the received signal on the subset $\mathcal{P} = \{q_1, ..., q_p\}$ of pilot sub-carriers can be modelled as follows for a single OFDM symbol:

$$\mathbf{r}_k = \mathbf{s}(\tau_k) + \mathbf{w}_k \tag{1}$$

where

$$\mathbf{r}_k = [r_{kq_1}, ..., r_{kq_P}]^T \tag{2}$$

$$\mathbf{w}_k = [w_{kq_1}, ..., w_{kq_P}]^T \tag{3}$$

and

$$\mathbf{s}(\tau_k) = [s_{q_1} e^{-j2\pi \frac{q_1 \tau_k}{QT}}, ..., s_{q_P} e^{-j2\pi \frac{q_P \tau_k}{QT}}]^T \tag{4}$$

with τ_k standing for $\tau_k(x, y)$. In the previous expressions, w_{kq} is the noise affecting sub-carrier q at base station k and s_q is the transmitted symbol on sub-carrier q. We suppose Additive White Gaussian Noise (AWGN) of variance $\sigma^2_{w_k}$.

We omit the extra phase rotation due to the carrier frequency in our model since it is corrected during the compensation of the phase difference between transmit and receive local oscillators.

III. Low Complexity Iterative Localization

The principle of our low complexity localization system is very similar to [5]. The algorithm makes use of the Bayes framework [6] to take into account prior knowledge from the previous iteration. Delay and position estimates are transmitted together with an indication of their reliability between the two steps. The position computed during a step of the algorithm is translated to a delay used as prior information by the next iteration.

A. Delay Estimation

We consider a Bayesian delay estimator that refines prior information received on the delay (mean and variance) using the pilot sub-carriers of the received OFDM signal. The complexity of the delay estimator can be drastically reduced compared to [5] by not considering the prior information on the delay to be Gaussian distributed but uniformly distributed on $[\tau_{k_{\min,i}}, \tau_{k_{\max,i}}]$ where i is the iteration index. Using the latter assumption and after some simplification, the posterior distribution of the delay reduces to:

$$p(\tau_k|\mathbf{r}_k) = \frac{p(\mathbf{r}_k|\tau_k)}{\int_{\tau_{k_{\min,i}}}^{\tau_{k_{\max,i}}} p(\mathbf{r}_k|\tau_k)d\tau_k} \quad (5)$$

$$= \frac{\exp\left(\frac{1}{\sigma^2_{w_k}}\Re\{\mathbf{r}_k^H \cdot \mathbf{s}(\tau_k)\}\right)}{P(\tau_{k_{\max,i}}) - P(\tau_{k_{\min,i}})} \quad (6)$$

where $P(\tau_k') = \int_{-\infty}^{\tau_k'} \exp(\frac{1}{\sigma^2_{w_k}}\Re\{\mathbf{r}_k^H \cdot \mathbf{s}(\tau_k)\})d\tau_k$. This integral represents the remaining terms of the cumulative distribution function of $p(\tau_k|\mathbf{r}_k)$ after some simplification and can be pre-computed before iterating. Each of the K base stations independently computes the mean and variance of τ_k by respectively assessing statistical expectations $\mu_{\tau_k|\mathbf{r}_k} = \mathcal{E}[\tau_k|\mathbf{r}_k]$ and $\sigma^2_{\tau_k|\mathbf{r}_k} = \mathcal{E}[(\tau_k - \mu_{\tau_k|\mathbf{r}_k})^2|\mathbf{r}_k]$ based on the knowledge of $p(\tau_k|\mathbf{r}_k)$. Those expectations require a numerical integration but the integration intervals are limited to $[\tau_{k_{\min,i}}, \tau_{k_{\max,i}}]$ that rapidly decreases with the iteration index. Mean values $\mu_{d_k} = c\mu_{\tau_k|\mathbf{r}_k}$ provide the Minimum Mean Square Error (MMSE) distance estimates used by the position estimator, i.e. $\hat{d}_k = \mu_{d_k}$. Those mean values are transmitted to the position estimator together with their variances $\sigma^2_{d_k} = c^2\sigma^2_{\tau_k|\mathbf{r}_k}$.

B. Position Estimation

The fusion centre independently makes an estimation of the MS position for each base station, based on the distance estimates of the $K-1$ other BS's. Those distance estimates are corrupted by an error e_k assumed as being of zero mean and of variance $\sigma^2_{d_k}$. Gathering observed distances of the $K-1$ other BS's, we get the following model for base station k:

$$\hat{\boldsymbol{d}}^k = \boldsymbol{d}^k(x,y) + \boldsymbol{e}^k \quad (7)$$

where

$$\hat{\boldsymbol{d}}^k = [\hat{d}_1, ..., \hat{d}_{k-1}, \hat{d}_{k+1}, ..., \hat{d}_K]^T \quad (8)$$

$$\boldsymbol{d}^k(x,y) = [d_1(x,y), ..., d_{k-1}(x,y), d_{k+1}(x,y), ...d_K(x,y)]^T \quad (9)$$

$$\boldsymbol{e}^k = [e_1, ..., e_{k-1}, e_{k+1}, ..., e_K]^T \quad (10)$$

The index of the target base station should be absent in those vector expressions.

Elements of $\boldsymbol{e}^k$ are independent and of possibly different variances. The noise covariance matrix is therefore diagonal and given by $C_{\boldsymbol{e}^k} = \mathrm{diag}\big([\sigma^2_{d_1}, ..., \sigma^2_{d_{k-1}}, \sigma^2_{d_{k+1}}, ..., \sigma^2_{d_K}]\big)$.

The Bayesian MMSE position estimator used in [5] can be replaced by the low complexity Weighted Linear Least Square (WLLS) estimator presented in [2]. This estimator simply solves a linearized version of the system of circular equations (7). After some manipulations, the latter system can be rewritten in the following matrix form:

$$\mathbf{b}_k = \mathbf{A}_k\boldsymbol{\theta}_k + \mathbf{q}_k \quad (11)$$

where $\boldsymbol{\theta}_k = [x \ y \ x^2 + y^2]^T$ is the vector to be estimated. The system matrix is $\mathbf{A}_k = [-2\mathbf{x}^k \ -2\mathbf{y}^k \ \mathbf{1}_{K-1}]$ where vectors $\mathbf{x}^k = [x_1, ..., x_{k-1}, x_{k+1}, ..., x_K]^T$ and $\mathbf{y}^k = [y_1, ..., y_{k-1}, y_{k+1}, ..., y_K]^T$ gather the coordinates of the $K-1$ base stations involved in the computation of the user position for BS k. Symbol $\mathbf{1}_{K-1}$ denotes a $K-1 \times 1$ unit vector. The observation vector in (11) is $\mathbf{b}_k = [(\hat{\boldsymbol{d}}^k)^2 - (\mathbf{x}^k)^2 - (\mathbf{y}^k)^2]$ and $\mathbf{q}_k = [2e^k\boldsymbol{d}^k(x,y)]$ is the noise term. The $()^2$ operator in the definition of $\mathbf{b}_k$ applies to each single element of the vectors. The estimate of $\boldsymbol{\theta}_k$ and the corresponding covariance matrix are obtained as [6, chap. 6]:

$$\hat{\boldsymbol{\theta}}_k = \left(\mathbf{A}_k^T\mathbf{W}_k^{-1}\mathbf{A}_k\right)^{-1}\mathbf{A}_k^T\mathbf{W}_k^{-1}\mathbf{b}_k \quad (12)$$

$$C_{\hat{\boldsymbol{\theta}}_k} = \left(\mathbf{A}_k^T\mathbf{W}_k^{-1}\mathbf{A}_k\right)^{-1}. \quad (13)$$

It follows from the definition of the noise vector $\mathbf{q}_k$ that a practical choice for the weighting matrix is $\mathbf{W}_k = 4C_{\boldsymbol{e}^k}\mathrm{diag}\big((\hat{\boldsymbol{d}}^k)^2\big)$. The user coordinate estimates $\hat{x}$ and $\hat{y}$ are respectively given by the first and second elements of $\hat{\boldsymbol{\theta}}_k$. Variances of the coordinates $\sigma^2_{x|\hat{\boldsymbol{d}}^k}$ and $\sigma^2_{y|\hat{\boldsymbol{d}}^k}$ correspond to the first two diagonal elements of $C_{\hat{\boldsymbol{\theta}}_k}$ while the covariance term is equal to $\Gamma_{xy|\hat{\boldsymbol{d}}^k} = C_{\hat{\boldsymbol{\theta}}_k,12}$.

C. Position to Delay Conversion

Those position informations are easily converted to a delay information used as prior information at the next iteration. By linearizing the relationship between the user coordinates and the delay to the k^{th} base station around the estimated position, we get similarly to [5]:

$$\hat{\tau}_k \approx \frac{1}{c}d_k(\hat{x},\hat{y}) \quad (14)$$

$$\sigma^2_{\tau_k} \approx \frac{1}{c^2 d_k^2(\hat{x},\hat{y})}\begin{bmatrix} x_k - \hat{x} \\ y_k - \hat{y}\end{bmatrix}^T \cdot \begin{bmatrix} \sigma^2_{x|\hat{\boldsymbol{d}}^k} & \Gamma_{xy|\hat{\boldsymbol{d}}^k} \\ \Gamma_{xy|\hat{\boldsymbol{d}}^k} & \sigma^2_{y|\hat{\boldsymbol{d}}^k}\end{bmatrix} \cdot \begin{bmatrix} x_k - \hat{x} \\ y_k - \hat{y}\end{bmatrix}. \quad (15)$$

Limits of the uniform delay distribution used as prior information at BS k are deduced from the delay estimate and its variance:

$$\tau_{k_{\min}} = \hat{\tau}_k - \sqrt{3\sigma^2_{\tau_k}} \quad (16)$$

$$\tau_{k_{\max}} = \hat{\tau}_k + \sqrt{3\sigma^2_{\tau_k}} \quad (17)$$

IV. Results

To estimate the performance of our low complexity iterative localization system, we assume a two-dimensional cell layout with an inter BS distance of 100 m and an hexagonal structure. The MS arbitrarily lies in the gray zone of Fig. 1 defined around a centre base station and communicates in the

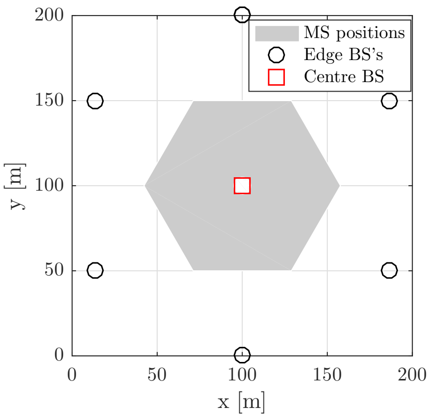

Fig. 1: Simulation scenario.

uplink with the 7 closest BS's using the OFDM modulation over a bandwidth of 40 MHz. Pilot symbols are distributed on 64 equispaced sub-carriers among 1024. We consider an AWGN channel and model the SNR at base station k as [7]:

$$\mathrm{SNR} = \xi d_k(x, y)^{-n} \tag{18}$$

where ξ describes the relationship between the SNR and the euclidian distance separating the MS from base station k. Coefficient ξ depends on the hardware, the transmit power, the carrier frequency, the temperature and the communication bandwidth. The path-loss exponent n is considered equal to 3. Algorithm performances depicted in this paper are averaged over 1000 MS position and noise realisations.

Fig. 2 depicts the average localization error as a function of the SNR at the centre BS. Our original (or MMSE) iterative formulation [5] is compared to the low complexity version of Section III and to the conventional two-step and direct approaches. The low complexity iterative estimation performs very close to the original formulation. The convergence of the low complexity solution is illustrated in Fig. 3. The performance gain becomes negligible after six iterations.

In terms of implementation complexity, our original algorithm formulation requires for each iteration to compute posterior PDF's on a fine and fixed grid for both delay and position estimations. This makes the complexity of the original formulation close to the DPE solution after a few iterations. Replacing the Bayesian MMSE position estimator of [5] by the WLLS estimator of Section III drastically reduces the complexity increment of each iteration in our low complexity formulation. Considering a uniformly distributed prior for the delay estimator allows to shrink the search intervals with the iteration index. Although the cost of the reformulated delay estimation step dominates for the first iteration, it becomes negligible compared to the WLLS position estimator after a few iterations. This makes the complexity increment linked to iterations two to six negligible compared to the first iteration, the computational cost of this first iteration being comparable to the two-step approach.

V. Conclusion

In this paper, we propose a low complexity formulation of a time-of-flight based iterative localization method. Our approach drastically reduces the computational effort compared to the formulation proposed in literature and proves to have an implementation cost comparable to the traditional

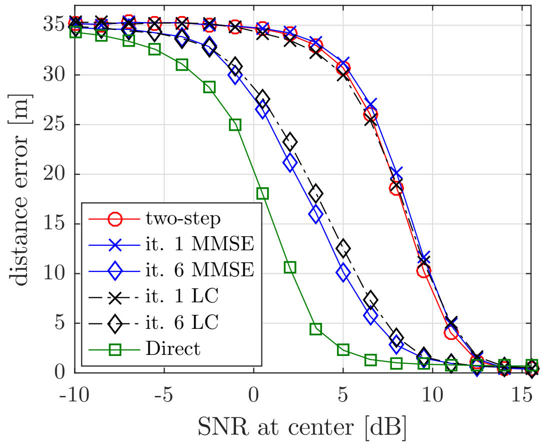

Fig. 2: Performance comparison of the localization algorithms. Average distance error as a function of the SNR at the centre BS.

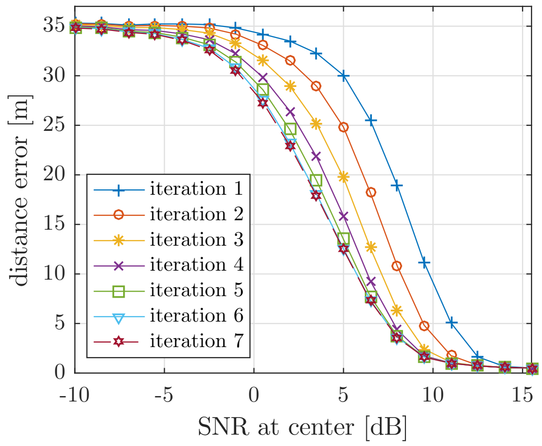

Fig. 3: Convergence of the low complexity iterative localization algorithm.

two-step localization. We show by numerical simulations that our efficient algorithm performs close to the optimal direct localization solution at a much lower computational cost.

Acknowledgment

The authors would like to thank the FNRS/FRIA for financial support.

References

[1] C. Cox, *An Introduction to LTE: LTE, LTE-Advanced, SAE and 4G Mobile Communications*. Wiley, 2012.

[2] R. Zekavat and R. M. Buehrer, *Source Localization: Algorithms and Analysis*, pp. 25–66. Wiley-IEEE Press, 2012.

[3] P. Closas, C. Fernandez-Prades, and J. A. Fernandez-Rubio, "Maximum likelihood estimation of position in gnss," *IEEE Signal Processing Letters*, vol. 14, pp. 359–362, May 2007.

[4] A. Amar and A. J. Weiss, "New asymptotic results on two fundamental approaches to mobile terminal location," in *2008 3rd International Symposium on Communications, Control and Signal Processing*, pp. 1320–1323, March 2008.

[5] F. Horlin, M. V. Eeckhaute, T. V. der Vorst, A. Bourdoux, F. Quitin, and P. D. Doncker, "Iterative ToA-based Terminal Positioning in Emerging Cellular Systems," in *IEEE International Conference on Communications*, pp. 1–5, May 2017.

[6] S. Kay, *Fundamentals of Statistical Signal Processing: Detection theory*. Prentice Hall Signal Processing Series, Prentice-Hall PTR, 1993.

[7] H.-L. Song, "Automatic vehicle location in cellular communications systems," *IEEE Transactions on Vehicular Technology*, vol. 43, pp. 902–908, Nov 1994.