Graz University of Technology

Institut für Statistik

**Seminarvortrag**

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# Optimal transport between random measures, totally convex functions and the Brenier theorem


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Abstract

The first part of this talk is devoted to recall the basics on the Optimal Transport problem. In particular, the existence of optimal transport plans and their characterization through $c$-concavity functions. When we apply this theory to the Euclidean setting, using the squared distance as cost, $c$-concavity boils down to usual convexity, and this allows us to prove the celebrated Brenier theorem about the existence and uniqueness of optimal transport maps. In the second part, I will present recent results concerning the Optimal Transport problem between laws of random measures in the Wasserstein-on-Wasserstein space $\mathcal{P}_2(\mathcal{P}_2(H))$, built over a Hilbert space $H$. Despite the lack of smoothness of the cost, the fact that the space $\mathcal{P}_2(H)$ is not Hilbertian, and the curvature distortion induced by the underlying Wasserstein metric, we will show how to recover, at the level of random measures in $\mathcal{P}_2(\mathcal{P}_2(H))$, the same deep and powerful results linking Euclidean Optimal Transport problems in $\mathcal{P}_2(H)$ and convex analysis. Our approach relies on the notion of totally convex functionals, on their total subdifferentials, and on their Lions-Lagrangian liftings in the space of square-integrable $H$-valued maps $L^2(Q, \mathbb{M}; H)$. With these tools, we identify a natural class of regular measures on $\mathcal{P}_2(H)$ for which the Monge formulation of the OT problem has a unique solution, and, when $H$ has finite dimension, we will show that this class includes relevant examples of measures with full support arising from the push-forward of nondegenerate Gaussian measures in $L^2(Q, \mathbb{M}; H)$. The talk is based on a joint work with Giuseppe Savaré.


G. Pammer