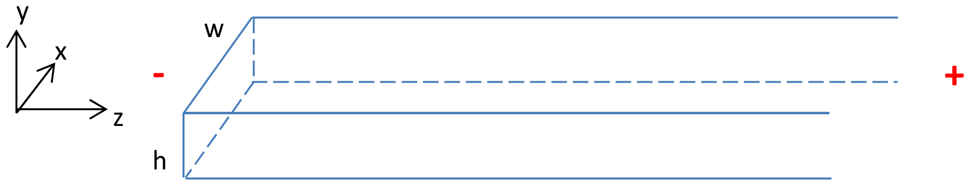
Exercise 4: coupling electrokinetics and hydrodynamics

Introduction

In this exercise session, we focus on electrokinetics of microsystems: electro-osmosis allow for generating plug flows in micro-canals preventing Taylor-Aris dispersion while electrophoresis is used to separate small particles/molecules having different size/charge ratios by applying an electrical field. We propose in this exercise session to review these typical principles and to extend their application to trap particles by coupling the electrokinetics transport to the hydrodynamics one.

Particle transport and separation in straight microchannels of uniform cross-section

In this first part we consider straight microchannels of rectangular cross section with low aspect ratio (h<<w) as sketched in the figure below. The channel as a length L and is filled with a fluid which has a viscosity µ.



Pressure driven flow

1) Under which conditions can the problem be treated in 2D (xz plan)? In this case, what is the velocity profile generated by applying a pressure gradient of ∆P/L where ∆P=P(z=0)-P(z=L)>0. In which direction is the fluid flowing? For ∆P=1mbar, 2mbar and 4mbar, L=3cm, h=20µm, w=80µm and 300µm which maximal velocities $v_{p,max}$ are expected? For the narrowest channels, with a D-shaped cross section, velocities of 130 and 265 µm/s are reported for 1 and 2mbar. Does this agree with your findings?

Electro-osmotic flow

An external electric field of intensity E is applied (cathode in z=0, anode in z=L). The channel is made of glass deprotonated with borate buffer and shows a negative surface charge density.

2) What are the signs of the adsorbed and mobile counter ions in the electrical double layer? In which direction is the fluid flowing under electro-osmosis effects? Recall the

relation giving the electro-osmosis velocity noting ζ_w the zeta potential at the plane of shear in the electrical double layer near the wall. Considering the liquid properties are similar to those of water, which typical velocities would you obtain applying 100, 250 and 375V? Compare to the values obtained for pressure driven flows.

Electrophoretic migration of charged particles in applied electric field

We consider suspended particles whose radius R is large by comparison to κ^{-1}, the Debye length.

3) Give the expression of the electrophoretic velocity of such a particle. We note ζ_p the particle zeta potential. What is the apparent mobility of such a particle in an electro-osmotic flow? How can you express the resulting velocity?

Superposition of transport mechanisms: results and discussion

We consider suspended particles with negative ζ_p and applied a pressure gradient $\Delta P/L$.

4) In which direction is the net displacement of such particles? Is it in agreement with the results obtained by Jellema et al. and reproduced below? Can particles be separated using such a device? Can they be trapped?

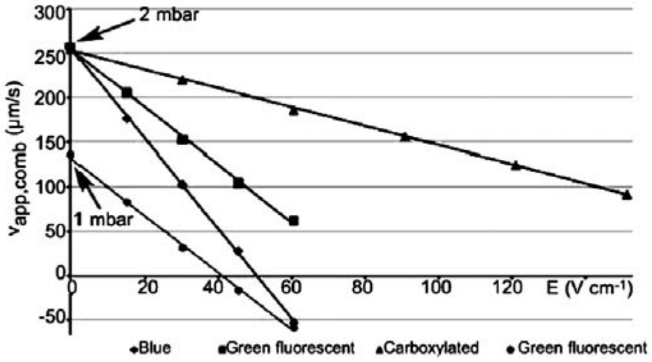


Fig. 4 Curves of apparent bead velocities, $v_{app,comb}$, under conditions of combined applied pressure and electric field, as a function of applied electric field (V cm^{-1}). The curves were constructed from PIV data for bead velocities, obtained independently for electrokinetic and pressure-driven bead transport (see text for details). Curves for three bead types, namely blue, green fluorescent, and carboxylated polystyrene beads, are presented for 2 mbar applied pressure (starting $v_{app,comb} = 255$ µm s^{-1}). A second curve for green fluorescent particles is also shown for 1 mbar pressure, for reasons stated in the text ($v_{app,comb} = 133$ µm s^{-1}). Positive values of bead velocity indicate movement in the direction of PF, negative values in the direction of EOF. Buffer used: 10 mM borate, pH 9.2.

Lap Chip, 2009, 9, 1914-1925
Jellema et al.

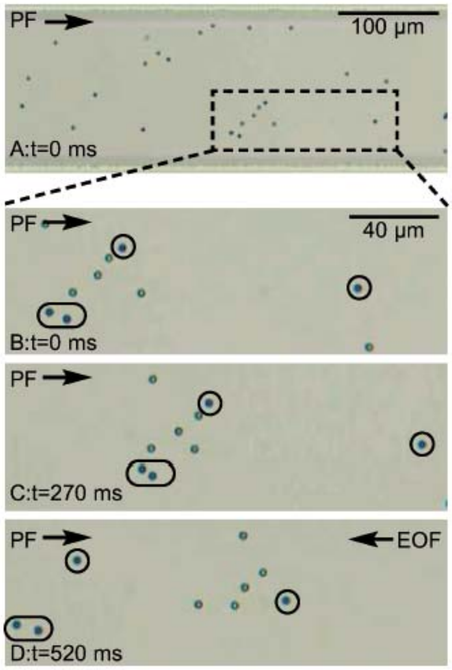
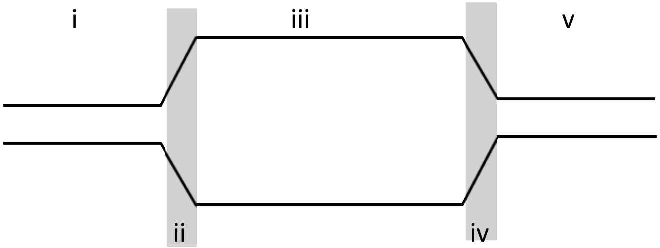


Fig. 5 Frames taken from the video of a separation experiment of two bead types with $\Delta\zeta$ of 35 mV in a straight channel. Pressure applied: 4 mbar. Solution: 10 mM borate buffer, pH 8.9. (A) Beads are transported with the PF in a 135 µm-wide microchannel. (B and C) Close-up of low-ζ_p (blue, circled) and high-ζ_p (white) beads transported with PF. (D) Low-ζ_p and high-ζ_p beads are transported with EOF and PF, respectively, when applying 350 V.

Particle trapping in diverging and converging channel



We now consider a channel with a succession of “diverging” (ii) and focusing” (iv) sections separating straight channels of constant small and large cross sections (iii and i/v, respectively).

Pressure driven flow

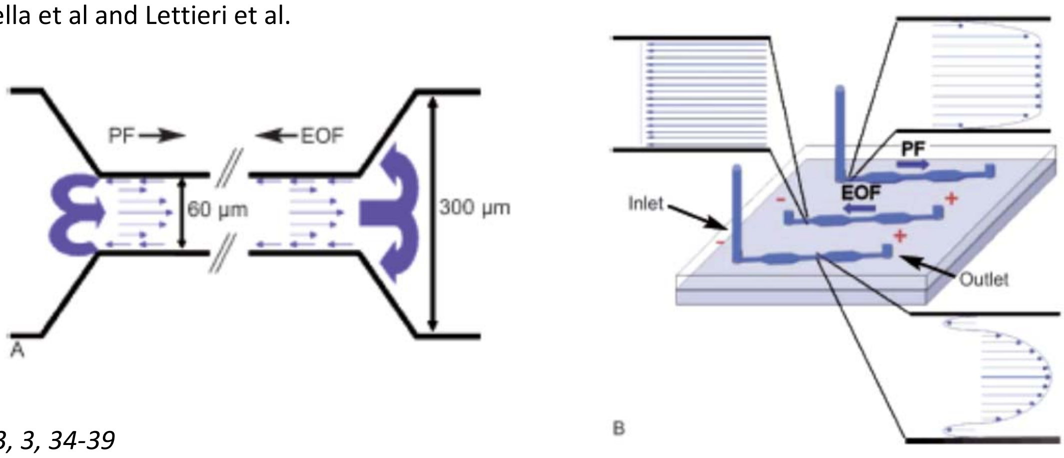
5) Give the expression of R_{hydro} the hydrodynamic resistance of the channel (height h) in a section i (broadness w_i)? In a section iii (broadness $w_{iii}>w_i$)? Qualitatively, how does the pressure evolves along a channel as the one depicted below? Draw the corresponding velocity profiles.

Osmotic flow

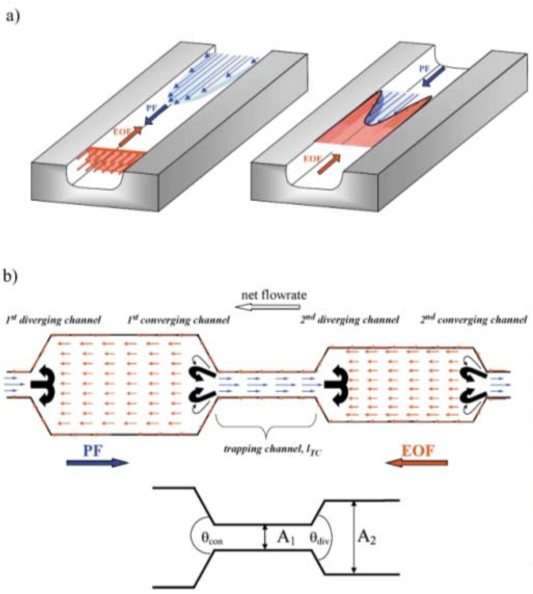
6) Using Ohm´s law, give the expression of E_j, the electric field in each section j as a function of V_j, L_j, R_j and i corresponding to the potential difference, the length, the electrical resistance of the segment j and the electrical current throughout the device. Can you express R_j as a function of the liquid resistivity ρ and about characteristic of the channel. What can you conclude the evolution of E along the channel? What does it imply for the electro-osmotic flow?

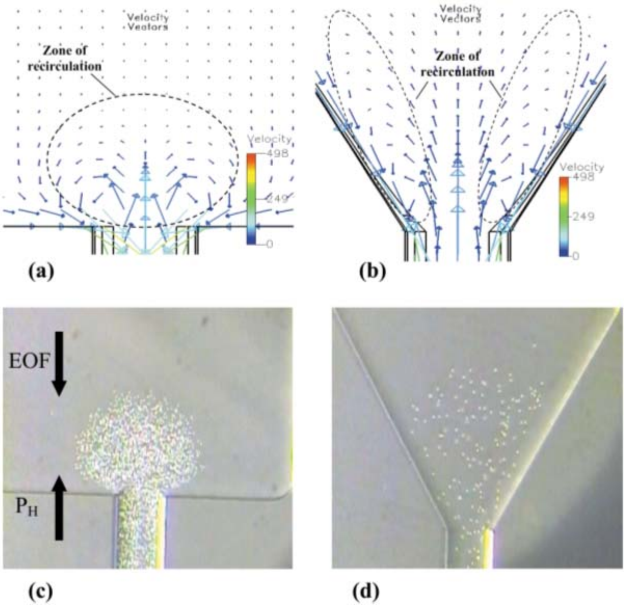
General flow field an trapping conditions

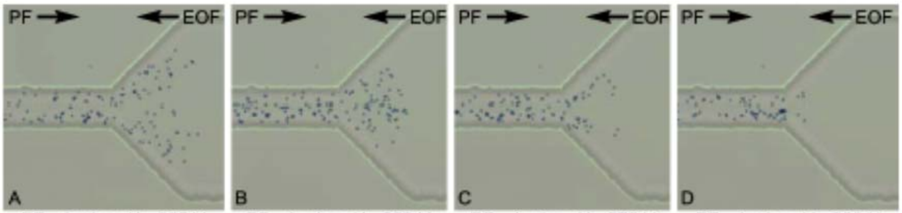
A qualitative description of the flow field is presented in the figure below together with some results from Jewella et al and Lettieri et al.



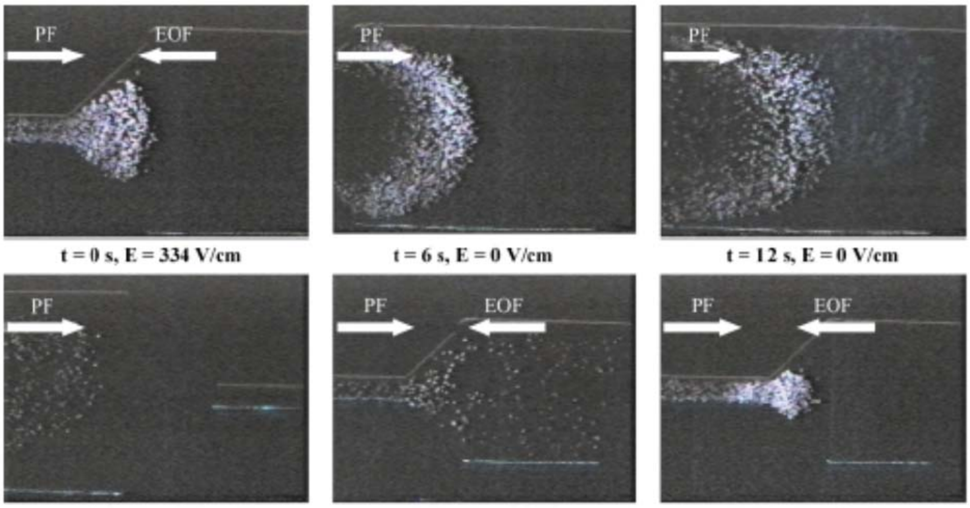
Lap Chip, 2003, 3, 34-39
Lettieri et al.







PF= 4 mbar, U= 225 V
PF= 4 mbar, U= 250 V
PF= 4 mbar, U= 275 V
PF= 4 mbar, U= 300 V



t = 0 s, E = 334 V/cm
t = 6 s, E = 0 V/cm
t = 12 s, E = 0 V/cm
t = 34 s, E = 0 V/cm
t = 61 s, E = 334 V/cm
t = 70 s, E = 334 V/cm

7) Can you explain the tunability of the trapping device (with varying E for a given ζ_p and a given geometry)? How would particles with a larger ζ_p behave? And those with a smaller ζ_p? Which applications could derive from this?