## 19. Steirisches Seminar über Regelungstechnik und Prozessautomatisierung

## 07. – 09. September 2015, Schloss Retzhof

### **CONFERENCE PROGRAMME**

## Monday 07.09.2015

- 17:00 18:00 Registration
- 18:00 Dinner

## Tuesday 08.09.2015

- 08:30 09:10 Alexander Barth Lyapunov-based Design of Adaptive Sliding Mode Controller
- 09:10 09:50 Leonid Fridman HOSM Based Observation, Identification and Output Based Control
- 09:50 10:30 Coffee Break
- 10:30 11:10 Jaime A. Moreno Discontinuous integral control for mechanical systems
- 11:10 11:50 Stefan Koch On Discretization of Sliding Mode Based Control Algorithms
- 12:00 13:30 Lunch

#### SOCIAL PROGRAMME

### Wednesday 09.09.2015

- 08:30 09:10 Robel Besrat Observers for non-linear MIMO descriptor systems
- 09:10 09:50 Alexander Schirrer; Stefan Jakubek Optimization-based formulations of absorbing boundary conditions in discrete-time wave propagation problems
- 09:50 10:30 Johann Reger Estimating Parameters and States Using Modulating Functions
- 10:30 11:00 Coffee Break

- 11:00 11:40 Mikulas Huba Comparing model-free and disturbance observer based control
- 11:40 12:20 Georg Stettinger Stability Analysis of interconnected Linear Systems with Coupling Imperfections
- 12:30 14:00 Lunch
- 14:00 14:40 Stefan Doczy Design and Validation of an SOH-Algorithm for Lithium Ion Batteries
- 14:40 15:20 Gernot Druml Effects of the nonlinear Arc of a 20-kV-net single-line cable fault on the Earth-Fault-Detection and -Control
- 15:20 15:50 Coffee Break
- 15:50 16:30 Christopher Zemann Model based control of a biomass fired steam boiler
- 16:30 17:10 Markus Freistätter Control-oriented Turbocharger Modeling
- 17:10 17:50 Soultana Vasileiadou Philon of Byzantium and his work on Pneumatic and Hydraulic Control Systems

Dinner

## Lyapunov-based Design of Adaptive Sliding Mode Controllers

#### Alexander Barth<sup>a</sup> Markus Reichhartinger<sup>b</sup> Johann Reger<sup>a</sup> Martin Horn<sup>b</sup> Kai Wulff<sup>a</sup>

Consider the nonlinear system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x}) (\Delta(\mathbf{x}, t) + u)$$
(1)

where  $\mathbf{x}(t) \in \mathbb{R}^n$  is the state,  $u(t) \in \mathbb{R}$  a scalar control input,  $\mathbf{f}$  and  $\mathbf{g}$  are known differentiable vector fields. Assume that the matched uncertainty  $\Delta$  may be decomposed as per

$$\Delta(\mathbf{x},t) = \Delta_{\mathrm{s}}(\mathbf{x}) + \Delta_{\mathrm{u}}(\mathbf{x},t) = \boldsymbol{\Theta}^{\mathrm{T}} \boldsymbol{\Phi}(\mathbf{x}) + \Delta_{\mathrm{u}}(\mathbf{x},t)$$
(2)

such that  $\Delta_{s}(\mathbf{x}) = \Theta^{T} \mathbf{\Phi}(\mathbf{x})$  represents the structured uncertainty, with unknown parameter vector  $\Theta$  and known base function  $\mathbf{\Phi}$ , and  $\Delta_{u}(\mathbf{x}, t)$  comprises the unstructured uncertainty and external disturbances.

Let a sliding variable  $\sigma = \sigma(\mathbf{x})$  be selected such that a desired dynamics is imposed on the sliding manifold  $\sigma \equiv 0$ . Further, let  $\sigma$  be of relative degree one wrt. the input u and assume the associated internal dynamics to be stable. Thus, along the solution the derivative reads

$$\dot{\sigma} = \underbrace{\Theta^{\mathrm{T}} \mathbf{a}_{1}(\mathbf{x}) + \mathbf{a}_{2}(\mathbf{x}, t)}_{=\phi(\mathbf{x}, t)} + \omega(\mathbf{x}, u) \tag{3}$$

where  $\mathbf{a}_1(\mathbf{x})$ ,  $\mathbf{a}_2(\mathbf{x}, t)$  as well as  $\omega(\mathbf{x}, u)$  may be expressed in terms of equations (1) and (2). Clearly, expression  $\phi(\mathbf{x}, t)$  captures the entire uncertainty and  $\omega$  is a known function, that given  $\mathbf{x}$ , is bijective wrt. the control input u.

The goal is to devise a controller  $\omega$  that stabilizes the origin of (3).

Neglecting the knowledge about the structure within the uncertainty and requiring that the entire uncertainty be uniformly bounded according to

$$|\phi(\mathbf{x},t)| \le \Omega_{\phi} |\sigma(\mathbf{x})|^{\frac{1}{2}} \tag{4}$$

for some  $\Omega_{\phi} > 0$  Shtessel et al. gave a solution to this problem [7, 8]. However, the square-root growth bound on the uncertainty may be restrictive in many practical situations.

Therefore, we propose to treat the structurally known part  $\Theta^{T} \mathbf{a}_{1}(\mathbf{x})$  of the uncertainty  $\phi(\mathbf{x}, t)$  separately from the unstructured part  $\mathbf{a}_{2}(\mathbf{x}, t)$ . This allows to relax the requirement (4) considerably such that no upper bound is needed for the uncertainty  $\phi(\mathbf{x}, t)$ . Only for the unstructured part  $\mathbf{a}_{2}(\mathbf{x}, t)$  we still require that

$$|\mathbf{a}_2(\mathbf{x},t)| \le \Omega_{a_2} \, |\sigma(\mathbf{x})|^{\frac{1}{2}} \tag{5}$$

uniformly for some  $\Omega_{a_2} > 0$ .

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**Main result:** In view of the super-twisting algorithm [4] we propose the following dynamic state-feedback with adaptive extension:

$$\omega = -k_1 |\sigma(\mathbf{x})|^{\frac{1}{2}} \operatorname{sign}(\sigma(\mathbf{x})) + \nu - \hat{\mathbf{\Theta}}^{\mathrm{T}} \mathbf{a}_1(\mathbf{x})$$
  

$$\dot{\nu} = -k_2 \operatorname{sign}(\sigma(\mathbf{x}))$$
  

$$\dot{\hat{\mathbf{\Theta}}} = \gamma k_2 \operatorname{sign}(\sigma(\mathbf{x})) \mathbf{a}_1(\mathbf{x})$$
(6)

with  $\nu(0) = 0$ ,  $\hat{\Theta}(0) = \hat{\Theta}_0$  for some  $\hat{\Theta}_0$ , controller state  $\nu$ , and parameters  $k_1, k_2, \gamma > 0$ .

In view of an adaptive extension of a Lyapunov-function presented in [6], employing the weak Lyapunov-function

$$V(\mathbf{x},\nu,\hat{\mathbf{\Theta}}) = k_2 |\sigma(\mathbf{x})| + \frac{1}{2}\nu^2 + \frac{1}{2\gamma}(\hat{\mathbf{\Theta}} - \mathbf{\Theta})^{\mathrm{T}}(\hat{\mathbf{\Theta}} - \mathbf{\Theta})$$
(7)

we show that the origin of the closed-loop system (3) is stable.

In our approach, the gains  $k_1$  and  $k_2$  of the controller (6) may be reduced significantly when compared, for example, with the adaptive-gain super-twisting algorithm [7] or its modification in [8]. For further details on our proposed approach, see [1].

As a general remark note that whenever given a Lyapunov-function for the nominal system that satisfies some continuity assumptions, e.g. in [5], the proposed approach may lead to novel families of adaptive sliding-mode controllers.

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## HOSM Based Observation, Identification and Output Based Control

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The key point of this study are *Robust Exact Differentiators* [1,2]. Such differentiators in the absence of noise, converge to the true derivatives of the signal after a finite time. Moreover, they provide best possible asymptotic precision with respect to sampling steps and measurement noise.

The paper presents The presentation contains the overview of results of Sliding Mode Control Laboratory on higher order sliding mode observation.

Fixed-time robust exact differentiators [3,4]. An arbitrary order differentiator that, in the absence of noise, converges to the true derivatives of the signal after a fixed time independent of the initial differentiator error is presented.

Robust exact observation of strongly observable and detectable systems [5-7]. A global observer is designed for strongly detectable LTI systems with bounded unknown inputs. The design of the observer is based on three steps. Firstly, the system is extended taking the unknown inputs (and possibly some of their derivatives) as a new state; then, using a HSOM differentiator, a new output of the system is generated in order to fulfil, what we will call, the strong observability condition, which finally decomposing the system, in new coordinates, into two subsystems; the first one being unaffected directly by the unknown inputs, and the state vector of the second subsystem is obtained directly from the original system output.

Such decomposition permits designing of a Luenberger observer for the first subsystem, which satisfies the strong observability condition, i.e. all the outputs have relative degree one w.r.t. the unknown inputs. This procedure enables one to estimate the state and the unknown inputs using the least number of differentiations possible.

Robust exact output control based on HOSM observation [8.9]. Semi-global finite-time exact stabilization of linear time-invariant systems with matched disturbances is attained using a dynamic output feedback, provided the system is controllable, strongly observable and the disturbance has a bound affine in the state norm. The novel non-homogeneous HOSM control strategy is based on the gain adaptation of both the controller and the differentiator included in the feedback. A robust criterion is developed for the detection of differentiator convergence to turn on the controller at a proper time.

Robust exact output control based on HOSM identification of perturbation[10-12]. The problem of robust exact output control for linear systems with smooth bounded matched unknown inputs is considered. The higher order sliding mode observers provide both theoretically exact observation and unknown input identification. A methodology is proposed to select the most adequate output control strategy for matched perturbations compensation. The possibility for theoretically exact uncertainties compensation using signals identified by HOSM observers. Towards this aim, we modify the hierarchical super-twisting observer in order to have the best possible observation and identification accuracy. Then,

two controllers are compared. The first one is an integral sliding mode controller based on the observed values of the state variables. The other strategy is based on the direct compensation of matched perturbations using their identified values. The performance of both controllers is estimated in terms of the deterministic noise upper bounds, sampling step and execution time. Based on these estimations, the designer may select the proper controller for the system.

A collection of the papers with different type of HOSM based observers, can be found on the web site http://verona.fi-p.unam.mx/~lfridman/

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## **Discontinuous integral control for mechanical systems**

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#### RESUMÉ

We consider a second order system

$$\dot{\xi}_{1} = \xi_{2} 
\dot{\xi}_{2} = f(\xi_{1}, \xi_{2}, t) + \rho(t) + \tau$$
(1)

where  $\xi_1 \in \mathbb{R}$  and  $\xi_2 \in \mathbb{R}$  are the states,  $\tau \in \mathbb{R}$  is the control variable,  $f(\xi_1, \xi_2, t)$  is some known function, while the term  $\rho(t)$  corresponds to uncertainties and/or perturbations. System 1 can represent a mechanical system, where  $\xi_1$  is the position and  $\xi_2$  is the velocity. An important control task is to track a smooth time varying reference r(t), i.e. if one defines the tracking error  $z_1 = \xi_1 - r$  and  $z_2 = \xi_2 - \dot{r}$ the objective is to asymptotically stabilize the origin of system

$$\dot{z}_{1} = z_{2} 
\dot{z}_{2} = f(\xi_{1}, \xi_{2}, t) + \rho(t) - \ddot{r}(t) + \tau.$$
(2)

With the control  $\tau = u - f(\xi_1, \xi_2, t) + \ddot{r}(t)$  the system becomes

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u + \rho(t) , \end{aligned}$$

$$(3)$$

where the perturbation  $\rho(t)$  is a time varying signal, not vanishing at the origin (i.e. when x = 0 the perturbation can still be acting). We notice that it is possible not to feed the second derivative of the reference  $\ddot{r}(t)$  to the control  $\tau$ . In this case it will be considered as part of the perturbation term  $\rho(t)$ .

Under the stated hypothesis it is well known that a continuous, memoryless state feedback u = k(x) is not able to stabilize x = 0. This is so, because the controller has to satisfy with the condition k(0) = 0, since the closed loop has to have an equilibrium at the origin for vanishing perturbation. But if the perturbation does not vanish, then the origin cannot be anymore an equilibrium point. Discontinuous controllers, as the first order Sliding Mode (SM) ones [3], [2] are able to solve the problem for non vanishing (or persistently acting) bounded perturbations. However, they require the design of a sliding surface that is reached in finite time, but the target x = 0 is attained only asymptotically fast, and at the cost of a high frecuency switching of the control signal (the so called *chattering*), that has a negative effect in the actuator, and excites unmodelled dynamics of the plant. Higher Order Sliding Modes (HOSM) [4], [7], [6], [5], [9] provide a discontinuous controller for systems of relative degree higher than one to robustly stabilize the origin x = 0 despite of bounded perturbations, but again at the expense of chattering. A natural alternative consists in adding an integrator, i.e. defining a new state  $z = u + \rho(t)$ , with  $\dot{z} = v$  and designing a third order HOSM controller for the new control variable v. This allows to reach the origin in finite time, and it will be insensitive to Lipschitz perturbations, i.e. with  $\dot{\rho}(t)$  bounded. In this form a continuous control signal u will be obtained, so that the chattering effect is reduced. However, this requires feedback not only the two states  $x_1$  and  $x_2$  but also the state z, which is unknown due to the unknown perturbation. Moreover, to implement an output feedback controller (assuming that only the position  $x_1$  is measured) it is necessary to differentiate two times the position  $x_1$ , with the consequent noise amplification effect.

In the case of (almost) constant perturbations  $\rho(t)$  a classical solution to the robust regulation problem is the use of integral action, as for example in the PID control [1]. The linear solution would consist of a state feedback plus an integral action,  $u = -k_1x_1 - k_2x_2 + z$ ,  $\dot{z} = -k_3x_1$ . This controller requires only to feedback the position and the velocity. For an output feedback it would be only necessary to estimate the velocity (with the D action for example). In contrast to the HOSM controller this PID control is only able to reject constant perturbations, instead of Lipschitz ones, and it will reach the target only exponentially fast, and not in finite time. By the Internal Model Principle it would be possible to reject exactly any kind of time varying perturbations  $\rho(t)$ , for which a dynamical model (an exosystem) is available. However this would increase the complexity (order) of the controller, since this exosystem has to be included in the control law.

Here we provide a solution to the problem, that is somehow an intermediate solution between HOSM and PID control. Similar to the HOSM control our solution uses a discontinuous integral action, it can compensate perturbations with bounded derivative ( $\rho(t)$  is Lipschitz) and the origin is reached in finite time. So it can solve not only regulation problems (where  $\rho$  is constant) but also tracking problems (with  $\rho$  time varying) in finite time and with the same complexity of the controller. Similar to the PID control the proposed controller provides a continuous control signal (avoiding chattering) and it requires only to feedback position and velocity. We also provide for a (non classical) D term, i.e. a finite time converging observer, to estimate the velocity. This basic idea has been already presented in our previous work [10]. In the present one we give a much simpler Lyapunov-based proof, and we also include an observer in the closed loop together with its Lyapunov proof. Our solution can be seen as a generalization of the Super Twisting control for systems of relative degree one [9], [4], [7], [5], [8] to systems with relative degree two. Extensions to arbitrary order is also possible and it will be briefly discussed.

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## On Discretization of Sliding Mode Based Control Algorithms

#### Stefan Koch<sup>a</sup>

Variable structure control techniques are one proper method to deal with control problems for uncertain or disturbed systems. In the last decades especially sliding mode concepts which can be used in controllers or observers have stood out over classical approaches, due to their simplicity and advantages such as robustness and finite time convergence. In order to achieve an ideal sliding mode, it is assumed that the switching of a discontinuous control input takes place at an infinitely high frequency. However, when it comes to digital implementations of such controllers this assumption cannot hold true anymore and switching delays limit the existence of a true sliding mode. Consequently, in a time discretized sliding mode system, the invariance property of the sliding manifold is deteriorated and trajectories form limit cycles around the sliding surface. The characteristics of this effects, which often are referred to as discretization chattering, are influenced by the discretization method, the selected sampling frequency as well as the choice of parameters [3, 4]. In order to maintain good control performance or estimation precision after digitization, the discrete time sliding mode system may be modified compared to its explicit discretized continuous time counterpart [1, 2].

This talk will focus on discretization effects in conventional (first-order) and super-twisting based sliding mode control systems. Recent approaches involving various discretization schemes of equivalent control based methods are summarized. The influence of the choice of the sampling frequency and controller parameters on the control precision are discussed.

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## Observers for non-linear MIMO descriptor systems

Robel Besrat<sup>a</sup> Felix Gausch<sup>a</sup> Nenad Vrhovac<sup>b</sup>

This article presents non-linear observers for exact I/O-linearizable descriptor systems with a specific focus on the semi-explicit form:

$$\dot{\boldsymbol{x}} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{z}) + \boldsymbol{B}(\boldsymbol{x}, \boldsymbol{z})\boldsymbol{u}$$
 (1a)

$$\mathbf{0} = \boldsymbol{p}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{u}) \tag{1b}$$

$$\boldsymbol{y} = \boldsymbol{c}(\boldsymbol{x}, \boldsymbol{z})$$
 (1c)

where  $\boldsymbol{x} \in \mathbb{R}^n$  and  $\boldsymbol{z} \in \mathbb{R}^p$  denote the vectors of the differential and the algebraic variables respectively. Furthermore, we assume that there are the same number of input variables  $\boldsymbol{u} \in \mathbb{R}^m$  and output variables  $\boldsymbol{y} \in \mathbb{R}^m$ .

In addition to the constraints explicitly specified in (1b) the differential algebraic equations (1a, 1b) include further implicit constraints for higher-index systems. All these constraints contained in (2) describe the manifold  $\mathcal{M}$ , where the solutions  $\boldsymbol{x}(t)$  and  $\boldsymbol{z}(t)$  of (1a, 1b) are located:

$$\mathbf{0} = \tilde{\boldsymbol{p}}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{u}) \,. \tag{2}$$

Following the preliminary work in [1] and [3], as part of the exact I/O-linearization for descriptor systems, an observer is developed that incorporates the impact of all the constraints during the estimation process.

To determine the observer structure, the explicit representation (3) of the non-linear descriptor system (1):

$$\dot{\boldsymbol{w}} = \begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{z}) + \boldsymbol{B}(\boldsymbol{x}, \boldsymbol{z}) \boldsymbol{u} \\ - \begin{bmatrix} \frac{\partial \boldsymbol{p}_{k-1}}{\partial \boldsymbol{z}} \end{bmatrix}^{-1} \frac{\partial \boldsymbol{p}_{k-1}}{\partial \boldsymbol{x}} (\boldsymbol{a}(\boldsymbol{x}, \boldsymbol{z}) + \boldsymbol{B}(\boldsymbol{x}, \boldsymbol{z}) \boldsymbol{u}) \end{bmatrix} = \boldsymbol{f}(\boldsymbol{w}) + \boldsymbol{G}(\boldsymbol{w}) \boldsymbol{u}$$
(3)  
$$\boldsymbol{y} = \boldsymbol{c}(\boldsymbol{w})$$

is transferred to Byrnes-Isidori normal form by choosing a suitable transformation (diffeomorphism)  $\boldsymbol{\xi} = \boldsymbol{\varphi}(\boldsymbol{w}) = [\boldsymbol{\zeta}^T, \boldsymbol{\eta}^T]^T$  and subsequently introducing a state feedback  $\boldsymbol{u} = (\boldsymbol{\zeta}, \boldsymbol{\eta}, \boldsymbol{v})$  with new inputs  $\boldsymbol{v}$ :

$$\dot{\boldsymbol{\zeta}} = \boldsymbol{A}\boldsymbol{\zeta} + \boldsymbol{B}\boldsymbol{v}$$
 (4a)

$$\dot{\boldsymbol{\eta}} = \boldsymbol{\vartheta} \left( \boldsymbol{\zeta}, \boldsymbol{\eta} \right)$$
 (4b)

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{\zeta} \,. \tag{4c}$$

The linear subsystem (4a, 4c) is observable independent of  $\boldsymbol{v}$  and  $\boldsymbol{\vartheta}$ , while the non-linear subsystem (4b) describes the non-observable internal dynamics. In case of a asymptotic

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minimum-phase system (3) the overall system (4) is detectable, and it is appropriate to design a high-gain observer for subsystem (4a, 4c) using the Luenberger method (5):

$$\hat{\boldsymbol{\zeta}} = \boldsymbol{A}\hat{\boldsymbol{\zeta}} + \boldsymbol{B}\hat{\boldsymbol{v}} + \boldsymbol{K}(\boldsymbol{y} - \hat{\boldsymbol{y}})$$
$$\hat{\boldsymbol{\eta}} = \boldsymbol{\vartheta}\left(\hat{\boldsymbol{\zeta}}, \hat{\boldsymbol{\eta}}\right)$$
$$\hat{\boldsymbol{y}} = \boldsymbol{C}\hat{\boldsymbol{\zeta}}.$$
(5)

In the original coordinates  $(\hat{\boldsymbol{w}} = \hat{\boldsymbol{\varphi}}^{-1}(\hat{\boldsymbol{\xi}}))$ , the observer gain  $\boldsymbol{K}_{HGR}$  is determined via the Moore-Penrose inverse of the reduced observability matrix, which is calculated from the time derivatives of the (known) output variables (4c):

$$\dot{\hat{\boldsymbol{w}}} = \boldsymbol{f}\left(\hat{\boldsymbol{w}}\right) + \boldsymbol{G}\left(\hat{\boldsymbol{w}}\right)\boldsymbol{u} + \boldsymbol{K}_{HGR}\left(\hat{\boldsymbol{w}}\right)\left(\boldsymbol{y} - \boldsymbol{c}\left(\hat{\boldsymbol{w}}\right)\right) 
\hat{\boldsymbol{y}} = \boldsymbol{c}\left(\hat{\boldsymbol{w}}\right).$$
(6)

With the correction approach suggested in [2] the convergence of the high-gain observer solution, against that of the system, can be improved. To achieve this, the observer (6) is extended by the implicit and explicit constraints contained in the vector  $\tilde{p}(w)$ :

$$\dot{\boldsymbol{w}} = \boldsymbol{f}\left(\hat{\boldsymbol{w}}\right) + \boldsymbol{G}\left(\hat{\boldsymbol{w}}\right)\boldsymbol{u} - \Delta \boldsymbol{P}_{\mathcal{M}}\left(\hat{\boldsymbol{w}}\right)\boldsymbol{M}\tilde{\boldsymbol{p}}\left(\hat{\boldsymbol{w}}\right) + \boldsymbol{K}_{HGR}\left(\hat{\boldsymbol{w}}\right)\left(\boldsymbol{y} - \boldsymbol{c}\left(\hat{\boldsymbol{w}}\right)\right) 
\hat{\boldsymbol{y}} = \boldsymbol{c}\left(\hat{\boldsymbol{w}}\right).$$
(7)

In this equation M denotes a free design parameter, while  $\Delta P_{\mathcal{M}}$  describes an orthogonal projection of the deviations in the constraints on the tangent bundle of the solution manifold  $\mathcal{M}$ .

Finally the extended observer (7) is implemented using a suitable example, and the results are discussed.

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# Optimization-based formulations of absorbing boundary conditions in discrete-time wave propagation problems

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This contribution shows methods to generate absorbing boundary conditions or layers in time-discrete dynamic systems with wave propagation phenomena. Such boundary conditions are useful when problems given on large or infinite domains should only be studied locally at confined regions of interest, such as the near-field solution in acoustic problems or local interaction effects near contact points in catenaries or cable dynamics. We consider time-discrete approximations of the solution represented by explicit time-marching schemes. This system representation is particularly useful for time-domain simulations or model-based control design.

A historic overview on developments in absorbing boundary conditions and the method of "perfectly matched layers" (PMLs) is given in [2]. Absorbing boundary conditions for various continuous as well as discretized forms of wave equations have been derived analytically, e.g. [3, 4]. A PML formulation for an Euler-Bernoulli bending beam with elastic support has been derived for a finite-element discretization in [1]. These results, however, are highly specific to the considered problem and may be computationally expensive. In this contribution, the optimization-based methods to construct absorbing boundary conditions or layers are generic in the sense that they essentially only require knowing the underlying dispersion relation.

The first method generates absorbing boundary conditions by direct optimization of the coefficients of the boundary stencil such that the reflection coefficient is optimized [6]. Stability criteria are proposed and considered either as constraint or as penalty term in the optimization problem in order to obtain optimal and stable boundary conditions.

The second method shown is inspired by the "perfectly matched layer". An optimal control problem is posed based on a modal decomposition of the current solution, where an optimal, spatially distributed feedback law is computed to emulate the impedance of a PML in the boundary region [5]. This convex optimization problem can be solved optimally in a closed form and shows high absorption performance even on badly conditioned discretization grids (see Fig. 1). Additionally, it allows the controlled modification of impedance in the boundary region, which is not realizable with other concepts such as traditional absorbing boundary conditions.

The proposed methods are widely applicable for linear system dynamics having wave propagation and lead to simple and computationally efficient model structures with the desired absorbing properties. These features are crucial in problems with real-time requirements, e.g. when creating design models of online model predictive control. One current industrial application for these models are real-time control tasks related to railway pantograph/catenary interaction.

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Abbildung 1: Displacement field of an Euler-Bernoulli bending beam under axial tension. On the left, a "Perfectly Matched Layer" of 5 m thickness is emulated by the optimized control law, while the right boundary is being excited by a sine sweep. High absorption performance over the entire frequency range is accomplished.

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## Estimating Parameters and States Using Modulating Functions

#### Johann Reger<sup>a</sup> Jerome Jouffroy<sup>b</sup>

We consider SISO-LTI systems of order n, given in input-output representation

$$y^{(n)} + a_{n-1}y^{(n-1)} + \dots + a_1\dot{y} + a_0y = b_{n-1}u^{(n-1)} + \dots + b_1\dot{u} + b_0u, \qquad (1)$$

or as often more compactly written with  $\mathbf{Y} = (-y, -\dot{y}, \dots, -y^{(n-1)}, u, \dot{u}, \dots, u^{(n-1)})^{\mathrm{T}}$  along

$$y^{(n)} = \mathbf{Y}^{\mathrm{T}} \boldsymbol{\theta} \,. \tag{2}$$

There  $\boldsymbol{\theta} = (a_0, a_1, \dots, a_{n-1}, b_0, b_1, \dots, b_{n-1})^{\mathrm{T}}$  holds unknown system parameters to be identified. For avoiding direct differentiation of probably noisy signals y(t) we employ the method of modulating functions [8]. We propose to slightly modify the definition given in [4, 5].

Let  $\varphi : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$  be a sufficiently smooth function and denote  $\varphi^{(i)}(t, t_1) := \frac{\partial^i \varphi}{\partial \tau^i}(\tau, t_1)|_{\tau=t}$ . A function  $\varphi(\cdot, \cdot)$  is called a *modulating function* (of order k) if for  $t_0 < t_1$  it satisfies

$$\varphi^{(i)}(t_0, t_1) \cdot \varphi^{(i)}(t_1, t_1) = 0, \qquad \forall i \in \{0, 1, \dots, k-1\}.$$
(3)

A modulating function for which  $\varphi^{(i)}(t_0, t_1) = 0$  and  $\varphi^{(i)}(t_1, t_1) \neq 0$  is called a *left* modulating function, while a modulating function for which  $\varphi^{(i)}(t_0, t_1) \neq 0$  and  $\varphi^{(i)}(t_1, t_1) = 0$  is called a *right* modulating function. A modulating function whose boundaries verify  $\varphi^{(i)}(t_0, t_1) = \varphi^{(i)}(t_1, t_1) = 0$  is called *total* modulating function.

In view of (3) for total modulating functions we may then enjoy the fundamental property

$$\int_{t_0}^{t_1} \varphi(\tau, t_1) \xi^{(i)}(\tau) d\tau = \int_{t_0}^{t_1} (-1)^i \varphi^{(i)}(\tau, t_1) \xi(\tau) d\tau$$
(4)

which allows both to avoid computing signal derivatives of  $\xi$  explicitly and to get rid of its unknown initial and final conditions [1, 2]. This way we may replace (2) with

$$z = \mathbf{w}^{\mathrm{T}} \mathbf{\theta} \tag{5}$$

where z and the vector **w** consists of integrals of the shape in (4). In order to obtain an estimate  $\hat{\boldsymbol{\theta}}$  of  $\boldsymbol{\theta}$ , we may gather a collection of  $m_{\phi} \geq n$  equations (5), each of them using a different modulating function  $\varphi_k(t)$ . We then get

$$\mathbf{z} = \mathbf{W}^{\mathrm{T}} \boldsymbol{\theta} \tag{6}$$

with  $\mathbf{z} = (z_1, z_2, \dots, z_m)^{\mathrm{T}}$  and regressor  $\mathbf{W} = (\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_m)$ . An estimate  $\hat{\boldsymbol{\theta}}^{\mathrm{T}} = (\hat{\mathbf{a}}^{\mathrm{T}}, \hat{\mathbf{b}}^{\mathrm{T}})$  is finally obtained by simple application of linear least squares, see for example [7, 6, 3]:

$$\hat{\boldsymbol{\theta}} = \left( \mathbf{W} \mathbf{W}^{\mathrm{T}} \right)^{-1} \mathbf{W} \mathbf{z} \,. \tag{7}$$

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**Main result:** Consider a state space representation of (1) where  $x_1 = y(t), x_2 = \dot{y}(t), \ldots$ Given a set of  $m_{\varphi} \ge n$  left modulating functions, the state vector  $\mathbf{x}(t_1)$ , as well as  $\mathbf{a}$  and  $\mathbf{b}$ , can be estimated in finite-time if  $\mathbf{W}$  and  $\boldsymbol{\Delta}$  have full rank. In this case, a state estimate is

$$\hat{\mathbf{x}}(t_1) = \left(\mathbf{\Delta}\mathbf{\Delta}^{\mathrm{T}}\right)^{-1} \mathbf{\Delta} \left(\mathbf{U}^{\mathrm{T}}\hat{\mathbf{b}} - \mathbf{Q}^{\mathrm{T}}\hat{\mathbf{a}} - \boldsymbol{\gamma}\right)$$
(8)

with  $\hat{\mathbf{a}}$ ,  $\hat{\mathbf{b}}$  in (7),  $\mathbf{U} = (\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{m_{\varphi}}), \mathbf{Q} = (\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_{m_{\varphi}}), \mathbf{\gamma} = (\gamma_1, \gamma_2, \dots, \gamma_{m_{\varphi}})^{\mathrm{T}}$  as per

$$\begin{split} \mathbf{\Delta} &= \begin{pmatrix} \mathbf{\phi}_{1}(t_{1},t_{1}) + \mathbf{\Gamma}_{1}^{\mathrm{T}} \hat{\mathbf{a}} \\ \mathbf{\phi}_{2}(t_{1},t_{1}) + \mathbf{\Gamma}_{2}^{\mathrm{T}} \hat{\mathbf{a}} \\ \vdots \end{pmatrix}, \qquad \gamma_{i} = \int_{t_{0}}^{t_{1}} (-1)^{n} \varphi_{i}^{(n)}(\tau,t_{1}) y(\tau) d\tau \\ \mathbf{u}_{i}^{\mathrm{T}} &= \int_{t_{0}}^{t_{1}} \varphi_{i}(\tau,t_{1}) \left( u(\tau), \dot{u}(\tau), \dots, u^{(n-1)}(\tau) \right) d\tau . \\ \mathbf{q}_{i}^{\mathrm{T}} &= \int_{t_{0}}^{t_{1}} \varphi_{i}(\tau,t_{1}), -\varphi_{i}^{(1)}(\tau,t_{1}), \dots, (-1)^{n-1} \varphi_{i}^{(n-1)}(\tau,t_{1})) y(\tau) d\tau \\ \mathbf{\phi}_{i}^{\mathrm{T}}(t_{1},t_{1}) &= \left( (-1)^{n-1} \varphi_{i}^{(n-1)}(t_{1},t_{1}), (-1)^{n-2} \varphi_{i}^{(n-2)}(t_{1},t_{1}), \dots, \varphi_{i}(t_{1},t_{1}) \right) . \\ \mathbf{\Gamma}_{i} &= \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \varphi_{i}(t_{1},t_{1}) & 0 & \cdots & 0 \\ \varphi_{i}(t_{1},t_{1}) & 0 & \cdots & 0 \\ -\varphi_{i}^{(1)}(t_{1},t_{1}) & \varphi_{i}(t_{1},t_{1}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ (-1)^{n-2} \varphi_{i}^{(n-2)}(t_{1},t_{1}) & (-1)^{n-3} \varphi_{i}^{(n-3)}(t_{1},t_{1}) & \cdots & 0 \end{pmatrix} \end{split}$$

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## Comparing model-free and disturbance observer based control

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The iP controller [1] represents the simplest intelligent (model free) PID controller. Based on the flatness theory, it may successfully be used for control of broad spectrum of nonlinear systems. Though, to demonstrate its properties, it will be analyzed in control of a simple integrator with an input disturbance.

Similarly, in Motion Control the frequently used control structures may be characterized as disturbance observer (DO) based PI control [5, 6, 10, 9, 7]. These are still explored from various points of view as robustness, or noise attenuation [8, 2, 3, 4]. Again, in the simplest case they may be illustrated by control of a single integrator with an input disturbance.

The paper deals with comparison of both types of control with focus on the performance and noise attenuation trade off.

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## Stability Analysis of interconnected Linear Systems with Coupling Imperfections

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This contribution deals with the stability analysis of interconnected systems using a model-based coupling approach to reduce effects of coupling imperfections. Analysis issues are discussed for linear SISO subsystems: first for the timeinvariant case, second for the time-invariant in combination with present nonlinearities and third for the time-varying case.

## 1 Introduction

This work deals with the stability analysis of a model-based coupling technique for interconnected linear systems. In general, a model-based coupling approach is introduced to overcome problems arising whenever the interconnections have a non-negligible influence on the overall system behavior. The interaction between two interconnected subsystems via different imperfect communication media is commonly characterized by introduced communication time-delays which degrade the coupling data exchange, see Abb. 1. These time-delays (labeled with  $(t_{s,1}, t_{s,2})$  and  $(t_{r,1}, t_{r,2})$ ) represent dead-times in the closed-loop system and result in so called round-trip-times, which may also be time-variant depending on the communication media utilization. Another important aspect are noisy coupling signals, introduced



Abbildung 1: Model-based coupling element (MBC) inserted between two coupled subsystems  $(S_1, S_2)$  to handle non-negligible coupling imperfections [2, 1]

by sensors installed in real-time systems, see Abb. 1. Such noisy measurement signals are

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problematic especially if signal-based coupling schemes are used. In this case the noise level is typically amplified, which in turn potentially leads to unstable simulations. Coupling data losses (see Abb. 1), caused by disturbances or overload of communication capacity are additional phenomena occurring in coupled systems [2].

## 2 Model-based coupling approach

The model-based coupling scheme in form of a coupling element (labeled with MBC) is inserted between two coupled subsystems, see Abb. 1. To ensure a time-correct coupling the output signal  $u_k$  of subsystem 1 ( $S_1$ ) should arrive at the input of subsystem 2 ( $S_2$ ) without any delay. This is also true for the output  $y_k$  of subsystem 2. However, significant dead-times due to imperfect communication media impose a serious limitation on the control performance of closed-loop systems. To overcome this limitation the proposed model-based coupling algorithm extrapolates the coupling signals ( $u_{k-t_{r,1}}, y_{k-t_{r,2}}$ ) to ( $\hat{u}_{k+t_{s,1}}, \hat{y}_{k+t_{s,2}}$ ) based on recursively identified subsystem models. Therefore, at discrete time instant k the extrapolated values  $\hat{u}_k$  and  $\hat{y}_k$  are already present at the subsystem inputs. This way, the effect of the round-trip-time is compensated. Coupling data losses have a very similar effect as dead-times and can therefore be compensated via additional model-based extrapolations according to the amount of lost data. Furthermore, noise corrupted coupling signals can be considered via adequate parametrizations of the recursive system identification algorithms to perform model-based filtering [1].

## 3 Stability Analysis

Important questions via the use of the model-based coupling approach arise: How do the introduced MBC elements effect the stability of the closed-loop? Can the closed-loop stability properties, without the coupling element, be preserved? In general this stability analysis is a complex task since one has to deal with nonlinear control loops independent of the introduced coupling elements. Therefore, in a first step, the stability analysis is restricted to linear closed-loop dynamics including the proposed coupling elements. The stability analysis is divided into three parts with increasing complexity: first MBC using time-invariant (pre-identified) subsystem models, second the influence of additional nonlinearities at the plant input together with these pre-identified internal models and finally time-varying closed-loop dynamics caused by parameter adaptations of the subsystem models and/or time-varying subsystem dynamics.

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## Design and Validation of an SOH-Algorithm for Lithium Ion Batteries

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The usage of modern lithium ion batteries in the hybrid powertrain of buses and commercial vehicles allows significant fuel savings. Therefore the reliable prediction of the usable energy window and the power limits over life time of the battery is very important for the optimal energy management of the vehicle.

In the presented project a high voltage (700V) and high power (120kW) battery for hybrid electric vehicles was developed. For the battery management a state of health (SOH) algorithm was designed to estimate the current ageing status, which is described by several values. Among these, the battery capacity, limited by the minimum cell capacity, is representative for the usable energy window and will decrease over life time. The internal charge resistance and discharge resistance are limiting charge power and discharge power ability and will increase over life time. For an accurate validation of the ageing status the battery must perform reference tests on a suitable test bench (i.e. static capacity test, hybrid pulse power characterisation).



Figure 1: Reference test results for battery capacity (left) and ageing status (right)

Life time testing of batteries on test bench, using a customer defined power profile, started in an early phase of the project, but was not finished at start of production. For validation reasons several batteries used in vehicles were provided by the customer for reference tests. Test results for some of these batteries are shown in Figure 1.

The SOH algorithm is slightly overestimating the ageing mechanisms of the batteries used in the vehicles and therefore prevents the systems from being overloaded, which was an important project target.

The life time requirements defined at start of the project were achieved by batteries used in vehicles and on test bench for more than five years.

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## Effects of the nonlinear Arc of a 20-kV-net single-line cable fault on the Earth-Fault-Detection and -Control

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#### Abstract

The results of field tests have shown, that the behaviour of the restriking earthfault in a cable is completely different in isolated and well-tuned compensated networks. In this paper, the differences will be considered in detail and the impact on the earthfault detection and the control of the Petersen-Coil will be explained.

Due to the different behaviour, there are new possibilities to reduce the current via the fault location.

Actually the control of the Petersen-Coil is a preventive action during the healthy network operation. On the other side the control of the Petersen-Coil is currently blocked during the earth-fault. Therefore a change of the network-size during the earthfault will increase the current via the fault location. In this paper the new method "faulty phase earthing" will be presented, which finally allows the correct tuning of the Petersen-Coil even under earth-fault conditions.

In addition, the earth fault detection will be influenced by this new method. These influences and the new demands on the earth fault detection will be presented in detail.

### 1 Physic of the earthfault

The characteristic impedance of the lossless line (surge impedance) has a value of about 335 Ohm for overhead lines and about 40 Ohm for cables. This surge impedance limits the discharge current via the fault location in a cable to about 400 A (*Fig. 1*).



Fig. 1 Discharge current of a loaded cable (11.5 kV \* sqrt(2), 20km, NA2XS2Y 150mm2)

## 2 Restriking earthfault in a cable-section

*Fig. 2* shows a line-to-ground-fault (LG-fault) in a cable. Due to the fault, an air-channel is developing with higher pressure. The isolation level of the air gap is smaller than the one of the dielectric.



#### 2.1 Results of field tests

The increasing restrikes at the beginning of the earthfault record shown in *Fig. 3* are due to the slow earth-fault-switch. After few seconds the detuning of the Petersen-Coil started. It is also remarkable, that at certain detuning the restriking pulses all have the same polarity. Details will be explained in the paper



Fig. 3 Restriking fault at damaged high voltage cable and tuning of the Petersen-Coil

Fig. 2 LG-fault in a cable section.

The major results from the records in (Fig. 4 and Fig. 5) are:

- The amplitude of the discharge-current via the fault location is limited by the surge impedance of the network. The shape of the pulse is rectangular and is not a decreasing e-function.
- The length of the pulse depends on the network length and is in this network about 1.4 ms.



Fig. 4 Restriking fault in a cable section of a well-tuned compensated network after about 4s

The main results from the field tests are the following items:

- ⇒ The amplitude of the discharge-current depends on the igniting voltage and is limited by the surge impedance.
- ⇒ The burning time of the arc is, due to the well-tuned Petersen-Coil, reduced from ≥10 ms to 1.4 ms
- ⇒ Due to the limited amplitude of the discharge current, due to the reduction of the burning time and due to the increase of the restriking time, the converted energy at the fault location is reduced to some 100 W. Therefore also the probability of the extension of the LG-fault to an LLG-fault at the fault location is reduced dramatically.

An additional result of the field tests is, that in the future we should classify networks according to the point, where the fault occurs:

- Earth-fault in the cable section => restriking
- Earth-fault in the OHL section => quasi-stationary

In case of a restriking earthfaults it is additional possible to improve the function of the Petersen-Coil with the Faulty-Phase-Earthing (FPE).

For both items more detailed information will be presented in the paper.



Fig. 5 Isolated network

#### **3** Summary

Due to the new relays and the new classification of earth-faults:

- Earth-fault in the cable section => restriking

- Earth-fault in the OHL section => quasi-stationary a different handling of the fault situation can be initiated.

If the fault is in the range of the OHL, the well-known advantages of the Petersen-Coil can be used.

If the fault occurs in the cable-section, the fault can be moved from the faulty cable-section to the substation by using the Faulty-Phase-Earthing (FPE) device. Due to the nonlinearity and the high restriking voltage of the cablefault, there will be no restriking and therefore no fault-current via the original fault-location.

The FPE ideally complements the advantages of the Petersen-Coil in OHL lines in cases of earthfaults in cable sections.

In addition, using the FPE a tuning of the Petersen-Coil also during the earthfault is now possible.

This new concept influences also the earthfault detection. Some solutions are also presented in the paper.

## Model based control of a biomass fired steam boiler

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#### Introduction

In terms of system theory, a biomass furnace with steam boiler is a non-linear, coupled multivariable system with several inputs and outputs. Conventional control strategies currently applied to these plants usually consist of decoupled linear sub-controllers, just partially or even not at all considering the coupled and non-linear behaviour of furnace and boiler. The goal of this work was to develop a new control strategy that utilizes a mathematical model of the process to improve the plant's operating behaviour and apply this control to an industrial plant for the production of process steam.

#### Plant description

The industrial plant, illustrated in Figure 1, has a nominal capacity of 6 MW and is designed for the combustion of wood chips. The fuel is fed onto a declining reciprocating grate by a hydraulic stoker. There it is heated up due to the high temperatures in the furnace. As a result, the water that is bound in the fuel is evaporated and the fuel's volatile components are released to the gas phase (devolatilization). The remaining charcoal is burned with primary air supplied beneath the grate, providing the heat necessary for the evaporation and devolatilization. The resulting flue gas moves from the primary combustion chamber to the secondary combustion chamber where additional air (secondary air) is added, ensuring a complete burnout of the fuel. Both the primary and the secondary combustion chambers are surrounded by refractory lining which physically resembles a heat storage continuously exchanging heat with the flue gas passing through the combustion chamber. Finally, the flue gas enters the steam boiler where most of the heat released by the combustion is transferred to water, leading to its evaporation. A part of the flue gas is recirculated into the primary combustion chamber, thus enabling the control of the temperature in the combustion chamber.

#### Modelling and control

The modelling of biomass furnace and steam boiler investigated has been performed separately for all relevant parts, namely the fuel bed, the gas phase combustion, the heat storage effect of the refractory lining and the evaporation of the water in the boiler. The mathematical model for the fuel bed consists of two first-order ordinary differential equations representing mass balances for the mass of water and dry fuel on the grate. They describe the correlation between the supplied primary air and fuel mass flows as well as the fuel composition and the mass flows of evaporated water and thermally decomposed dry fuel. The static model for the gas phase combustion equals a standard combustion calculation. It considers the water and fuel released from the fuel bed, the supplied air and the recirculated

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flue gas and provides the adiabatic combustion temperature as well as the mass flow and the chemical composition of the resulting flue gas. The heat storage effect of the refractory lining is modeled by one linear first order ordinary differential equation describing the correlation of the adiabatic combustion temperature and the temperature of the flue gas at the end of the secondary combustion chamber. The evaporation of water in the steam boiler as well as the heat transfer from the flue gas to the water is described by two first order non-linear ordinary differential equations.

Similar to conventional control strategies, the variables controlled by the developed strategy are the steam pressure, the water level in the boiler, the temperature of the flue gas at the end of the secondary combustion zone and the oxygen content of the flue gas at boiler outlet. The developed control is based on the method of Exact Input-Output Linearization in combination with an Extended Kalman Filter.

#### Results of the implementation at an industrial plant

The control strategy developed has been implemented and validated at the industrial biomass plant described previously where it led to significant improvements of the plant's operating behaviour, with all controlled variables being kept closer to the desired values. In particular, a better control of the oxygen content in the flue gas has been achieved, indicating a more regular and complete burnout of the fuel. Furthermore, the new control reacts more quickly to the rapid changes of the heat demand occurring in this plant, providing an overall more stable plant behaviour and consequently a more stable steam pressure as illustrated in Figure 2.



Figure 1: Biomass fired steam boiler



Figure 2: Stabilization of steam pressure

## Control-oriented Turbocharger Modeling

#### Markus Freistätter<sup>ab</sup> Robert Bauer<sup>b</sup> Nicolaos Dourdoumas<sup>a</sup> Wilfried Rossegger<sup>b</sup>

The use of turbochargers in the automotive industry has grown significantly over the past years. Almost every Diesel engine and an increasing number of gasoline engines are equipped with a turbocharger. Research and testing of turbochargers is often done using hot gas test beds. In such an unit a mixture of natural gas and air is burnt in a combustion chamber and the hot burnt gas is used to drive the turbine. Models are needed of the test bench itself as well as of the device under test (the turbocharger).

There already exists a variety of turbocharger models of differing complexity. Kessel presents in [1] very detailed models for turbine and compressor. However, they don't fit the measured data obtained on the test bench very well. The newly developed model resembles one of the methods of Moraals overview in [2] as physical modeling is combined with curve fitting. The modeled quantities include mass flow, pressure ratio, efficiency and temperatures of compressor and turbine. Compressor instabilities, the influence of Wastegate valves and Variable Geometry Turbines are considered as well.

- [1] KESSEL, JENS-ACHIM: Modellbildung von Abgasturboladern mit variabler Turbinengeometrie an schnellaufenden Dieselmotoren. Doktorarbeit, TU Darmstadt, Mai 2004.
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## Philon of Byzantium and his work on Pneumatic and Hydraulic Control Systems

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Philon of Byzantium lived in the third century B.C. in Alexandria and taught in its famous Museum. He is credited with being the author of the most important technical handbook of Hellenistic antiquity, the so-called  $M\eta\chi\alpha\nu\kappa\eta$   $\Sigma \acute{\nu}\tau\alpha\xi\iota\varsigma$  – Mechanical Syntaxis, i.e., a compendium of mechanical sciences. This manual contained nine books, from which the only surviving book of *Pneumatics* includes some of the most important applications of pneumatic and hydraulic mechanisms.

Indicative examples are the following:

#### 1. Proof of the materiality of air

The invisible air has a material substance, which becomes apparent through its pressure. So, if someone inverts a vessel and plunges it into the water, no water penetrates into the vessel [3, II, pp.458-461].

#### 2. Regulation of water level by means of air

The proof of the materiality of air allowed Philon to design a pneumatic-hydraulic control system, which regulates the water level of a vessel, despite the outflow of water, by means of the air pressure. Philon introduces here the important invention of closed loop control systems with feedback, as first Ktesibios and later Heron have also described [3, XII, pp.482-485].

#### 3. A woman servant offers automatically wine and water

Philon describes an automatic control mechanism in the form of a woman servant, made from bronze or silver, who offers wine and water. She holds with her right hand a pitcher, while the left hand remains free, until someone places on it an empty cup. Immediately wine and water, in a prescribed proportion, flow from the pitcher in this cup. The entire pneumatic-hydraulic control mechanism is hidden in the inner hollow space of the woman servant. This space includes a wine and a water chamber, whose air pressure is mechanically controlled by valves and tubes [4, 30]. This example is a notable inventive idea that opens the way to robotics.

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#### **Historical Bibliography**

[1] The original work of Philon of Byzantium " $\Pi \epsilon \rho i \Pi v \epsilon v \mu \alpha \tau i \kappa \omega v$  - About Pneumatics" in ancient Greek is not saved.

[2] It is followed by an Arabic translation and the Latin editions of Constantinople and Oxford, entitled *"Liber de Philonisch ingeniis spiritualibus –* Philon's Book about Inventions of Pneumatics".

[3] A part of the above mentioned work is translated in German and included in the book "Heron of Alexandria, Volume 1" by Wilhelm Schmidt, Leipzig, Teubner Verlag, 1899, under the title "*Die Druckwerke Philons von Byzanz*".

[4] It is followed by the French translation by Baron Carra de Vaux, in Paris, 1903, published under the title "*Le livre des appareils Pneumatique et des machines hydrauliques*". Only this French translation includes the issue of the House Servant (Theme 30).