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Contents

Part I.

Tuesday

Robustness of Distributed Frequency Control in Modern Power Systems: Time Delays & Dynamic Communication Topology

Johannes Schiffer[a] **Florian Dörfler**[b] **Emilia Fridman**[c]

One of the most relevant control applications in power systems is frequency control. This control task is typically divided into three hierarchical layers: primary, secondary and tertiary control [6]. In the present talk, we focus on secondary control which is responsible for the regulation of the frequency to a nominal value in an economically efficient way and subject to maintaining the net area power balance. Traditionally, secondary frequency control has been carried out on the high-voltage transmission system by using large fossil-fueled power plants as actuators [6]. Yet, the increasing penetration of distributed renewable generation interfaced to the network via power inverters renders these conventional schemes inaproppriate, creating a clear need for robust and distributed solutions with plug-and-play capabilities [10].

Multi-agent systems (MAS) represent a promising framework to enable such solutions. A popular distributed control strategy for MAS is the distributed averaging-based integral (DAI) algorithm, also known as consensus filter [7], that relies on averaging of integral actions through a communication network. The distributed character of this type of protocol has the advantage that no central computation unit is needed and the individual agents, i.e., generation units, only have to exchange information with their neighbors [1]. DAI algorithms have been proposed previously to address the objectives of secondary frequency control in bulk power systems [11, 8] and also in microgrids (i.e., small-footprint power systems on the low and medium voltage level) [9, 1, 2].

The closed-loop DAI-controlled power system is a cyber-physical system whose stability and performance crucially relies on nearest-neighbor communication. Despite all recent advances, communication-based controllers (in power systems) are subject to considerable uncertainties such as message delays, message losses, and link failures [10] that can severely reduce the performance – or even affect the stability – of the overall cyber-physical system. Such cyber-physical phenomena and uncertainties have not been considered thus far in DAI-controlled power system analysis.

Motivated by this, we present conditions for robust stability of nonlinear DAI-controlled power systems under communication uncertainties. With regards to delays, we consider constant as well as fast-varying delays. The latter are a common phenomenon in sampled data networked control systems, due to digital control [4, 3] and as the network access and transmission delays depend on the actual network conditions, e.g., in terms of congestion and channel quality [5]. In addition to delays, in practical applications the topology of the communication network can be time-varying due to message losses and link failures [7]. This

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can be modeled by a switching communication network [7]. Thus, the explicit consideration of communication uncertainties leads to a switched nonlinear power system model with (time-varying) heterogeneous delays. For such systems, we provide sufficient delay-dependent conditions for robust stability by constructing a common Lyapunov-Krasovskii functional. Our stability conditions can be verified without exact knowledge of the operating state and reflect a fundamental trade-off between robustness and performance of DAI control. The effectiveness of the derived approach is illustrated on a numerical benchmark example, namely Kundur's four-machine-two-area test system [6, Example 12.6].

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Fractional-Order Observer for Integer-Order LTI Systems

Christoph Weise[a] **Kai Wulff**[a] **Johann Reger**[a]

We consider a completely observable n-order LTI system

$$\Sigma_{\mathrm{IO}}:\begin{cases}\dot{x}(t)=Ax(t)+Bu(t) & \text{(1a)}\\ y(t)=Cx(t) & \text{(1b)}\end{cases}$$

with initial condition $x(t_0)=x_0$. The solution of the free system ($u\equiv 0$) is given by the well-known matrix exponential function

$$\Phi(t,t_0)=\exp\left(A(t-t_0)\right) \tag{2}$$

as transition matrix, i.e. $x(t)=\Phi(t,t_0)\,x_0$. A fractional order system with a pseudo-state representation may be understood as an extension of the integer-order case. It takes the form

$$\Sigma_{\mathrm{FO}}:\begin{cases}\mathcal{D}^\alpha\tilde{x}(t)=A\tilde{x}(t)+Bu(t) & \text{(3a)}\\ \tilde{y}(t)=C\tilde{x}(t) & \text{(3b)}\end{cases}$$

with n states, the order of differentiation $\alpha\in(0,2)$ and the initial conditions $\tilde{x}(t_0)=\tilde{x}_0$. In this equation, $\mathcal{D}$ is the fractional-order derivative using Caputo's definition [3, 1]. Since this fractional differential operator is not local, the pseudo transition matrix given by the Mittag-Leffler-function $\mathcal{E}$ reads

$$\tilde{\Phi}(t,t_0)=\mathcal{E}_{\alpha,1}(A(t-t_0)^\alpha)=\sum_{i=0}^{\infty}\frac{(A(t-t_0)^\alpha)^i}{\Gamma(\alpha i+1)} \tag{4}$$

and depends on the complete past of the system, see [1]. In opposition to the exponential function the scalar Mittag-Leffler-functions exhibits an algebraic decay [2] which leads to a slow convergence for large times. At initial time t_0, however, the derivative is unbounded.

Our aim is to exploit this property for designing observers that show a faster convergence of the estimation error.

Main result

The connection of fractional-order systems with some class of time-varying integer-order systems has been discussed in previous works [4, 5]. However, in this contribution we derive a fractional-order system associated with the integer-order system to be observed, that is

$$\mathcal{D}^\alpha z(t)=\underbrace{\begin{pmatrix}0 & I & 0 & \cdots & 0\\ 0 & 0 & I & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & I\\ A & 0 & 0 & \cdots & 0\end{pmatrix}}_{\bar{A}}z(t)+\underbrace{\begin{pmatrix}0\\ 0\\ \vdots\\ 0\\ B\end{pmatrix}}_{\bar{B}}\bar{u}(t)\,, \tag{5}$$

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where $\alpha^{-1} = k \in \mathbb{N}$ is the rational order of differentiation and the new input is defined by a fractional-order integral with respect to the original input $\bar{u}(t) = \mathcal{I}^{(k-1)\alpha}u(t) = \mathcal{I}^{1-\alpha}u(t)$. The extended state z then contains the original state and its fractional order integrals, i.e.

$$z(t) = \begin{pmatrix} z_1(t) \\ z_2(t) \\ \vdots \\ z_{k-1}(t) \\ z_k(t) \end{pmatrix} = \begin{pmatrix} \mathcal{I}^{(k-1)\alpha}x(t) \\ \mathcal{I}^{(k-2)\alpha}x(t) \\ \vdots \\ \mathcal{I}^{\alpha}x(t) \\ x(t) \end{pmatrix}. \tag{6}$$

Choosing matching initial conditions

$$z(0)^\top = \begin{pmatrix} 0 & 0 & \cdots & 0 & x_0^\top \end{pmatrix}^\top \tag{7}$$

we conclude that $z_k(t) = x(t)$, thus the trajectories are identical. When initializing properly, this associated fractional-order system captures various properties of the original integer-order LTI system, e.g. system (5) inherits to be stable, observable or controllable if the original system (1) exhibits the corresponding property. We can also formulate a direct connection of the eigenvalues of the integer-order and associated fractional-order system.

Using this system we obtain an observer that shows a very fast convergence immediately after initialization and a poor convergence for large times. In order to overcome the latter problem we propose two strategies:

- Reinitialization of the observer in short intervals may lead to a convergence faster than exponential.
- The concept of impulsive observers [6] can be extended to fractional-order systems such that the observer converges in fixed time and the performance is increased in the first time interval.

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Longitudinal tunnel ventilation control: Dynamic feedforward control and non-linear disturbance observation

Nikolaus Euler-Rolle[a] **Stefan Jakubek**[a]

New constructions or refurbishments of road tunnels impose increasingly tight safety requirements on the electrotechnical tunnel equipment such as the ventilation system, as well as on its operation. Particularly in the event of an incident with fire and smoke spreading in the tunnel, adequate safety measures have to be taken without delay to protect life and health of the tunnel users. The main goal is to guarantee a minimum amount of time for persons in the tunnel to safely follow the escape routes with sufficient visibility available. For this purpose, tunnels exceeding a certain minimum length are equipped with ventilation systems.

In this contribution, non-linear longitudinal ventilation control is considered holistically in case of an emergency, where jet fans are used to induce fresh air into the tunnel through one portal and exhaust the smoke through the other. Since the spread of smoke in the tunnel cannot be measured, it is assumed that a safe condition is achieved by maintaining a prescribed average air flow velocity in the tunnel, which is high enough to convey smoke out of the tunnel, but not too high to save the naturally occurring smoke stratification from being destroyed. In this context, control is especially challenging for short tunnels due to their low inertia and the resulting highly dynamic behaviour. In particular, two key elements are investigated. First, the enhancement of classic linear control with a non-linear dynamic feedforward control of the jet fans is considered. Second, the observation and rejection of disturbances is treated. As there are several disturbance influences such as vehicles in the tunnel, buoyancy of hot gases, wind load onto the portals or meteorological pressure differences influencing the flow velocity, a sufficiently fast disturbance rejection capability is required to compensate the quickly changing air flow velocity in the tunnel. For this purpose, the dynamic feedforward control is expanded with the ability to take into account estimated disturbances that are fed back from a non-linear unknown input observer. Non-linear disturbance observation of external influences is achieved by applying a specially structured non-linear observer. Deviations between the measured flow velocity and its estimation are exclusively attributed to external disturbances. As a consequence, the proposed observer has a unique structure that allows to show the stability of the observer for a specific implementation for the open- and closed-loop system with few restrictive assumptions. Based on Lyapunov theory the convergence and stability of the implemented observer is independent of the control scheme. Simulations show significant improvements in the control performance with active disturbance rejection.

The proposed approach to obtain the dynamic feedforward control is based on feedback linearisation as a non-linear system transformation. Feedback linearisation is applied to the air flow model, and the resulting non-linear input transformation is used as model inverse

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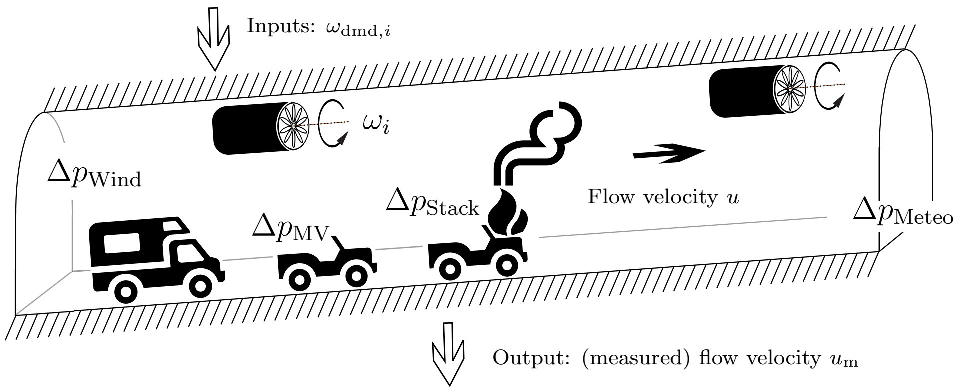

Figure 1: Schematic overview of the air flow model showing the inputs, the output and the individual sources and losses of momentum.

for feedforward control of the rotational speeds of the jet fans. In Figure 1 a schematic overview of the air flow model and the individual sources of momentum in form of equivalent pressure differences is given. The controlled input $\omega_{\mathrm{dmd},i}$ into the model is the demanded rotational speed of each jet fan and the output is the measured air flow velocity u_{m}. However, when applying feedback linearisation, controllability issues in combination with a bifurcation characteristic caused by the absolute values in the Bernoulli equation occasionally lead to implausible control signals. Thus, for a flawlessly robust operation in different conditions, these issues require a modified evaluation of the feedforward control. Instead of the original expression resulting from feedback linearisation, a modified robustness-oriented feedforward control is based on the state transformation for trajectory evaluation in combination with control loops to actuate the individual jet fans.

Both, the proposed non-linear dynamic feedforward control and the disturbance observer have been implemented and tested in the St. Ruprecht motorway tunnel on the Austrian Semmering motorway S6 in course of an encompassing tunnel refurbishment and modernisation. Thus, experiments could have been carried out while the tunnel was closed to traffic. During the final commissioning also a test with an actual fire has been conducted. All results show excellent control performance with significantly reduced correcting feedback control action. Further details and results can be found in [1] and [2].

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Model Predictive Control with Flatness Based Linear Programming for the Single Mast Stacker Crane

Anastasiia Galkina[a] **Kurt Schlacher**[a]

Introduction

This paper deals with model predictive control (MCP) for a stabilization of a time-optimal motion of a single mast stacker crane (SMC) (see Fig. 1), which is used for an automatic storage or retrieval of payloads in automated warehouses. The mathematical model of the plant is a distributed parameter one, but it admits an excellent approximation by a flat lumped parameter system. To reduce a working time and to increase the SMC productivity a time-optimal strategy is chosen. It is shown that the approximated system can be simplified further and a linear time-varying system can be considered for the control design. MPC is chosen to stabilize the trajectory and fulfill certain state and input constraints. MPC is derived by implementing linear parametric optimization, where flatness of the model is exploited. The linear time-varying model is parametrized by the flat output. The optimization task for the MPC is formulated in a form of a linear program, which is solved by means of an open-source optimization software LPSOLVE. Finally, simulation results are presented.

Modeling and time-optimal trajectory

Model of the SMC is presented in [4] and [3]. Although the system under consideration is a distributed parameter one, we use a lumped parameter system derived by help of the Rayleigh−Ritz approximation. According to this method the mast deflection $w\,(Y,\,t)$ is approximated by the first-order Ritz ansatz function

$$w^*\,(Y,\,t) = x_c\,(t) + \mathit{\Phi}_1\,(Y)\,\bar{q}^1\,(t)\,,$$

with the new generalized coordinate $\bar{q}_1$, and the spatial basis function

$$\mathit{\Phi}_1\,(Y) = 6\,(Y/L)^2 - 4\,(Y/L)^3 + (Y/L)^4\,.$$

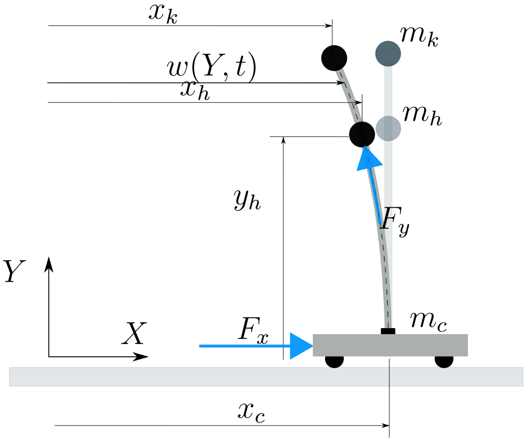

Abbildung 1: Single mast stacker crane

The nonlinear approximating equations of motion are

$$M_{\alpha\beta}(q^\alpha)\ddot{q}^\beta + C_\alpha(q_\alpha, \ddot{q}_\alpha) = G_{\alpha\xi}u^\xi, \tag{1}$$

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Here $q = [x_c \quad \bar{q}_1 \quad y_h]^T$ are the generalized coordinates. Assuming $\partial_Y \Phi_1(y_h) = 0$ and $\partial_Y^2 \Phi_1(y_h) = 0$, the nonlinear model (1) is simplified further. This simplification results in linear time-varying system

$$\begin{aligned} \dot{\bar{\mathbf{x}}} &= \mathbf{A}_1(y_{h,d}, \dot{y}_{h,d}, \ddot{y}_{h,d})\bar{\mathbf{x}} + \mathbf{b}_1(y_{h,d}, \dot{y}_{h,d})F_x, \\ \dot{\bar{\mathbf{y}}} &= \mathbf{A}_2\bar{\mathbf{y}} + \mathbf{b}_2\bar{v}, \end{aligned} \tag{2}$$

with $\bar{\mathbf{x}} = [x_c \quad \bar{q}_1 \quad \dot{x}_c \quad \dot{\bar{q}}_1]^T$, $\bar{\mathbf{y}} = [y_h \quad \dot{y}_h]^T$, $\bar{v} = (F_y - gm_h)/m_h$ and the optimal trajectory $(y_{h,d}, \dot{y}_{h,d}, \ddot{y}_{h,d})$.

Model predictive control with linear programming

The model (2) can be considered as two interconnected subsystems, where the first subsystem is a linear time-varying one and the second is a linear time-invariant one. The formulation of the optimization task for the MPC is based on the method, which was implemented in [1]. The subsystems of (2) are parametrized by the flat output $\mathbf{h} = [h_1 \quad h_2]^T$ using an exact time discretization and a transformation into the Brunovsky canonical form. The optimization task for the MPC in a form of a linear program (see e.g. [2]) can be divided into two independent optimization tasks corresponding to the subsystems of (2) and is given by

$$\begin{array}{rcll} & \min_{\bar{\mathbf{h}}^1} & \varepsilon_1 + \varepsilon_2 + \varepsilon_3 & \\ |M_{b,k}(\bar{\mathbf{h}}^1) - M_{b,d,k}| & \leq & \varepsilon_1 & k = 1, ..., N \\ |F_{x,k}(\bar{\mathbf{h}}^1) - F_{x,d,k}| & \leq & \varepsilon_2 & k = 1, ..., N \\ |x_{h,k}(\bar{\mathbf{h}}^1) - x_{h,d,k}| & \leq & \varepsilon_3 & k = 1, ..., N \\ & \min_{\bar{\mathbf{h}}^2} & \varepsilon_4 + \varepsilon_5 & \\ |y_{h,k}(\bar{\mathbf{h}}^2) - y_{h,d,k}| & \leq & \varepsilon_4 & k = 1, ..., N \\ |F_{y,k}(\bar{\mathbf{h}}^2), F_{y,d,k}| & \leq & \varepsilon_5 & k = 1, ..., N \end{array}$$

where ε_i, $i = 1, ..5$ are some positive values, $(M_{b,d,k}, F_{x,d,k}, x_{h,d,k})$ and $(y_{h,d,k}, F_{y,d,k})$ - desired optimal trajectory, $\bar{\mathbf{h}}^1 = [\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3 \quad h_1]^T$ and $\bar{\mathbf{h}}^2 = [\varepsilon_4 \quad \varepsilon_5 \quad h_2]^T$ are optimization vectors and N is an optimization horizon. An open-source optimization software LPSOLVE is used for solving both optimization tasks. Finally, simulation results of the trajectory stabilization for the nonlinear model (1), subject to parameter uncertainty and measurement noises are presented.

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Part II.

Wednesday morning session

Estimation of the Earthfault Distance in a 110-kV-Network using Traveling Waves – Results from Field Tests

Gernot Druml[a]

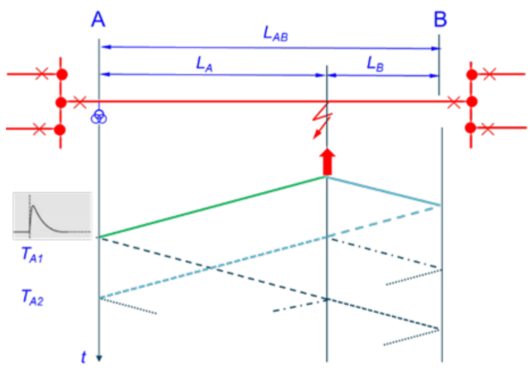
Due to today's increased demands on grid operation management, new methods for earth fault localization and detection are needed. The fault localization should be performed as quickly as possible and under the condition, that the fault current at the fault location will not be significantly increased.

The pros and cons of Distance Protection in solid grounded networks are well known. In case of a single-line earthfault, the current via the fault location in solid grounded networks is in the range of some kA.

In compensated networks the residual current Ires via the fault location is, in case of a single line fault, only few ampere. Very often this current is much smaller than the load current of the feeder. A distinction between load current, circulating currents in meshed networks and fault current is more or less not possible. Therefore, the Earthfault-Distance-Protection in compensated networks is usually switched off.

In a 110-kV-network the distance between two substations is in the range of 50 km to 200 km. Also if the feeder is identified correctly, it is still a challenge to locate the exact fault position on this feeder.

In this paper the results of field-test in a 110-kV-network using single ended measurements based on travelling waves with different fault impedances will be presented.



The advantage of using travelling waves is, that they are robust against some "disturbances" for example:

- load current
- circulating currents in meshed networks

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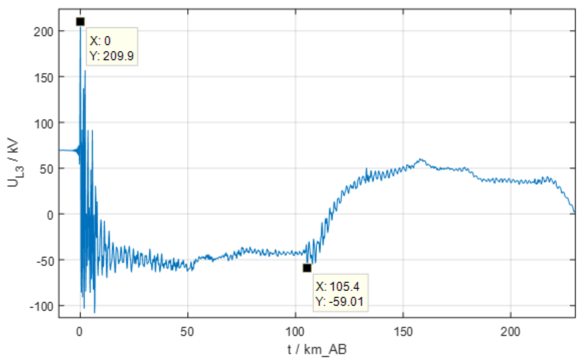
- distributed generation
- dependence of the zero-sequence-system due to the skin depth of the current
- independent of neutral treatment

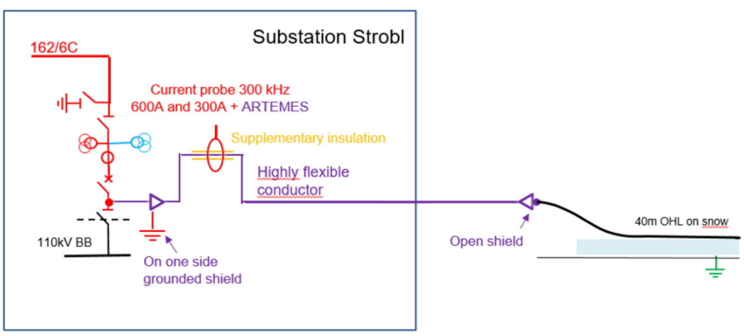
The disadvantage of travelling waves are:

- Reflexions on each change of the surge impedance for example junction, bus bar, etc.
- Behaviour of connected transformers as large capacitor (5 . . . 10 nF)
- Measurement equipment
- Measurement with high frequencies in the range of some MHz

From field-test we have learned, that there some special effects of the theory of travelling waves must be taken into account. Then a accuracy of the distance estimation in the range of 1% is possible, as shown in the following picture.



Also special effects of different faults will be presented, for example:



Over-Head-Line falling on 35 cm dry snow

AUTOMATED DESIGN, INSTANTIATION AND QUALIFICATION OF COMPLEX, HIGHLY FAULT TOLERANT AVIONIC SYSTEMS

Florian Kraus[a] **Reinhard Reichel**[a]

MOTIVATION. *The Flexible Platform (fp) technology* reflects its value in the effective development of complex safety-critical fly-by-wire systems, in particular, but not exclusively in the small aircraft domain (CS23) while significantly reducing risks and costs. It is embedded in the *AAA-Process*, which in turn – as the bigger picture – also covers the certification relevant documentation and testing aspect. Both were developed at the University of Stuttgart.

As shown in the submitted presentation, the core idea behind the *fp technology* results in a necessity to adapt generic components to a specific solution. Meeting this challenge emphasizes on use of Domain-Specific Models to represent the system being designed. These models are then used to automatically synthesize executable software code or to generate interfaces for subsequent analysis, documentation or testing purposes.

This presentation covers both, the *Flexible Platform technology* and the *AAA-Process* in general. It will discuss the motivation for

- making use of a platform approach,
- separating the application (e.g. flight control laws) from the platform management,
- embedding the application into a virtual, failure-free and simplex-minded environment and
- describing the system's properties at a very high level of abstraction using a domain-specific modeling language.

Finally the presentation focuses on the multistep refinement process that has proven within the scope of multiple research projects. Applied methods, tools and concepts are presented.

BACKGROUND. The *fp technology* in its core is characterized by a clear separation between the actual application (i.e. flight control laws) and a generic management software layer (*platform management software, plama*). This management layer comprises the whole signal and network communication as well as the entire redundancy management to operate the system in a failure-tolerant way. Complexity, distribution, failure tolerance and redundancy are transparent to the actual application. Once the interface is well defined, both the application and the *plama* can be developed widely independently in accordance with the well-established V-Process whereas the *plama* benefits of its platform based design that ensures a reuse of its generic components.

Adaption to an individual system (e.g. specialization) is done by composition and parametrization of these generic components. This results in many advantages but shifts the development effort towards this specialization. It is a tremendously challenging task since

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thousands of parameters have to be set properly and we cannot disregard their strong dependence on the application interface and the actual system's hardware topology. To meet the challenge of specializing the generic *fp* components we make use of the *Model Driven Engineering approach (MDE, also known as MDD)* and share the idea of using a *domain-specific modeling language (DSML, or more general: DSL)* that is tailored to a specific domain of interest, i.e. the Flexible Platform domain. This allows to describe the system's properties at a very high level of abstraction and is an intuitive and efficient way to express (e.g. to model) the system's specification manually. This specification is stored as a domain-specific model and is then used to automatically specialize the *platform management software* in a multi-step refinement process that generates implementation-level parameter data. The refinement process includes several complex model transformations. Those transformations – together with the *DSML* – carry all the system- and software-architectural knowledge required for the specialization task. Model transformations implemented in a traditional high-level, general-purpose programming language (e.g. Python) become inefficient and almost unmanageable in the illustrated case. Dedicated graph transformation languages overcome this problem. In this approach we found a suitable solution that allows the formal specification of model transformations using elements from the source and target domain-specific languages.

In a nutshell, a high-level system specification is used to adapt the platform management software automatically which finally leads to executable software code. The underlying modeling language is tailored to the specific needs of the *fp* respectively fly-by-wire domain and therefore implicates its expressiveness on all levels of abstraction.

Multi-sensor data fusion for automated driving

Daniel Watzenig[a]

Automated vehicle technology has the potential to be a game changer on the roads, altering the face of driving as we experience it by today. Many benefits are expected ranging from improved safety, increased energy efficiency, reduced congestion, lower stress for car occupants, and better road utilization due to optimal integration of private and public transport. Automated driving is characterized by a computer-based derivation and execution of appropriate driving maneuvers based on the current traffic situation captured by a multitude of sensors. The basis for evaluating the traffic situation, however, is knowledge about all relevant entities in a vehicle's environment, including traffic participants, road infrastructure (lane markings, traffic signs, traffic lights) or obstacles. For acquiring such knowledge, numerous sensors have to be used that permanently provide relevant data. As effective sensors are, they have some drawbacks such as

- Limited range
- Performance is susceptible to common environmental conditions (rain, fog, varying lighting conditions)
- Range determination not as accurate as required
- Detection of artefacts, so-called "false positives"

In order to overcome these drawbacks, multi-sensor fusion plays an important role in order to increasing reliability and safety and hence user acceptance of automated vehicles. Model-based sensor fusion is the combining of sensory data or data derived from disparate sources such that the resulting information has less uncertainty. The term uncertainty reduction implies

- Increased object classification accuracy (Higher detection rate, fewer false alarms, enhanced level of detail of object description)
- Improved state estimation accuracy
- Improved robustness for instance in adverse weather conditions
- Increased availability
- Enlarged field of view

For many automated driving functions, information from different sensors are required (e.g., the velocity of an object measured by a radar and its class determined by a camera). Model-based sensor fusion inherently combines data from all available sensors in order to provide both a higher quantity and a higher quality of available information. For a single sensor, it is difficult to determine the current error of its measurement. The model-based approach, however, can exploit the redundancy between sensors as well as the model-knowledge to

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estimate the current estimation errors. Only if the driving functions know about the current expected error of the environmental representation, it can take appropriate decisions. The most common way to describe these uncertainties is by means of probabilistic quantities.

This talk discusses state-of-the-art methods for model-based multi-sensor data fusion for automated driving in order to manage correlated, inconsistent, and imperfect data. Different algorithms (e.g. Bayes estimators, fuzzy reasoning, evidential belief reasoning, adaptive and hybrid techniques...), concepts (high level, intermediate level, low level), and classes (complementary, competitive, cooperative) will be analyzed and investigated in terms of applicability and performance by examples.

Model-based control of hydronic networks using graph theory

Daniel Muschick[a] **Viktor Unterberger**[a b] **Markus Gölles**[a]

Hydronic networks are networks using a liquid heat-transfer medium for heating or cooling purposes. They can range in size from small cooling applications in machines to large district heating networks with many producers and consumers.

Whatever their size, the main control problems always remain the same: heat needs to be transported from sources to sinks while ensuring that certain temperatures, mass flows and pressures in the system remain within given boundaries. The difficulty lies in the fact that the heat to be transported cannot be controlled directly; it depends on both the fluid mass flows and temperature levels in the system. In order to control a hydronic network, it is thus necessary to determine the fluid mass flows which result in the desired behavior; then the hydraulic components have to be controlled in such a way that the desired mass flows are realized. For this the network as a whole has to be considered and the mutual influences of the different components have to be taken into account. Currently, however, the components in hydronic networks are often controlled individually and only their local influence is considered.

This article presents a control architecture that automatically takes into account the links between the hydraulic and the thermal aspects of a hydronic network. The key to handle this issue efficiently and systematically is to represent the hydronic network in a structured way using graph theory. By doing so, it is possible to automatically generate mathematical models of both the stationary heat distribution, used for determining the necessary mass flows, and the hydraulic dynamics used for the control of these mass flows.

The graphs representing the networks consist of nodes $\mathfrak{N}$ connected via edges $\mathfrak{E}$. Each edge corresponds to a component in the network, e.g. a pipe, a heat producer or a valve. Each component has an influence on both the temperatures T and the pressures p in the network. This influence can be described by possibly non-linear functions ΔT for the temperature change and Δp for the pressure change depending on the type of component, the temperatures, the mass flows and the control signals in the case of actuators.

The individual edges and nodes are connected via node and mesh equations

$$\sum_{i\in\mathfrak{N},\, \dot{m}_{ij}>0} \dot{m}_{ij} = \sum_{k\in\mathfrak{N},\, \dot{m}_{jk}>0} \dot{m}_{jk} \quad \forall j \in \mathfrak{N}, \qquad \sum_{\{i,j\}\in\mathfrak{M}_k} \Delta p_{ij} = 0 \quad \forall\, \mathfrak{M}_k \in \mathfrak{M}, \tag{1}$$

where $\dot{m}_{ij}$ and Δp_{ij} denote the mass flow from node i to node j respectively the corresponding pressure difference $p_j - p_i$, and $\mathfrak{M}$ denotes the set of all loops or meshes in the network.

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Assuming constant heat capacities, a similar relation exists for describing the mixing of flows in nodes:

$$\sum_{i\in\mathfrak{N},\,\dot m_{ij}>0} \dot m_{ij}\,(T_i + \Delta T_{ij}) = \Big(\sum_{k\in\mathfrak{N},\,\dot m_{jk}>0} \dot m_{jk}\Big) T_j \quad \forall j \in \mathfrak{N} \tag{2}$$

The individual component equations can now be combined with these relations and written in a concise way to describe the stationary heat distribution.

Similarly, the hydraulic dynamics can be automatically obtained by combining the node and mesh equations with the simple differential equation

$$\frac{l_{ij}}{A_{ij}} \frac{\mathrm{d}\dot m_{ij}}{\mathrm{d}t} = \Delta p_{ij} \tag{3}$$

describing the acceleration of a fluid in every edge representing a pipe with length l_{ij} and cross section A_{ij}.

Finally, the models generated can now be used in the hydronic control strategy, which consists of three parts: First, the model describing the stationary heat distribution is used to calculate the mass flows in such a way that the desired temperature levels are reached and no mass flow limitations are violated. Second, a temperature controller compares the actual temperatures with the desired values and adapts the calculated mass flows $\dot{\mathbf{m}}_{\mathrm{ff}}$ in order to reach the desired values. Third, the resulting desired mass flows $\dot{\mathbf{m}}^*$ are used as references for a hydraulic controller to control all actuators (valves $\mathbf{u}_{\mathrm{V}}$ and pumps $\mathbf{u}_{\mathrm{P}}$) at the same time. A schematic overview of this cascading control structure is shown in Fig. 1.

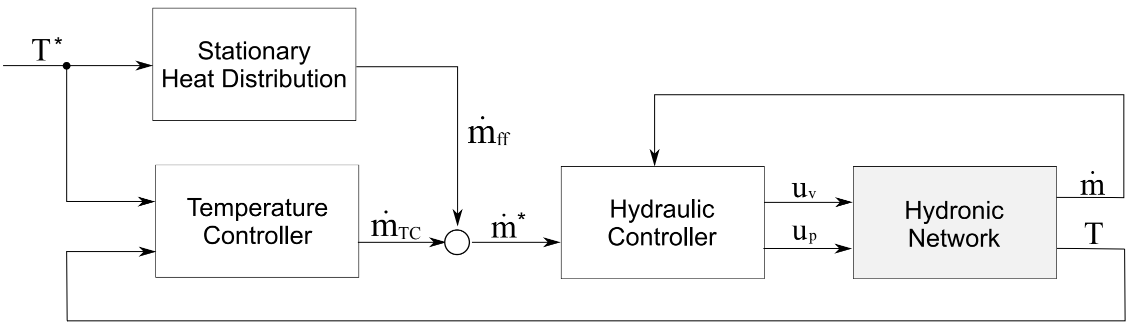

Figure 1: Cascading control structure for a hydronic network.

Model-Based Reference Shaping for Biomass Grate Boilers

Richard Seeber[a] **Markus Gölles**[b] **Nicolaos Dourdoumas**[a] **Martin Horn**[a]

The use of biomass as a sustainable fuel has become more and more important in recent years. One common use is its combustion in grate boilers ranging from small-scale boilers supplying heat to a single consumer to large-scale district heating plants. A model-based control approach has been shown to greatly improve the efficiency of such boilers' operation [2, 3, 4]. The approach is based on a fourth-order nonlinear model developed in [1, 2]. It consists of an input-output linearization to control relevant outputs in a decoupled manner, of a Kalman filter to reconstruct the plant states therefor required, and of PI-controllers to eliminate constant control deviations.

A challenge with this approach are actuator saturations. When active, nonlinear couplings are reintroduced and possibly undesired output deviations can occur; these may cause emergency shutdowns, reduced efficiency or increased emissions. In this talk a strategy is presented that prevents these effects by an appropriate modification of reference inputs.

For this purpose, an optimization problem is considered that aims for minimizing suitably weighted reference deviations. In general this leads to a nonlinear and non-convex problem; to avoid this a sequential minimization of the deviations is used, with the sequence being determined by requirements of the plant operation. A series of linear-fractional optimization problems is thus obtained that may ultimately be solved using linear programming.

The strategy was implemented and experimentially verified on a biomass grate boiler with a nominal capacity of 180 kW. Figure 1 exemplarily shows one result of this verification obtained by imposing an artificial lower limit on the primary air mass-flow after time t_1, which in a real-world scenario could be caused by an actuator failure. Without reference shaping, a highly undesired permanent deviation of the the hot water feed temperature T_f from its reference occurs. This corresponds to a surplus in power output. With the reference shaping strategy in place this deviation is avoided by instead accepting control deviations of less critical plant outputs such as the air ratio in the fuel bed.

The technique leads to several improvements. Undesired deviations of critical plant outputs such as the discussed surplus in power output or a too small flue-gas oxygen content are avoided to the greatest extent possible. Additionally, windup of the PI-controllers' integrators is mitigated and overall control performance during actuator saturations is improved. As a consequence, by alleviating overshoots, reducing variations and avoiding excessively large values of the flue-gas temperature, thermal stress on the furnace refractory lining is reduced, thus increasing its lifetime. Furthermore, carbon monoxide emissions may also be reduced under certain operating conditions.

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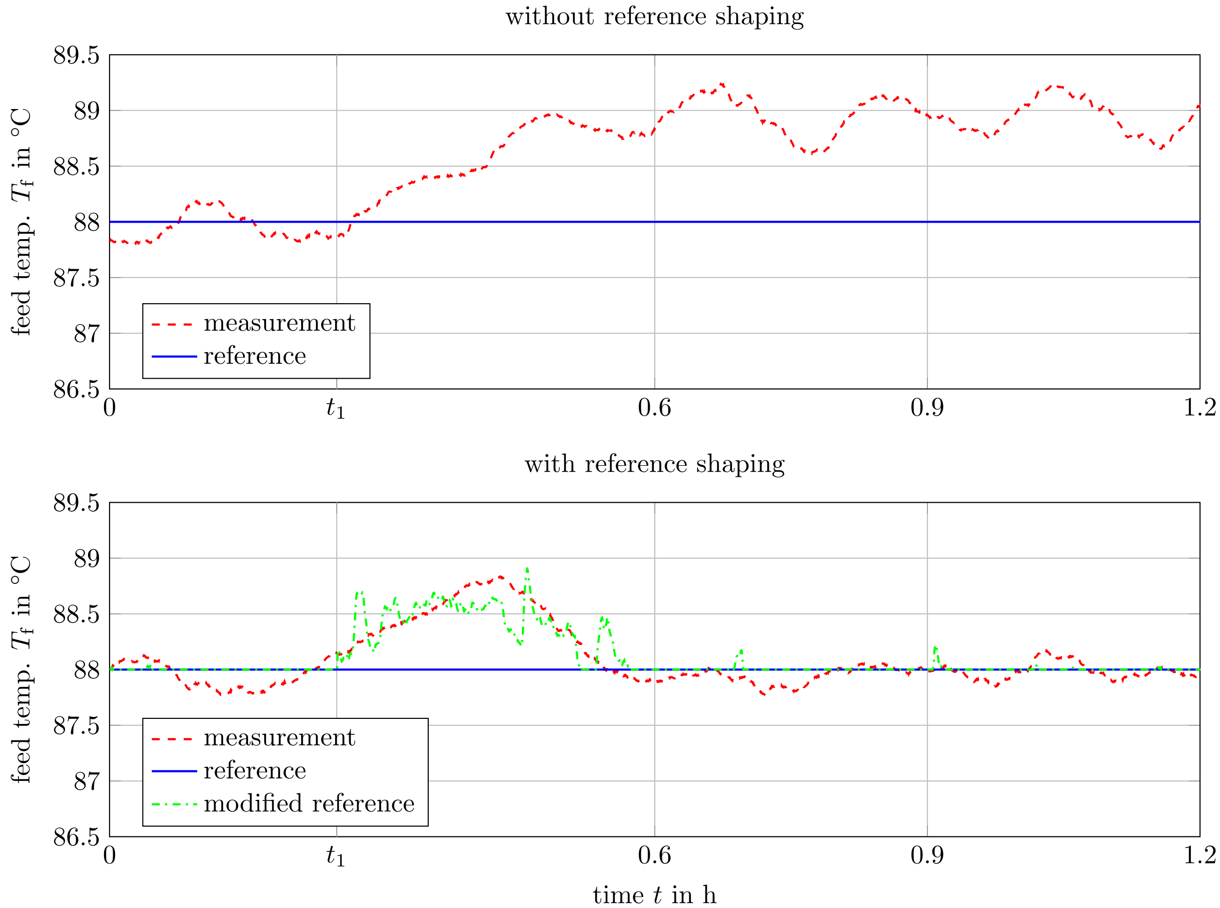

Figure 1: Hot water feed temperature T_f with an artificial upper limit being imposed on the primary air mass-flow starting at time instant t_1

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Part III.

Wednesday afternoon session

Is It Reasonable to Substitute Discontinuous SMC by Continuous HOSMC?

Leonid Fridman[a]

Professor Utkin proposed an example showing that the amplitude of chattering caused by the presence of parasitic dynamics in systems governed by First- Order Sliding-Mode Control (FOSMC) is lower than the obtained using Super-Twisting Algorithm (STA). This example served to motivate this research reconsidering the problem of comparison of chattering magnitude in systems governed by FOSMC that produces a discontinuous control signal and by STA that produces a continuous one, using Harmonic Balance (HB) methodology. With this aim the Averaged Power (AP) criteria for chattering measurements is revisited. The STA gains are redesigned to minimize amplitude or AP of oscillations predicted by HB. The comparison of the chattering produced by FOSMC and STA with redesigned gains is analyzed taking into account their amplitudes, frequencies and values of AP allowing to conclude that:

(a) for any value of upper bound of disturbance and Actuator Time Constant (ATC) there exist a bounded disturbance for which the amplitude and AP of chattering produced by FOSMC is lower than the caused by STA

(b) if the upper bound of disturbance and upper bound of time-derivative disturbance are given, then for all sufficiently small values of ATC the amplitude of chattering and AP produced by STA will be smaller than the caused by FOSMC

(c) critical values of ATC are predicted by HB for which the parameters, amplitude of chattering and AP, produced by FOSMC and STA are the same. Also the frequency of self exited oscillations caused by FOSMC is always grater than the produced by STA.

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Adaptive Extension for Higher Order Sliding Mode Controllers

Alexander Barth[a] **Johann Reger**[a]

This contribution deals with nonlinear systems given by

$$\begin{aligned} \dot{x}_i &= x_{i+1}, i = 1, ..., n-1, \\ \dot{x}_n &= f(x) + g(x)\big(\Delta(x,t) + u\big) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}$ a scalar control input, f and g are known vector fields. The uncertainty $\Delta(x,t)$ is a composition of two terms

$$\Delta(x,t) = \Delta_{\mathrm{s}}(x) + \Delta_{\mathrm{u}}(x,t) = \Theta^{\mathrm{T}}\phi(x,t) + \Delta_{\mathrm{u}}(x,t)\,, \tag{2}$$

namely the structured uncertainty $\Delta_{\mathrm{s}}(x) = \Theta^{\mathrm{T}}\phi(x,t)$ with an unknown parameter vector Θ and known regressor $\phi(x,t)$. The term $\Delta_{\mathrm{u}}(x,t)$ represents an unstructured uncertainty.

The control objective is to find a suitable control law u such that substituted in (1) the origin is stable even in the presence of the uncertainties in shape of (2).

Sliding-mode control is an established design method to achieve this goal. In general it requires little knowledge about the system or the uncertainties and therefore may simplify the controller design. The main idea is to define a manifold such that the corresponding sliding variable has relative degree one with respect to the control input u. The sliding mode controller ensures that the state of system (1) is forced to this manifold in finite time. Due to a suitable selection of the manifold, the internal dynamics is stable and the state converges to the origin.

Various design approaches [3, 5], including adaptive controllers [6, 1, 2], have been presented to solve this task.

Recently in [4], Moreno proposed a class of control designs that do not require the relative degree one condition and allow the stabilization of the overall system (1) in finite time. These controllers require the cumulative uncertainty to be bounded by

$$|\Delta(x,t)| \leq \Omega_\Delta \tag{3}$$

for some $\Omega_\Delta > 0$.

Main result: We extend the approach by Moreno by an adaptive part to improve the robustness of the proposed class of controllers. The structurally known part $\Theta^{\mathrm{T}}\phi(x,t)$ is compensated by an adaptive extention separately from the unstructured part. As a result, the requirements (3) on the uncertainty may be relaxed to

$$|\Delta_{\mathrm{u}}(x,t)| \leq \Omega_u \tag{4}$$

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with $\Omega_u > 0$ such that the sliding mode part has to handle only the unstructured part of uncertainty.

Exemplarily for a second order system we show the respective design method. However, the method can be extended to an arbitrary order system.

The controller including the adaptation law is given by

$$\begin{aligned} u &= -k_2 \operatorname{sign}\left(\lceil x_2 \rfloor^2 + k_1 x_1\right) - \hat{\Theta}^{\mathrm{T}} \phi(x,t) \\ \dot{\hat{\Theta}} &= \gamma\, g(x)\, \left(\lceil x_2 \rfloor^2 + k_1^2 \lceil x_1 \rfloor^2\right) \phi(x,t) \end{aligned} \tag{5}$$

with $\hat{\Theta}(0) = \hat{\Theta}_0$ for some $\hat{\Theta}_0$, parameters $k_1, k_2, \gamma > 0$ and $\lceil x \rfloor^\rho = \operatorname{sign}(x)\ |x|^\rho$.

The stability of the origin in the closed-loop system is proofed using the Lyapunov function

$$V(x_1, x_2, \hat{\Theta}) = \frac{2}{3}|x_1|^{\frac{3}{2}} + \frac{2}{3}|x_2|^{\frac{3}{2}} + k_1^{\frac{1}{2}} \lceil x_1 \rfloor^{\frac{1}{2}} x_2 + \frac{2\, k_1^{\frac{3}{2}}}{3}|x_1|^{\frac{3}{2}} + \frac{1}{2\,\gamma}\left(\Theta - \hat{\Theta}\right)^{\mathrm{T}} \left(\Theta - \hat{\Theta}\right), \tag{6}$$

orininally proposed by Moreno in [4], and extending it by a quadratic term regarding the estimation error $\Theta - \hat{\Theta}$. In view of the certainty equivalence principle, we may then obtain the adaptation law shown in (5).

Since the sliding mode part only has to cover the unstructured part of the uncertainty, we expect to significantly reduce the gains k_1 and k_2 of the controller (5) compared to the conventional approach if $\Omega_{\mathrm{u}} << \Omega_{\Delta}$. Moreover, the aproach does not require a fixed upper bound on the structured uncertainty $\Delta_{\mathrm{s}}(x,t)$, which allows to compensate larger class of uncertainties.

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Design of Saturated Sliding Mode Control with Continuous Actuating Signal

Mohammad Ali Golkani[a] **Markus Reichhartinger**[a] **Martin Horn**[a]

Having applied a sliding mode control scheme, a closed-loop's satisfactory robust performance is achieved despite the presence of a particular class of plant uncertainties and external disturbances. Conventional sliding mode control, i.e. first-order sliding mode approach, guarantees a saturated and discontinuous control input. Second-order sliding mode techniques such as the twisting as well as super-twisting algorithms provide absolutely continuous control signals in the case that the relative degree of the system with respect to a defined sliding function is one [3]. Furthermore, the continuous twisting controller introduces a continuous actuating signal if this relative degree is two [4]. In general, these high-order sliding mode algorithms improving the sliding accuracy of the standard sliding mode are able to counteract perturbations, which are Lipschitz continuous, and recorded in the literature as the chattering attenuation strategies. However, for systems with saturating actuators, it is difficult to tune the aforementioned controllers such that the control inputs do not exceed given saturation bounds. Moreover, in the case that a state variable is not measurable and therefore a high-order sliding mode observer is also applied, fairly restrictive assumptions need to be made. It is revealed exemplarily in this presentation that the super-twisting controller based on the high-order sliding mode observer, which is considered in [1] to implement the control system with a mathematical justification, cannot be employed in some scenarios.

This talk presents a twisting-based control law, in which estimate information provided by a second-order robust exact differentiator is incorporated [2]. The class of uncertainties and disturbances dealt with here is much larger than in [1]. It is shown that the actuating signal is Lipschitz continuous and its absolute value is bounded by a known constant. Results of further investigations, which have been conducted into how other algorithms can be adopted in order to introduce saturated signals, are discussed.

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A Check of Flatness for time continuous Systems by Computer Algebra Methods

Kurt Schlacher[a] **Markus Schöberl**[a]

Introduction

Since the late 80's of the last century a simple test for time continuous systems is known, whether one can derive an exact linear input to state behavior by static feedback. If this test is met, a system of linear PDEs or equivalently of nonlinear ODEs and a set of nonlinear equations must be solved to derive the linearizing feedback. For the class of systems, which allow a linear behavior by dynamic feedback, several approaches are known, but it is more challenging to derive a test without solving PDEs or nonlinear ODEs. In this contribution we propose a computer algebra approach, with does not require the solution of DEs.

An Old Result

Consider the locally reachable system

$$x_t^i = f^i\left(t, x, u\right) \ , \quad i = 1, \ldots, n \tag{1}$$

with the vector field $f = \partial_t + f^i \partial_{x^i}$ and the involutive distribution $U = \operatorname{span}\left(\{\partial_{u^1}, \ldots, \partial_{u^m}\}\right)$. The system is input to state linearizable by static feedback, iff together with

$$\begin{aligned} D_0 &= U & D_k &= D_{k-1} + [D_{k-1}, f] \\ D_1 &= D_0 + [D_0, f] & &\vdots \\ &\vdots & D_{l+1} &= D_l \end{aligned}$$

the distributions D_s, $s = 0, \ldots, s-1$ are involutive and $\dim\left(D_l\right) = n + m$ is met.

Let us consider the first step of this test, where we use the shortcut $B = [U, f]$. Involutivity of D_1 implies $[U, B] \subset D_1$. This is equivalent to the existence of an input transformation $\bar{u} = g\left(t, x, u\right)$ to an AI system

$$x_t^i = a^i\left(t, x\right) + B_j^i\left(t, x\right) \bar{u}^j \ , \quad j = 1, \ldots, m \ . \tag{2}$$

Involutivity of D_1 implies $[B, B] \subset D_1$, too. This is equivalent to the existence of a state transformation $\bar{x} = h\left(t, x\right)$ to an AI system

$$\bar{x}_t^i = a^i\left(t, x\right) + \bar{B}_{\bar{j}}^i a_j^{\bar{j}}\left(t, x\right) \bar{u}^j \ , \quad \bar{j} = 1, \ldots, m \ , \tag{3}$$

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where the first $\bar{m} \leq m$ columns of $\left[\bar{B}^i_{\bar{j}}\right]$ are unit and the remaining are zero vectors. In addition $\left[a^{\bar{j}}_j\right]$ is invertible. Now the system (3) can easily been split in a trivial and second one that needs further investigations. It is worth mentioning, that the tests here require differentiation and solving linear equations in the ring of e.g. smooth functions, whereas the transformations from (1) to (3) requires the solution of DEs and nonlinear equations. See e.g. [1] for one of the first publications in this topic.

The Dynamic Feedback Case

A necessary condition for a system like (1) to be flat is, one can transform it to a Partial Affine system, or PAI-system, like

$$x^i_t = a^i\left(t, x, v\right) + B^i_{\alpha_w}\left(t, x, v\right) w^{\alpha_w} \ . \tag{4}$$

Such an input transformation exists only, iff there exists an involutive distribution $W \subset U$ such that $[W, [W, f]] \subset [W, f] + W$ is met. W corresponds to the variables w_{α_w}, $\alpha_w = 1, \ldots, m_w$ and the involutive complement V, $U = W \oplus V$ to v_{α_v}, $\alpha_v = 1, \ldots, m_v$ with $m_w + m_v = m$. It is worth mentioning that the determination of W is already a non trivial task. To derive V one has to solve DEs and nonlinear equations, but its existence can be shown in a straightforward manner. With help of the distributions $B = \operatorname{span}\left(\left\{B^i_{\alpha_w}\partial_{x^i}\right\}\right)$, $\alpha_w = 1, \ldots, m_w$ and $V = \operatorname{span}\left(\left\{\partial_{v^{\alpha_v}}\right\}\right)$, $\alpha_v = 1, \ldots, m_v$ we get further cases.

- Let $[V, B] \subset B + V$ be met, them B has a basis independent on v. If B is involutive, we are back to the previous section, otherwise on has to construct an involutive distribution $\bar{B} \subset B$ and can partially follow the previous steps.
- If $\left[V, \bar{B}\right] \subset \bar{B} + V$is met for $\bar{B} \subset B$, one applies the previous item to $\bar{B}$.
- Otherwise, we propose to prolong the system (4) in the following trivial manner:

$$\begin{aligned} x^i_t &= a^i\left(t, x, v\right) + B^i_{\alpha_w}\left(t, x, v\right) w^{\alpha_w} \\ v^{\alpha_v}_t &= v^{\alpha_v}_1 \ , \end{aligned} \tag{5}$$

  It is shown in [2], how one derives flat output by helps of the representation (5). But the proposed algorithm requires solving DEs and nonlinear equations.

In this contribution we present a computer algebra approach for a flatness test, which avoids solving DEs and nonlinear equations, whenever it is possible. This is an significant extension of [2]. The algorithms are implemented in the computer algebra System Maple 2016.

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