Robust State Estimation for Linear Time Varying Lateral Vehicle Dynamics with Unknown Road Curvature

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Abstract— **This paper presents a cascaded observer scheme for state estimation of the lateral vehicle dynamics. A velocity dependent linear time varying model where the road curvature acts as an unknown input is considered. The proposed observer structure is based on an $\mathcal{H}_\infty$ filter in combination with a higher order sliding mode compensator and yields exact finite time state estimation. The observer design is carried out step by step and simulation results show the applicability of the proposed approach.**

I. INTRODUCTION

Since a few decades, advanced driver assistant systems (ADAS) have been evolving very fast and are a popular topic in theoretic and applied research [1]. ADAS and active safety systems need accurate information on the vehicle's state and its environment. Thus several recent contributions deal with the problem of state estimation of the lateral vehicle dynamics, see e.g. [2, 3, 4]. As pointed out in [5], the road curvature is one of the main disturbances acting on the vehicle's lateral position, e.g., in a lane change maneuver. However, for control purposes it is important to correctly estimate the vehicle's state, which is the motivation for the present contribution.

For strongly observable LTI systems, [6] proposed to use a classical Luenberger observer which yields a bounded estimation error in the presence of a bounded unknown input. A step-by-step differentiation algorithm using a sliding mode differentiator was then applied to the observer's innovation to reconstruct the estimation error in finite time yielding exact state estimates. This concept was extended to a higher order sliding mode (HOSM) algorithm in [7] and [8] generalized this work to linear time varying systems by using a deterministic least squares filter as stabilizer for the error system which then is reconstructed in finite time.

This paper proposes a cascaded observer structure to solve the problem of state estimation for the time varying vehicle lateral dynamics despite of the unknown road curvature. The observer is based upon an $\mathcal{H}_\infty$ filter to achieve a bounded estimation error. A HOSM differentiator is then used together with structural properties of the system to correct the perturbed estimates obtained by the $\mathcal{H}_\infty$ filter. This leads to a theoretically exact reconstruction of the true states in finite time.

The contribution is structured as follows. The problem statement is outlined in section II. In section III, preliminaries which are needed in the subsequent observer design are recalled. In section IV, a cascaded $\mathcal{H}_\infty$-HOSM observer is derived for a general linear time varying (LTV) system with unknown inputs. Section V represents an LTV model for the vehicle lateral dynamics which is affected by the unknown road curvature. In VI, the developed observer is applied for robust state estimation. A conclusion is drawn in section VII.

II. PROBLEM STATEMENT

Consider a linear time varying (LTV) system

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) + D(t)w(t), \\ y(t) &= C(t)x(t), \qquad x(t_0) = x_0,\ t \geq t_0 \in \mathbb{R}, \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the measurement vector, $u(t) \in \mathbb{R}^q$ is the known (control) input vector and $w(t) \in \mathbb{R}^m$ is an unknown input vector. The matrices $A(t)$, $B(t)$, $C(t)$ and $D(t)$ are known time-varying matrices of suitable dimensions. The following statements are assumed to hold throughout the paper:

(A1) The triple $(A(t), D(t), C(t))$ is a constant rank system representation, see [9].

(A2) The output $y(t)$ and all matrices in (1) are bounded and continuously differentiable with bounded derivatives up to the $(\nu-1)$-th derivative, where ν is the (constant) observability index of $(A(t), D(t), C(t))$.

(A3) The unknown input $w(t)$ is bounded with bounded derivatives up to $w^{(\nu-1)}(t)$ and has a known Lipschitz constant L such that

$$\|w^{(i)}(t)\| \leq L,\ \forall\, i = 0, \ldots, \nu - 1 \tag{2}$$

holds.

(A4) The triple $(A(t), D(t), C(t))$ is strongly observable.

The goal of the present work is to design a state estimation algorithm which converges to the true states in finite time independent of the unknown input. This algorithm will then be applied to the problem of state estimation for a LTV lateral vehicle dynamics model with unknown road curvature.

III. PRELIMINARIES

In this section, observability and strong observability for the class of so-called constant rank systems is recapitulated. As the known input $u(t)$ can always be canceled out in the observer error dynamics, without loss of generality $B(t) \equiv 0$ and only the triple $(A(t), D(t), C(t))$ is considered subsequently. First, the definitions for observability and strong observability are recalled:

Definition 1 (strong observability [10, 11]):

i) The pair $(A(t), C(t))$ is called observable, if $\dot{x} = A(t)x$, $C(t)x(t) \equiv 0$ on some time interval always implies that $x(t) \equiv 0$ on this interval.

ii) The triple $(A(t), D(t), C(t))$ is called strongly observable if $\dot{x} = A(t)x + D(t)w(t)$, $C(t)x(t) \equiv 0$ on some time interval always implies that $x(t) \equiv 0$ on this interval for any input $w(t)$.

The generalized controllability and observability matrices for $(A(t), D(t), C(t))$ can be defined as

$$\begin{aligned} Q_k(t) &= \begin{bmatrix} P_0(t) & P_1(t) & \cdots & P_{k-1}(t) \end{bmatrix}, \\ R_k^T(t) &= \begin{bmatrix} C_0^T(t) & C_1^T(t) & \cdots & C_{k-1}^T(t) \end{bmatrix}, \end{aligned} \tag{3}$$

as shown in [9]. The elements $P_i(t)$ and $C_i(t)$, $i = 0, \ldots, k$ are recursively defined according to

$$\begin{aligned} P_{k+1}(t) &= A(t)P_k(t) + \dot{P}_k(t),\ P_0(t) = D(t), \\ C_{k+1}(t) &= C_k(t)A(t) + \dot{C}_k(t),\ C_0(t) = C(t). \end{aligned} \tag{4}$$

The generalized controllability and observability matrices are now used to define the class of constant rank systems.

Definition 2 (Constant rank system, [12]): The system $(A(t), D(t), C(t))$ is a constant rank system representation if and only if there exist positive integers μ, ν, q_c and q_0 such that $D(t)$ and $C(t)$ are μ respectively ν times continuously differentiable and $A(t)$ is $\max(\mu, \nu) - 1$ times continuously differentiable, such that

$$\begin{aligned} \operatorname{rank} Q_\mu(t) &= \operatorname{rank} Q_{\mu+1}(t) = q_c \leq n,\ \forall t, \\ \operatorname{rank} R_\nu(t) &= \operatorname{rank} R_{\nu+1}(t) = q_0 \leq n,\ \forall t. \end{aligned} \tag{5}$$

The smallest integers μ and ν for which relation (5) holds are termed controllability respectively observability index. Note that in the LTV-case, these integers may be strictly greater than n.

The concept of observability for LTV systems is extended to strong observability in [10]. The necessary and sufficient condition proposed there will be directly used to design the robust observer in the present contribution. Therefore, the following matrices depending on the observability index are introduced

$$\begin{aligned} \mathcal{D}_{\alpha,\alpha-1}(t) &= C_0(t)D(t) && \text{for } 2 \leq \alpha \leq \nu, \\ \mathcal{D}_{\alpha,1}(t) &= C_{\alpha-2}(t)D(t) + \dot{\mathcal{D}}_{\alpha-1,1}(t) && \text{for } 3 \leq \alpha \leq \nu, \\ \mathcal{D}_{\alpha,\beta}(t) &= \mathcal{D}_{\alpha-1,\beta-1}(t) + \dot{\mathcal{D}}_{\alpha-1,\beta}(t) && \text{for } 3 \leq \beta < \alpha \leq \nu, \end{aligned} \tag{6}$$

with C_i as the corresponding entries of the generalized observability matrix (3), see [10, 8]. The condition for strong observability is summarized in the next theorem.

Theorem 1 (Strong observability, [10]): The constant rank system $(A(t), D(t), C(t))$ is strongly observable on any interval if and only if it is observable and

$$\operatorname{rank} S(t) = \operatorname{rank} S^*(t) \tag{7}$$

holds for

$$S(t) = \begin{bmatrix} R_\nu(t) & J_\nu(t) \end{bmatrix},\quad S^*(t) = \begin{bmatrix} I_n & 0 \\ R_\nu(t) & J_\nu(t) \end{bmatrix} \tag{8}$$

with

$$J_\nu(t) = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \mathcal{D}_{2,1} & 0 & \cdots 0 & \\ \mathcal{D}_{3,1} & \mathcal{D}_{3,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{D}_{\nu,1} & \mathcal{D}_{\nu,2} & \cdots & \mathcal{D}_{\nu,\nu-1} \end{bmatrix}, \tag{9}$$

and ν as the observability index.

Proof: The strong observability condition holds on any interval because of the constant rank assumption. The rest of the proof is presented in [10]. ■

The strong observability condition can be directly used to reconstruct the states by using the system output and its derivatives which is summarized in the next proposition.

Proposition 1 (State reconstruction, [10]): Under assumption (A2), define a matrix $K(t) \in \mathbb{R}^{p\nu \times p\nu}$ such that

$$\operatorname{Ker} K(t) = \operatorname{Im} J_\nu(t)\ \forall t \tag{10}$$

and furthermore let

$$H(t) = R_\nu^T(t)K^T(t)K(t)R_\nu(t). \tag{11}$$

Then, $H(t)$ is invertible and

$$x(t) = H^{-1}(t)R_\nu^T(t)K^T(t)K(t)\hat{y}(t) \tag{12}$$

holds, with

$$\hat{y}(t) = \begin{bmatrix} y^T(t) & \dot{y}^T(t) & \cdots & y^{(\nu-1)T}(t) \end{bmatrix}^T. \tag{13}$$

This implies that if the system is strongly observable, the states can be reconstructed by using the output and its derivatives, which is a generalization of the LTI case [13].

IV. ROBUST OBSERVER DESIGN

In this section, an observer for system (1) which yields exact state reconstruction in finite time is derived. The presented concept is based on the ideas of [6, 8, 13] and is described in detail subsequently.

A. Cascaded Observer Design

The idea of the cascaded observer structure is depicted in Fig. 1. The $\mathcal{H}_\infty$ stabilizer should provide a bounded estimation error despite the unknown input. Using the bounded observer innovation and its derivatives which are generated by a HOSM differentiator, the error system can be reconstructed in finite time because of the strong observability property. Thus, the estimates generated by the $\mathcal{H}_\infty$ filter can be corrected and all the system states are reconstructed in finite time. Both parts of the cascaded observer are now explained in detail.

B. $\mathcal{H}_\infty$ Stabilizer

In the present approach, the least-squares filter in [8] is replaced by an $\mathcal{H}_\infty$ filter as a stabilizer of the estimation error [14, 15]. Thus, results for the time varying finite horizon $\mathcal{H}_\infty$ filter for a general LTV system (1) are recalled. An $\mathcal{H}_\infty$ filter which gives an estimate $\tilde{x}(t)$ for the state $x(t)$ of (1) is designed following [15] by utilizing the next proposition.

Proposition 2 ($\mathcal{H}_\infty$ filter, [15]): Assume that $w(t) \in \mathcal{L}_2$, the initial condition x_0 of (1) is unknown, the worst case performance bound is defined according to

$$J := \sup_{\|w\|_2 \neq 0} \frac{\|e\|_2^2}{\|w\|_2^2 + x_0^T R x_0}, \tag{14}$$

where $e(t) = x(t) - \tilde{x}(t)$ is the estimation error, $R = R^T$ is a positive definite matrix. Then

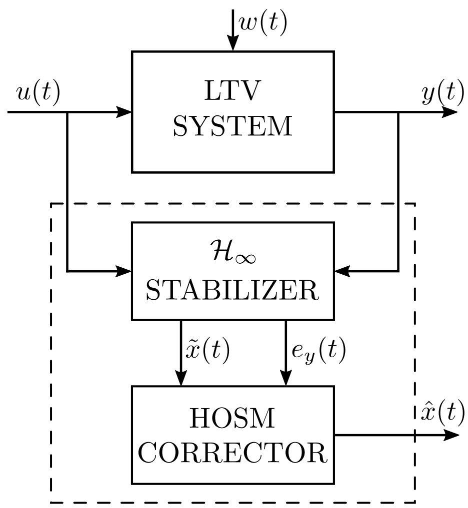

Fig. 1. Cascaded observer structure

1) there exists a filter

$$\dot{\tilde{x}}(t) = A(t)\tilde{x}(t) + P(t)C^T(t)\left[y(t) - C(t)\tilde{x}(t)\right]. \quad (15)$$

such that

$$J < \gamma^2, \; \gamma > 0 \quad (16)$$

holds for the performance bound (14) on a time interval $t \in [0, t_1]$ with $t_1 \in \mathbb{R} < \infty$ if and only if there exists a uniformly bounded positive semidefinite solution $P(t)$ to the Riccati differential equation

$$\begin{aligned}\dot{P}(t) =& A(t)P(t) + P(t)A^T(t) - \\ & P(t)\left(C^T(t)C(t) - \frac{1}{\gamma^2}I\right)P(t) + D(t)D(t)^T\end{aligned} \quad (17)$$

with $P(0) = P_0 = R^{-1}$.

2) the unforced error system

$$\dot{e}(t) = \left[A(t) - P(t)C^T(t)C(t)\right]e(t) \quad (18)$$

is exponentially stable if $w(t) \equiv 0$.

Proof: See [15]. ■

However, according to assumption (A2), the unknown input is not necessarily in $\mathcal{L}_2$. Thus, the convergence properties of the error system under assumption (A2) are explicitly stated in the following proposition.

Proposition 3: If the unknown input is bounded according to assumption (A2), then the estimation error $e(t)$ of the perturbed error system

$$\dot{e}(t) = \left[A(t) - P(t)C^T(t)C(t)\right]e(t) + D(t)w(t) \quad (19)$$

converges into a bounded neighborhood around the equilibrium $e(t) = 0$.

Proof: Consider the Lyapunov function candidate

$$V(e) = e^T P^{-1} e. \quad (20)$$

Under the assumption that P is positive definite, V is also positive definite. Its time derivative

$$\begin{aligned}\dot{V} &= \dot{e}^T P^{-1} e - e^T P^{-1}\dot{P}P^{-1}e + e^T P^{-1}\dot{e} \\ &= e^T\left[\left(A - PC^TC\right)^T P^{-1} - P^{-1}\dot{P}P^{-1} + \right. \\ & \quad \left. P^{-1}\left(A - PC^TC\right)\right]e + 2e^TP^{-1}Dw \\ &= e^T[-C^TC - \frac{1}{\gamma^2}I - P^{-1}DD^TP^{-1}]e + \\ & \quad 2e^TP^{-1}Dw\end{aligned} \quad (21)$$

is negative definite for $w(t) = 0$ and thus due to the boundedness of $w(t)$ the estimation error $e(t)$ converges exponentially into a bounded neighborhood around the equilibrium $e(t) = 0$. ■

Remark 1: If P is positive semidefinite, the proof can be modified by using a perturbed Riccati equation, which has a unique bounded positive definite solution for a sufficiently small perturbation ϵI, $\epsilon > 0$, see [14].

Remark 2: P_0 is in some sense the confidence in the knowledge of the initial state and thus can be regarded as a tuning parameter in practical applications. A suboptimal $\mathcal{H}_\infty$ filter can be designed for system (1) by selecting γ and checking the existence of (17). Searching for the minimal γ for which (17) has a positive semidefinite solution iteratively yields the suboptimal $\mathcal{H}_\infty$ filter.

C. HOSM Corrector

To correct the state estimates, the robust differentiator by Levant [16] is utilized in the observer. Thus, it is briefly summarized here. Consider an unknown smooth signal $f_0(t)$ where the r-th derivative $f_0^{(r)}(t)$ exists and has a known Lipschitz constant i.e. $|f_0^{(r+1)}(t)| < L$. Then, the differentiator [16] is defined in the recursive form

$$\begin{aligned}\dot{z}_0 &= v_0 = -\lambda_r L^{1/(r-1)}|z_0 - f_0(t)|^{r/(r+1)}\text{sign}(z_0 - f(t)) + z_1, \\ \dot{z}_1 &= v_1 = -\lambda_{r-1}L^{1/r}|z_1 - v_0|^{(r-1)/r}\text{sign}(z_1 - v_0) + z_2, \\ &\vdots \\ \dot{z}_r &= -\lambda_0 L\text{sign}(z_r - v_{r-1}),\end{aligned} \quad (22)$$

with sufficiently large parameters λ_i. One possible choice of parameters for a differentiator up to order 5 is $\lambda_0 = 1.1$, $\lambda_1 = 1.5$, $\lambda_2 = 2$, $\lambda_3 = 3$, $\lambda_4 = 5$, $\lambda_5 = 8$, as proposed in [16]. A tuning procedure to determine parameters for arbitrary order differentiators is proposed in [17]. It is shown in [16] that in the absence of noise the equations

$$|z_i - f_0^{(i)}(t)| = 0, \; i = 0, \ldots, r \quad (23)$$

holds after a finite transient time and thus this differentiator can be used to exactly reconstruct the derivatives of $f_0(t)$. The differentiator is now used to reconstruct the observer error $e(t)$ of (19). The output error is defined as

$$e_y(t) = y(t) - C(t)\tilde{x}(t), \quad (24)$$

which is bounded with bounded derivatives, see [8].

Proposition 4: The estimation error $e(t)$ can be reconstructed by using the output error $e_y(t) = C(t)e(t)$ and its derivatives such that

$$\tilde{e}(t) = H_e^{-1}(t) R_{\nu,e}^T(t) K^T(t) K(t) \hat{e}_y(t), \tag{25}$$

with $\hat{e}_y = \begin{bmatrix} e_y^T & \dot{e}_y^T & \cdots & e_y^{(\nu-1)T} \end{bmatrix}^T$ and $H_e(t)$ designed for the error system according to proposition 1 and $R_{\nu,e}$ as the observability matrix for the triple $(\tilde{A}(t), D(t), C(t))$. $\tilde{A}(t) = A(t) - P(t)C^T(t)C(t)$ is the dynamic matrix of the error system.

Proof: It is shown in [8] that if system (1) is strongly observable, then the error system can be reconstructed by applying the relation in proposition 1 to the triple $(\tilde{A}(t), D(t), C(t))$. ■

Theorem 2 (Cascaded Observer): Let (A1)-(A4) hold. Then it is possible to design a cascaded observer for system (1) consisting of a $\mathcal{H}_\infty$ stabilizer according to Proposition 2 and a HOSM corrector according to proposition 4 with the state estimate

$$\hat{x}(t) = \tilde{x}(t) + \tilde{e}(t). \tag{26}$$

This yields an exact state reconstruction such that

$$e_c(t) = x(t) - \hat{x}(t) \equiv 0 \tag{27}$$

holds for all $t \geq t_f$ where t_f represents a finite time instant.

Proof: The proof is analogous to the proof of [8, Theorem 5.1], except by using Proposition 3 to show that the estimation error of the $\mathcal{H}_\infty$ filter is bounded. ■

V. LTV MODEL OF VEHICLE LATERAL DYNAMICS

The model of the vehicle lateral dynamics is derived in [18]. It is an extension of the linearized bicycle model for a time varying longitudinal velocity $v(t)$ and can be written in the form of system (1) with

$$\begin{aligned} x(t) &= \begin{bmatrix} y_r & \dot{y}_r & \psi & \dot{\psi} \end{bmatrix}^T, \\ A(t) &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_1/v(t) & -a_1 & a_2/v(t) \\ 0 & 0 & 0 & 1 \\ 0 & a_3/v(t) & -a_3 & a_4/v(t) \end{bmatrix}, \\ B(t) &= \begin{bmatrix} 0 & b_1 & 0 & b_2 \end{bmatrix}^T, \\ D(t) &= \begin{bmatrix} 0 & (a_2 - v^2(t)) & 0 & a_4 \end{bmatrix}^T, \\ C &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \end{aligned} \tag{28}$$

The states of this model are the vehicle's relative lateral position y_r, velocity $\dot{y}_r$, relative yaw angle ψ and its derivative $\dot{\psi}$. The unknown input is $w(t) = 1/\rho(t)$ where $\rho(t)$ is the road curvature and the known control input $u(t) = \delta(t)$ is the steering angle. The measured outputs are the relative lateral position and the relative yaw angle. The (constant) parameters are

$$\begin{aligned} a_1 &= \frac{-2(C_{s_f} + C_{s_r})}{m}, & a_2 &= \frac{(C_{s_r} l_r - C_{s_f} l_f)}{m}, \\ a_3 &= \frac{C_{s_r} l_r - C_{s_f} l_f}{I_z}, & a_4 &= \frac{-2(C_{s_f} l_f^2 + C_{s_r} l_r^2)}{I_z}, \\ b_1 &= \frac{2C_{s_f}}{m}, & b_2 &= \frac{2 l_f C_{s_f}}{m} \end{aligned} \tag{29}$$

with C_{s_f} and C_{s_r} as the front respectively rear cornering stiffness parameter. The vehicle mass is denoted by m and I_z is the corresponding yaw inertia. Moreover, l_f and l_r are the distances from the front respectively rear axle to the center of gravity. The normal load distribution on each tire is assumed to be static. This assumption is valid for a slow varying velocity; a more complex model where the effect of acceleration dependent tire forces is considered can be found in [19]. However, the approach presented here can be extended to the model of [19].

VI. OBSERVER DESIGN FOR LATERAL VEHICLE DYNAMICS

In this section, the robust observer design is carried out for the vehicle lateral dynamics model presented in section V. The model parameters are taken from [20] and are $m = 1564\,\text{kg}$, $C_{s_f} = C_{s_r} = 14 \times 10^4\,\text{Nm/rad}$, $l_f = 1.268\,\text{m}$, $l_r = 1.620\,\text{m}$ and $I_z = 2230\,\text{kgm}^2$. For the simulation it is assumed, that the velocity is time varying with

$$v(t) = 25 + \sin(t)\ \text{m/s}, \tag{30}$$

and the steering angle is $\delta(t) = 0$. The unknown road curvature is also time varying according to

$$w(t) = \frac{1}{\rho(t)} = 0.5 \sin(1.2t)\ \text{m} \tag{31}$$

which is quite large compared to real operating conditions (see e.g. [5]). However, the goal is to show that the state estimates converge to the true values independent of the unknown input.

A. Observer Design

It is straightforward to see that the observability index is $\nu = 2$ as the observability matrix yields

$$\text{rank}(R_2) = \text{rank} \begin{bmatrix} C \\ CA \end{bmatrix} = \text{rank} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = n. \tag{32}$$

Thus, as $C(t)D(t) = 0$, the matrix $J_\nu(t)$ is the zero matrix and it can directly be stated that the system is strongly observable. First, the $\mathcal{H}_\infty$ filter is designed to stabilize the error system. Therefor, the Riccati equation (17) corresponding to (28) is solved numerically for different values of γ trying to iteratively find the smallest γ for which a positive semidefinite solution exists. In the present example, P_0 as the identity matrix and $\gamma = 25$ yields a positive definite solution of the Riccati equation over the simulation duration. Thus, one can implement the suboptimal $\mathcal{H}_\infty$ filter as a stabilizer of the error dynamics according to (15). To design the reconstruction from proposition 4, it is clear that $\mathrm{Im}J_\nu = \{0\}$ and thus $K(t)$ can be chosen as the identity matrix. The time varying matrices $R_{\nu,e}(t)$ and $H_e(t)$ are determined during runtime by utilizing the solution $P(t)$ of the Riccati equation from the $\mathcal{H}_\infty$ stabilizer. The first derivative of the output error $e_y(t)$ is then estimated by using algorithm (22). The parameters are taken from section IV-C and the Lipschitz constant was determined in simulation and set to $L = 1$. As roads are constructed out of road segments with linear curvature (so-called clothoids), theoretically, only the first derivative of the unknown input is Lipschitz bounded and thus, a differentiator with order $r = 1$ is applied to this problem. If also higher order derivatives than $\nu - 1$ are Lipschitz bounded, a higher order differentiator could be utilized to increase the accuracy as pointed out in [16].

B. Simulation Results

For simulation, the least squares filter [21, 8] is compared to the presented $\mathcal{H}_\infty$ approach. The parameters of the $\mathcal{H}_\infty$ filter are $\gamma = 25$ and P_0 as the identity matrix. For the least squares filter, P_0 also was chosen as identity matrix and Q was tuned such that the peak in norm of the estimation error $\|e_c(t)\| = \sqrt{e_c^T(t)e_c(t)}$ during the initial transient is approximately the same for both filters, which yields $Q = 1000\,I$. In Fig. 2 the norm of the estimation error for both filters with and without the HOSM compensator is depicted. It can be seen that without the HOSM compensator, both, the least squares (LS) and the $\mathcal{H}_\infty$ filter have a bounded estimation error. However, the bound of the least squares filter is about twice as large for this specific example. Including the HOSM corrector, the estimation errors for both concepts converge to zero in a finite time. The error of the cascaded $\mathcal{H}_\infty$-HOSM observer converges slightly faster. In

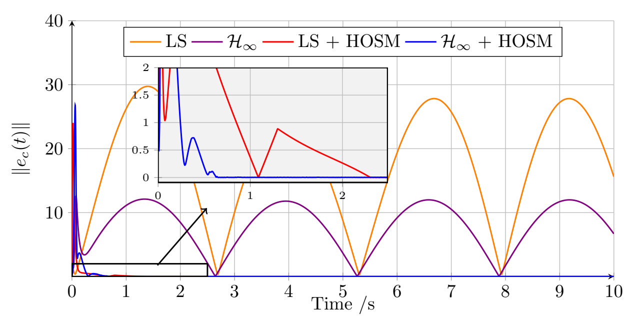

Fig. 2. Norm of the estimation error for the reduced error system.

Fig. 3, the logarithmic norm of the estimation error of the cascaded $\mathcal{H}_\infty$-HOSM estimator is shown for different orders of the robust differentiator (22). Only the first derivative is used in the compensator which yields better accuracy of the state reconstruction for higher order differentiators. This coincides with the expected behavior [16].

VII. Conclusion and Outlook

This paper presented a linear time-varying cascaded observer for the state estimation of the lateral vehicle dynamics.

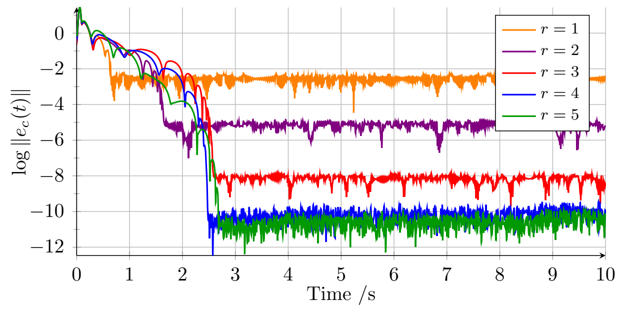

Fig. 3. Logarithmic estimation error for the cascaded $\mathcal{H}_\infty$/HOSM observer for different orders of the differentiator (only the first derivative is used in the observer).

Compared to a previously proposed cascaded scheme [8] based on a least squares filter, this contribution utilizes an $\mathcal{H}_\infty$ filter which shows faster convergence to the true states in the simulation example. The presented method could be extended to more complex and accurate vehicle models such as, e.g., the one presented in [19]. Open research questions are the sensitivity of this method to model uncertainties and measurement noise.

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