Discrete-Time Super Twisting Controller for Networked Control Systems ⋆

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Abstract: In this paper, an integral sliding mode based control concept for networked control systems in the presence of variable time delays and external perturbations is proposed. The control concept consists of a buffering mechanism and a discrete time integral sliding mode based control law. This setting offers the possibility to use discrete-time sliding mode control techniques designed for relative degree one systems. As a consequence, networked control algorithms can be developed based on recent results from this active field of research. Using a discretized version of the super twisting algorithm, the so called matching algorithm, increases the accuracy due to the absence of discretization chattering. The effectiveness of this control strategy is demonstrated by means of laboratory experiments.

Keywords: Time-varying delay, Sliding-mode control, Discretization, Closed-loop control

1. INTRODUCTION

The feedback loop of modern control systems is very often closed via a network. This means that a sensor transmits its measurements via a network to the controller which computes the actuating signal and sends it via the same or a second network to the actuator. This architecture benefits from increased flexibility and adaptability because it offers the possibility to easily replace components or even add additional ones. Furthermore, large spatial distances can be overcome with very little wiring effort or even without wires using wireless communication technologies. Nevertheless, using those technologies in control engineering comes also with additional challenges in the design of the control algorithms due to network imperfections. One of these imperfections is, that packets could get lost. There are several scientific publications concerning the design of control algorithms considering this type of imperfection (see e.g. Schenato et al. (2007); Xiong and Lam (2007); Azimi-Sadjadi (2003)).

The second challenge is to design control algorithms, which are robust with respect to delays induced by the network. There are some approaches, which considers time varying delays and guarantees stability (see Liu and Fridman (2012); Liu et al. (2012)) using Lyapunov-Krasovskii analysis. Other approaches are based on prediction and achieve compensation of communication delays and data losses (see Liu (2010)). Some approaches make use of event-triggered control to reduce the network load, which should indirectly lead to smaller time delays and less packet loss. This idea of using event-triggered control is already combined with sliding mode control and can be found e.g. in Behera and Bandyopadhyay (2016); Incremona et al. (2017). An overview of existing techniques for networked control are given in Zhang et al. (2016); Heemels and van de Wouw (2010).

In Ludwiger et al. (2017) a method was proposed, which considers time varying delays and uses discrete-time first order sliding mode control to alleviate perturbations. An extension of this approach is given in Ludwiger et al. (2018), where two discrete-time sliding mode methods using higher relative degree sliding variables are proposed.

In the present paper a control scheme for networked control systems (NCS) based on integral sliding mode control is presented which offers the opportunity to cast the problem into a form where discrete-time sliding mode control laws designed for relative degree one systems can be applied. As a consequence recently developed algorithms as the discrete-time equivalent super twisting controller Koch and Reichhartinger (submitted) can be applied. The properties of the proposed algorithm will be shown by means of laboratory experiments. In the following section the problem statement is explained and the assumptions are pointed out. Afterwards the proposed approach is described in detail starting with the buffering mechanism, followed by the plant modeling and ending with the control design. The effectiveness of the proposed approach is then exemplified by means of laboratory experiments. Final conclusions and an outlook is given in the last section.

2. PROBLEM STATEMENT

In this paper, a linear time-invariant system

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{A}_c\boldsymbol{x} + \boldsymbol{b}_c(u(t) + f(t)) \tag{1}$$

with state vector $\boldsymbol{x} \in \mathbb{R}^n$, scalar input $u \in \mathbb{R}$ is considered. Function $f \in \mathbb{R}$ represents a matched perturbation. Matrix $\boldsymbol{A}_c$ and input vector $\boldsymbol{b}_c$ have appropriate dimensions. All

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states $\boldsymbol{x}$ of the plant are sampled periodically with the constant sampling time T, i.e.$\boldsymbol{x}(kT) = \boldsymbol{x}_k$ with $k \in \mathbb{N}_0$, and are sent via a network to the controller, see Fig. 1. Due to the imperfection of the network, the measurements are received at the controller delayed by $\tau_k^s \in \mathbb{R}^+$. A delay induced by the computing time for evaluating the control law, can also be considered by a variable time delay $\tau_k^c \in \mathbb{R}^+$. The resulting actuating signal is then sent over the network to a zero order hold (ZOH) element at the input of the actuator. The delay induced by this transmission is denoted as τ_k^a. The following assumptions are made:.

Assumption 1. The given plant (1) is controllable and the constant sampling time T is not equal to the pathological sampling time (see Kalman et al. (1963))

Assumption 2. No packet loss occur and the sum of all the time delays

$$\tau_k^s + \tau_k^c + \tau_k^a \leq dT \qquad d \in \mathbb{N} \tag{2}$$

is bounded. Please note that this overall time delay could be larger than the sampling time T.

Assumption 3. The matched perturbation $f(t)$ is piecewise constant, i.e.,

$$f(t) = f(kT) = f_k \quad kT \leq t < (k+1)T, \quad k \in \mathbb{N}_0 \tag{3}$$

and the change rate is bounded, i.e.

$$\sup \left| \frac{f_{k+1} - f_k}{T} \right| = L < \infty. \tag{4}$$

No dropouts occur, therefore it is possible to lump all time delays together to the round trip time

$$\tau_k = \tau_k^s + \tau_k^c + \tau_k^a. \tag{5}$$

The main goal is to robustly stabilize the origin of (1) despite of time varying delays and external perturbations.

3. PROPOSED APPROACH

The architecture of the proposed approach is depicted in Fig. 1. A buffer is implemented at the actuator side of the plant to keep the round trip time (the delay from the sensor to the actuator) constant. In the following subsections, the buffering mechanism, the model of the NCS as well as the controller are presented in more detail.

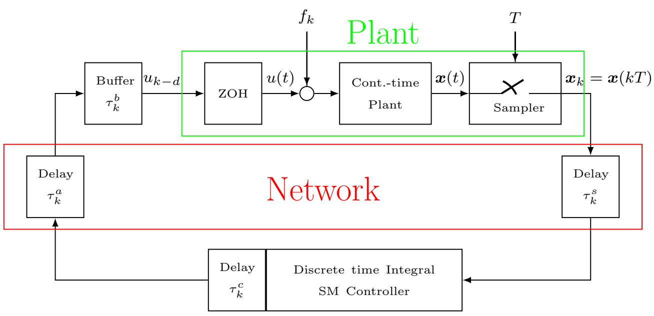

Fig. 1. Architecture of proposed NCS approach

3.1 Buffering Mechanism

The sampler attaches a time stamp to each measurement before transmission, which is not altered by the controller. Due to a time synchronization between the sampler and the buffer, the current time delay τ_k could be determined. The buffer stores the received packets and forwards them after an additional delay of

$$\tau_k^b = dT - \tau_k \tag{6}$$

to ensure a constant round trip time. This buffering mechanism is a very common approach in multimedia applications (see e.g. Ramjee et al. (1994); Atzori and Lobina (2006)) but has also been introduced in the control community e.g. in Quevedo and Nesic (2011).

3.2 Modelling of the Plant with Buffered Network

Considering the sampling time T, the constant time delay dT achieved by the buffer and plant (1), the discrete-time model of the NCS can be stated as

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{b}(u_{k-d} + f_k) \tag{7}$$

with $\boldsymbol{A} = e^{\boldsymbol{A}_c T}$ and $\boldsymbol{b} = \int_0^T e^{\boldsymbol{A}_c s}\boldsymbol{b}_c ds$. Defining the lifted state vector

$$\boldsymbol{\xi}_k = \left[\boldsymbol{x}_k^T \; u_{k-1} \; u_{k-2} \; \ldots \; u_{k-d}\right]^T \tag{8}$$

results in the lifted model

$$\boldsymbol{\xi}_{k+1} = \underbrace{\begin{bmatrix} \boldsymbol{A} & \mathbf{0} & \ldots & \mathbf{0} & \boldsymbol{b} \\ \mathbf{0} & 0 & \ldots & 0 & 0 \\ \mathbf{0} & 1 & 0 & \ldots & 0 \\ \vdots & & \ddots & & \vdots \\ \mathbf{0} & 0 & \ldots & 1 & 0 \end{bmatrix}}_{\hat{\boldsymbol{A}}} \boldsymbol{\xi}_k + \underbrace{\begin{bmatrix} \mathbf{0} \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{\hat{\boldsymbol{b}}} u_k + \underbrace{\begin{bmatrix} \boldsymbol{b} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{\hat{\boldsymbol{b}}_f} f_k. \tag{9}$$

Remark 1. In contrast to (1) the perturbation in (9) does not fulfill the matching condition, although it is fulfilled in (7). Nevertheless, it is possible to design a sliding mode control law in order to robustly stabilize the origin of the state vector $\boldsymbol{x}_k$ but not the state vector $\boldsymbol{\xi}_k$, which is common for unmatched uncertainties. However, this is a desired behavior in this NCS applications, because the elements $u_{k-1} \; u_{k-2} \; \ldots \; u_{k-d}$ have to compensate for the matched perturbation.

3.3 Controller Design

The basic concept of integral sliding mode control is to robustify a control loop, which was designed for the nominal case i.e. $f_k = 0, \forall k$. This is achieved by defining the control signal as

$$u_k = u_{N,k} + u_{S,k} \tag{10}$$

where $u_{N,k}$ denotes a nominal control signal, designed for the nominal case, and $u_{S,k}$ representing the sliding mode based part of the control law. The control law $u_{S,k}$ is designed to compensate for the matched perturbation in order to keep the desired properties, ensured by the nominal control law, also in the perturbed case. The design of these two parts will be explained in more detail in the following sections.

Nominal Control Law
The nominal lifted model

$$\hat{\boldsymbol{\xi}}_{k+1} = \hat{\boldsymbol{A}}\hat{\boldsymbol{\xi}}_k + \hat{\boldsymbol{b}}u_{N,k} \tag{11}$$

with the nominal lifted state vector

$$\hat{\boldsymbol{\xi}}_k = \left[\boldsymbol{x}_k^T \; u_{N,k-1} \; u_{N,k-2} \; \ldots \; u_{N,k-d}\right]^T \tag{12}$$

results from the lifted model (9) and lifted state vector (8) for $f_k = u_{S,k} = 0, \forall k$. Applying, e.g., classical approaches like assigning $n+d$ eigenvalues by means of a linear state control law

$$u_{N,k} = -\boldsymbol{k}^T \hat{\boldsymbol{\xi}}_k \qquad \text{with } \boldsymbol{k}^T \in \mathbb{R}^{n+d}. \tag{13}$$

This control law is designed to achieve stability and desired performance of the unperturbed NCS.

Sliding Mode Control Law

In the remainder of this paper, a discrete time equivalent of the super-twisting algorithm, the so called matching algorithm, which was recently proposed in Koch and Reichhartinger (submitted) is used. The basic idea of this algorithm will be explained in the following paragraph. Consider a continuous time perturbed integrator

$$\begin{aligned} \frac{\mathrm{d}\sigma}{\mathrm{d}t} &= \tilde{u}_S + \varphi \\ \frac{\mathrm{d}\varphi}{\mathrm{d}t} &= \Delta(t) \end{aligned} \tag{14}$$

with $\sup(\Delta(t)) = L_\varphi < \infty$ and applying the super twisting algorithm proposed in Levant (1993) results in the well-known closed loop dynamic

$$\begin{aligned} \frac{\mathrm{d}\sigma}{\mathrm{d}t} &= -\lambda_1 \lceil \sigma \rfloor^{\frac{1}{2}} + \sigma_2 \\ \frac{\mathrm{d}\sigma_2}{\mathrm{d}t} &= -\lambda_2 \lceil \sigma \rfloor^{0} + \Delta(t) \end{aligned} \tag{15}$$

where $\lceil y \rfloor^\gamma = |y|^\gamma \operatorname{sign}(y)$. As proposed in Reichhartinger and Spurgeon (2018) this closed loop dynamics can be written in the pseudolinear form

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\sigma \\ \sigma_2\end{bmatrix} = \boldsymbol{M}(\sigma)\begin{bmatrix}\sigma \\ \sigma_2\end{bmatrix} + \begin{bmatrix}0 \\ \Delta(t)\end{bmatrix} \tag{16}$$

with

$$\boldsymbol{M}(\sigma) = \begin{bmatrix} -\lambda_1 |\sigma|^{-\frac{1}{2}} & 1 \\ -\lambda_2 |\sigma|^{-1} & 0 \end{bmatrix}, \tag{17}$$

its characteristic polynomial (almost everywhere)

$$w(s) = s^2 + \lambda_1 |\sigma|^{-\frac{1}{2}} s + \lambda_2 |\sigma|^{-1} \tag{18}$$

and the resulting state dependent eigenvalues

$$s_1(\sigma) = p_1 |\sigma|^{-1/2} \qquad s_2(\sigma) = p_2 |\sigma|^{-1/2} \tag{19}$$

with parameters $p_1, p_2 \in \mathbb{C}$. Such that

$$\lambda_1 = -(p_1+p_2), \qquad \lambda_2 = p_1 p_2. \tag{20}$$

Performing Euler forward discretization of (14) with sampling time T results in

$$\sigma_{k+1} = \sigma_k + T\tilde{u}_{S,k} + T\varphi_k \tag{21a}$$

$$\varphi_{k+1} = \varphi_k + T\Delta_k. \tag{21b}$$

The basic idea of the matching algorithm is to design a control law

$$\begin{aligned} \tilde{u}_{S,k} &= \frac{1}{T}\left(u_{1,k}\sigma_k - \sigma_k\right) + \nu_k \\ \nu_{k+1} &= \nu_k + u_{2,k}\sigma_k \end{aligned} \tag{22}$$

with $u_{1,k}$ and $u_{2,k}$ such that the state dependent eigenvalues of the discrete-time closed loop system matches the state dependent discretized eigenvalues

$$q_i = \begin{cases} e^{Ts_i} & \sigma_k \neq 0 \\ 0 & \sigma_k = 0 \end{cases} \qquad i = 1, 2. \tag{23}$$

of the continuous-time closed loop system (15). Applying (22) to (21a) results in the discrete-time closed loop system

$$\begin{bmatrix}\sigma_{k+1} \\ \sigma_{2,k+1}\end{bmatrix} = \boldsymbol{M}_d \begin{bmatrix}\sigma_k \\ \sigma_{2,k}\end{bmatrix} + \begin{bmatrix}0 \\ \Delta_k\end{bmatrix} \tag{24}$$

with $\sigma_{2,k} = \nu_k + \varphi_k$ and

$$\boldsymbol{M}_d = \begin{bmatrix} u_{1,k} & T \\ u_{2,k} & 1 \end{bmatrix}. \tag{25}$$

Computing $det(z\boldsymbol{I} - \boldsymbol{M}_d)$ results in the characteristic polynomial

$$w(z) = z^2 - (u_{1,k}+1)z + u_{1,k} - Tu_{2,k}. \tag{26}$$

Specifying the desired characteristic polynomial

$$z^2 - (q_1+q_2)z + q_1 q_2 \tag{27}$$

and using the desired state dependent eigenvalues (23) results in

$$u_{1,k} = q_1 + q_2 - 1 \tag{28}$$

$$u_{2,k} = \frac{1}{T}\left[u_{1,k} - q_1 q_2\right] \tag{29}$$

as proposed in Koch and Reichhartinger (submitted). This algorithm has some appealing properties. First of all, no discretization chattering appears using this algorithm. Secondly, overestimating the desired controller gains has very little negative impact on the accuracy of the closed loop system compared to the explicit Euler discretized version of the super twisting algorithm (see Livne and Levant (2014)). These properties will be demonstrated by means of experiments in the next section.

Remark 2. An other possible choice for the desired state dependent eigenvalues would be

$$q_i = 1 + s_i T \qquad \text{with } i = 1, 2 \tag{30}$$

which would lead to the explicit Euler discretized version of the super twisting algorithm. The disadvantage of this algorithm is, that the closed loop accuracy is deteriorated by discretization chattering.

In the following theorem, the second part of the control law is designed based on the previously explained matching algorithm and the integral sliding mode control approach.

Theorem 3. Considering continuous-time plant (1), the sampling with constant sampling time T and the constant round trip time ensured by the buffer (6) results in the lifted model (9) with state vector $\boldsymbol{\xi}_k$. Define the discrete-time integral sliding variable

$$\sigma_k = \boldsymbol{m}^T \boldsymbol{\xi}_k + w_k \tag{31}$$

with

$$\boldsymbol{m}^T = \begin{bmatrix}\boldsymbol{m}_1^T & 1 & \boldsymbol{0}\end{bmatrix}, \qquad \boldsymbol{m}_1^T \boldsymbol{b} = 1 \tag{32}$$

where $\boldsymbol{m}_1^T \in \mathbb{R}^n$. Let the nominal control law in (10) equal (13) and

$$w_{k+1} = \boldsymbol{l}^T \boldsymbol{\xi}_k, \qquad \boldsymbol{l}^T = -\boldsymbol{m}^T(\hat{\boldsymbol{A}} - \hat{\boldsymbol{b}}\boldsymbol{k}^T). \tag{33}$$

Design a discrete-time sliding mode control law

$$u_{S,k} = \sigma_k + T\tilde{u}_{S,k} \tag{34}$$

with $\tilde{u}_{S,k}$ given in (22) and choose

$$\lambda_1 = 1.5\sqrt{\Lambda} \qquad \lambda_2 = 1.1\Lambda \qquad \Lambda \geq \frac{L}{T} \tag{35}$$

with the change rate L as in (4). Then the origin of (1) is robustly stabilized.

Proof. Consider the forward increment of (31) and using (9), (10) and (32) results in

$$\begin{aligned} \sigma_{k+1} &= \boldsymbol{m}^T \boldsymbol{\xi}_{k+1} + w_{k+1} \\ &= \boldsymbol{m}^T \hat{\boldsymbol{A}} \boldsymbol{\xi}_k + \boldsymbol{m}^T \hat{\boldsymbol{b}} u_k + \boldsymbol{m}^T \hat{\boldsymbol{b}}_f f_k + w_{k+1} \\ &= \boldsymbol{m}^T (\hat{\boldsymbol{A}} \boldsymbol{\xi}_k + \hat{\boldsymbol{b}} u_{N,k}) + u_{S,k} + f_k + w_{k+1} \end{aligned}$$

Applying (33) and (13) results in

$$\sigma_{k+1} = u_{S,k} + f_k \tag{36}$$

which is equivalent to (21a) using (34) and $f_k = T\varphi_k$. The parameters λ_1 and λ_2 should be set for the matching algorithm using the very well-established parameter setting

$$\lambda_1 = 1.5\sqrt{L_\varphi} \qquad \lambda_2 = 1.1 L_\varphi \tag{37}$$

with

$$\sup \left| \frac{\varphi_{k+1} - \varphi_k}{T} \right| = L_\varphi \tag{38}$$

which was initially proposed in Levant (1998) and the stability was recently proven in Seeber and Horn (2017). The change rate (38) can easily be computed from (4) using

$$\sup \left| \frac{\varphi_{k+1} - \varphi_k}{T} \right| = \sup \left| \frac{f_{k+1} - f_k}{T^2} \right| = L_\varphi = \frac{L}{T}. \tag{39}$$

Therefore, the gains can be designed using (35). ■

Remark 4. The sliding variable defined in (31) is the discrete time equivalent to the well-known continuous time sliding variable

$$\sigma(t) = \boldsymbol{m}^T \boldsymbol{\xi}(t) + \int_0^t \boldsymbol{l}^T \boldsymbol{\xi}(\tau) d\tau. \tag{40}$$

4. LABORATORY EXPERIMENT

In this section the effectiveness of the previously proposed algorithm will be verified by means of a laboratory experiment. Figure 2 shows a picture as well as the mechanical scheme of the used spring mass laboratory experiment. The setup consists of a mass m attached to a spring with linear spring characteristic via a pulley using a nylon cord. Friction in the pulley is assumed to be viscous (proportional to the velocity). The other side of the spring is connected to a wheel, which can be actuated using a speed controlled electrical motor. The motor as well as the pulley are equipped with encoders to measure the position $z(t)$ and the mass position $y(t)$. Using the state vector

$$\boldsymbol{x}^T = \begin{bmatrix} y & \frac{\mathrm{d}y}{\mathrm{d}t} & z \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

results in the mathematical model

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{c}{m} & -\frac{k}{m} & \frac{c}{m} \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} (u + f) \tag{41}$$

with mass $m = 0.18\,\mathrm{kg}$, spring constant $c = 3.840\,\mathrm{N\,m^{-1}}$, friction coefficient $v = 0.042\,\mathrm{kg\,s^{-1}}$ and input gain $b = 0.086\,\mathrm{m\,s^{-1}\,V^{-1}}$. In order to verify the performance of the perturbed NCS, a perturbation

$$f_k = \frac{1}{3}\left[\sin(kT+5) + \sin\left(\frac{1}{\pi}(kT+5)\right) + 1\right] \tag{42}$$

consisting of two sinusoidals and a constant is applied. The exact change rate can be derived as

$$L = 0.440\,\mathrm{V\,s^{-1}}. \tag{43}$$

The states $x_{1,k} = x_1(kT)$ and $x_{3,k} = x_3(kT)$ are sampled with the constant sampling time

$$T = 20\,\mathrm{ms}, \tag{44}$$

the state $x_{2,k} = x_2(kT)$ is obtained using a differentiating filter. Imperfections of sensors and differentiating filter are neglected in the further investigations. The networked induced delay is implemented using two Simulink delay blocks, one to delay the sensor measurements and the other one to delay the control signal. These blocks are configured such that the worst case round trip time

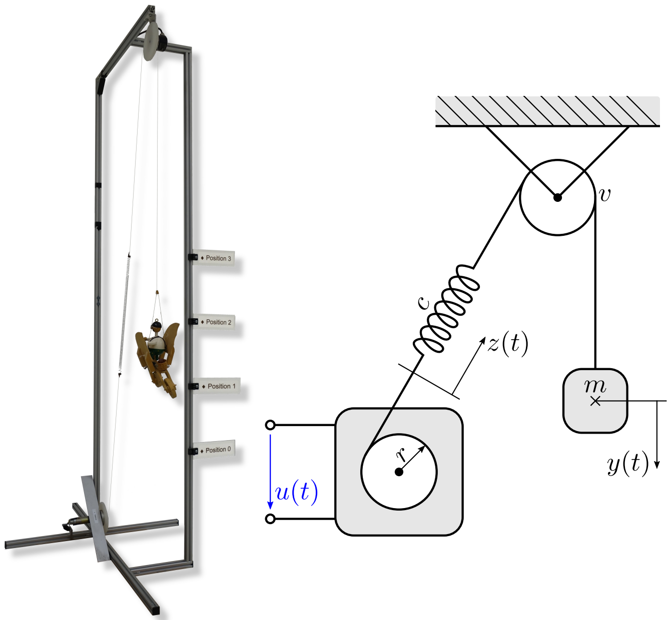

Fig. 2. Spring mass laboratory experiment (left) and mechanical scheme (right)

$$\tau_k \le 200\,\mathrm{ms} = 10T \tag{45}$$

is achieved, which results in $d = 10$. The nominal control law (13) was designed by placing the $n + d = 13$ poles to $z_i = e^{-10T}$, $i = 1, 2, \ldots, 13$ using Ackermann's formula. Vector $\boldsymbol{m}^T$ is designed according to (32). One possible choice is

$$\boldsymbol{m}_1^T = \begin{bmatrix} 1.143 & -9.157 & 584.020 \end{bmatrix}. \tag{46}$$

Theoretically the choice of $\boldsymbol{m}^T$ is only restricted by (32) but due to measurement noise, unmodeled dynamics in real world applications it is necessary to tune those parameters. To verify the quality of the current setting, choose $u_{S,k} = 0$, $\forall k$ and evaluate the sliding variable σ_k. According to (36), the sliding variable should then equal the one step delayed perturbation.

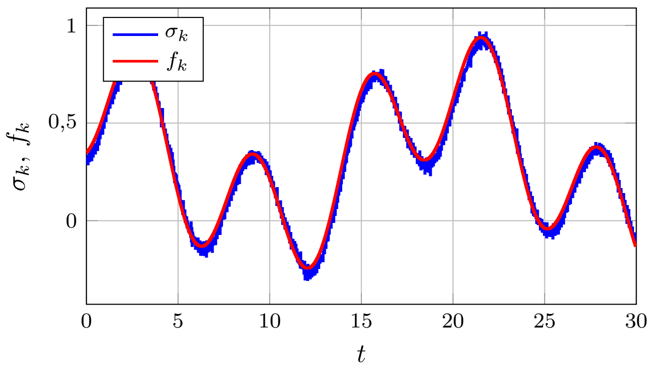

Fig. 3. Sliding variable σ_k for $u_{S,k} = 0$ and perturbation f_k.

The result after this tuning process is depicted in Fig. 3. This figure shows that using the vector $\boldsymbol{m}^T$ given in (46) is a reasonable choice, because the sliding variable equals the perturbation very well. To design the gains λ_1 and λ_2 of the super twisting algorithm according to (35), the change rate L_φ of the perturbation $\varphi_k = \frac{f_k}{T}$ has to be known or estimated. In real world applications usually this change rate is not known exactly and therefore has to be estimated. As too low values could lead to instability, typically the value is estimated very high. In this paper it is assumed that the estimated change rate $\Lambda = 132$ was about six times higher than the actual value which leads to

$$\lambda_1 = 17.274 \qquad \lambda_2 = 145.877 \tag{47}$$
$$p_1 = p_2^* = -8.637 - 8.443i$$

where p_2^* denotes the conjugate complex of p_2.

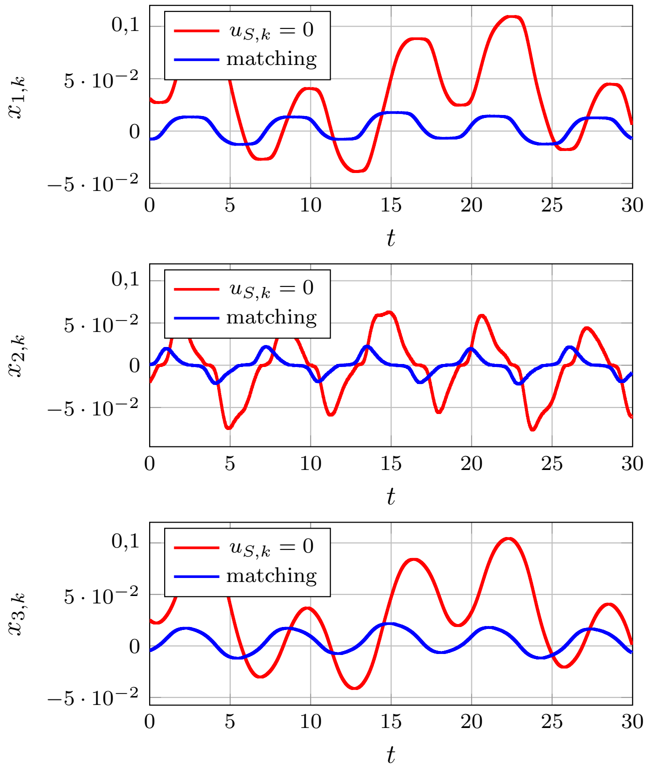

Fig. 4. States for $u_{S,k} = 0$ in red and using the matching algorithm in blue.

The evolution of the states during the experiment using the proposed method is shown with blue lines in Fig. 4. In order to show the effectiveness of the sliding mode based part of the control law, the results obtained using only the nominal control law is shown in red. Comparing these two lines show a significant increase of accuracy. This increased accuracy is also visible in the sliding variable, because for $\sigma_k = 0\ \forall k$ the perturbation would be compensated exactly, which is not possible for discrete time systems. Therefore, the magnitude of the sliding variable is a measure for the accuracy of the closed loop system.

Figure 5 shows in blue the value of the sliding variable σ_k corresponding to the results depicted in Fig. 4 using the proposed approach. Comparing figures 3 and 5 exemplifies that the magnitude of the sliding variable using the matching sliding mode control law is about ten times smaller than when using only the nominal control law. Additionally, Fig. 5 shows in red the evolution of the sliding variable using the explicit Euler discretized version

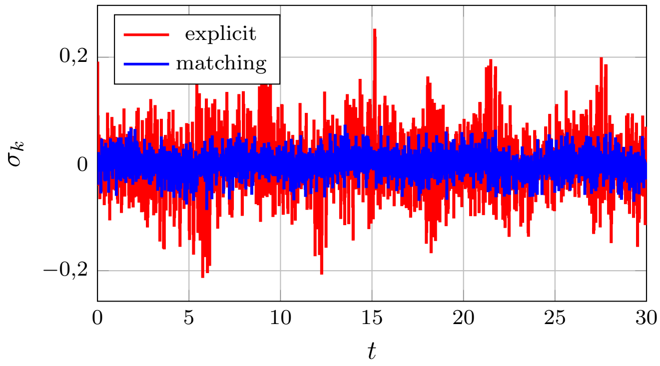

Fig. 5. Comparison of sliding variable for matching algorithm in blue and explicit Euler discretized algorithm in red

of the super twisting algorithm. One can clearly see that the magnitude of σ_k is about three times smaller using the matching algorithm. This is due to rapidly increasing amplitude of discretization chattering for increasing values of the estimated change rate Λ.

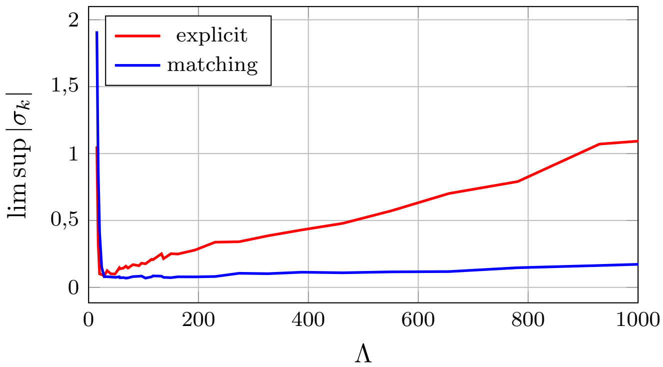

Fig. 6. Magnitude of sliding variable with increasing values of the estimated change rate Λ.

This phenomenon is illustrated in Fig. 6 where the magnitude of the sliding variable σ_k is shown for both algorithms and increasing values of the estimated change rate Λ. One can clearly see, that the curve representing the explicit Euler discretized scheme ascents much faster than the one representing the matching algorithm. The influence of the parameter Λ on the magnitude of the sliding variable using the matching algorithm is very low, which is a huge advantage as the exact value might be unknown.

5. CONCLUSIONS AND OUTLOOK

An integral sliding mode based control strategy for NCS with time varying delays have been proposed in this paper. Using the integral sliding mode strategy, offered the possibility to apply recently developed discrete-time sliding mode control approaches designed for relative degree one systems. Laboratory experiments verified the effectiveness of this approach using the matching discretization technique for the super twisting algorithm. The results of these experiments are also compared to the ones obtained using the explicit Euler discretization technique for the super twisting algorithm. These experiments showed clearly the advantages of the matching approach because the accuracy is much less sensitive to overestimating the change rate of the perturbation, whose exact value is usually not known in practical applications. In future research it would be very interesting to extend the proposed approach in order to explicitly consider additional and more complex network effects.

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Place acknowledgments here.

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