An Anti-Windup Scheme for the Super-Twisting Algorithm

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***Abstract*— In this paper, a system, which is subject to perturbations and unknowns, with a saturating actuator is considered. In order to design a robust feedback control law based on the sliding mode approach, the standard super-twisting algorithm is modified adopting an anti-windup technique. The control signal introduced into the system is continuous everywhere and the performance of the conventional super-twisting controller is significantly improved in the case that the initial condition of the system is far away from the origin. Global finite-time stability properties of the closed-loop are investigated, which gives a parameter setting for the controller. Having employed numerical simulations, feasibility and effectiveness of the scheme are indicated.**

I. Introduction

The Super-Twisting Algorithm (STA) as a well-known second-order sliding mode control method has been successfully applied to systems affected by disturbances, which are Lipschitz continuous. For systems of relative degree more than one, a sliding function needs to be defined such that the relative degree of the system with respect to this function is one. Having used this algorithm, the time derivative of the sliding variable is not incorporated into the control law design and an absolutely continuous control signal is provided [1], [2]. Moreover, if the actuator dynamics are fast enough, the asymptotic accuracy of the sliding variable under discrete-time measurements is improved and the chattering effect is reduced comparing with the first-order sliding mode control [3].

However, in the case that the control input is introduced to the system through a saturating actuator, the signal produced by the conventional super-twisting controller may exceed the saturation bounds. This causes the windup effect since the discontinuous integral action exists therein. In [4], the largest domain of attraction for such a system under the aforementioned control is computed and the finite-time stability within this domain is guaranteed. It is shown that in the case the initial condition of the closed-loop system belongs to this domain, the control signal remains within the bounds and the windup does not occur. It becomes evident when the initial values are outside this domain, the closed-loop satisfactory performance may be degenerated.

In order to attenuate the windup effect, switching between two control strategies based on the saturation limits is included in the sliding mode control laws designed in [5], [6]. This may result in high frequency switching on the limits and therefore undesirable zigzag motion in the control input as well as the system trajectory. In [7], [8], a saturated super-twisting technique with at most one switch between two different sliding mode control algorithms is proposed. Although, this tackles the aforementioned problem, a neighborhood of the origin needs to be defined to make the switch based on that and a disturbance estimator needs to be employed in order to deal with perturbations with the bounds close to the saturation limits. The latter makes the sliding mode control scheme redundant due to the fact that both the controller and estimator reconstruct perturbations. A different version of saturated super-twisting control law is designed in [9], which is compact in the sense that switching from one algorithm to another one and the estimator are not used. However, this modification makes a fairly restrictive assumption on the bounds and class of disturbances.

The significant contribution of this paper is to introduce a comprehensive second-order sliding mode control strategy adopting an anti-windup scheme, in which the properties of the standard STA are retained, the disturbance estimator is not incorporated, and no additional constraint on the bounds and class of perturbations are imposed. The rest of the paper is organized as follows: the problem and the objective are explained in Section II. The proposed control law design is described in Section III. The stability analyses of the closed-loop system are carried out in Section IV. Simulation results are illustrated in Section V followed by a conclusion given in Section VI.

II. Problem Statement

Consider ~~that~~ a system is described by

$$\frac{dz}{dt} = b(t)\mathrm{sat}_\rho(u) + a(t)\,, \tag{1}$$

where the output of the system is denoted by $z \in \mathbb{R}$ and u is the scalar control input. The actuator is saturated if $|u| \leq \rho$ does not hold, where ρ is a known constant. This is realized from the definition

$$\mathrm{sat}_\eta(y) = \begin{cases} y & \text{for } |y| \leq \eta\,, \\ \eta \lceil y \rfloor^0 & \text{for } |y| > \eta\,, \end{cases} \tag{2}$$

where the notation

$$\lceil y \rfloor^0 = \mathrm{sgn}(y) \quad \text{as a particular case of} \quad \lceil y \rfloor^\gamma = |y|^\gamma \mathrm{sgn}(y)$$

is used. The possibly time-varying function $a(t)$ represents the effect of perturbations and $b(t)$ denotes a multiplicative unknown corresponding to the input u, which may also be a function of time t.

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Assumption 1. The functions $a(t)$ and $b(t)$ are globally bounded and Lipschitz continuous, i.e.

$$\begin{aligned} |a(t)| &\leq a_M \,, & |\frac{da}{dt}| &\leq L_a \,, \\ 0 < b_m \leq b(t) &\leq 1 \,, & |\frac{db}{dt}| &\leq L_b \,, \end{aligned} \tag{3}$$

where a_M, L_a, b_m, and L_b are some known constants and without loss of generality, the upper bound of $b(t)$ is one. Furthermore, $\rho > \frac{a_M}{b_m}$ is satisfied.

The objective is to design a feedback control law for system (1) such that

- the system state z converges to the origin in a finite time despite the presence of disturbances and unknowns;
- the control signal u is continuous everywhere and in the case that its absolute value is not confined to the saturation limit ρ, the windup effect is mitigated.

In order to achieve that, a novel control approach based on the STA is introduced in the next section.

III. Proposed Anti-Windup Strategy

The conventional super-twisting controller is designed as

$$u = -k_1 \lceil z \rfloor^{\frac{1}{2}} + \nu \,, \tag{4a}$$

$$\frac{d\nu}{dt} = -k_2 \lceil z \rfloor^{0} \,, \tag{4b}$$

where k_1 and k_2 are the positive constants to be tuned. An anti-windup technique is incorporated into the aforementioned control law as

$$u = -k_1 \lceil z \rfloor^{\frac{1}{2}} + \nu \,, \tag{5a}$$

$$\frac{d\nu}{dt} = -k_2 \lceil z \rfloor^{0} - k_3 \beta \nu \,, \qquad |\nu_0| \leq \frac{k_2}{k_3}, \tag{5b}$$

where the initial value of the state variable $\nu(t=0) = \nu_0$, similarly to the saturated STA considered in [9], is chosen appropriately. The value of the binary variable β is assigned as

$$\beta = \begin{cases} 1 & \text{if} \quad |u| > \rho \,, \\ 0 & \text{if} \quad |u| \leq \rho \,. \end{cases}$$

It is shown later that at most, one switch in β from 1 to 0 occurs if the positive gains k_1, k_2, and k_3 are selected appropriately. This implies that in the case the produced control signal satisfies $|u| \leq \rho$, the actuator saturation does not happen afterwards. Sufficient conditions for choosing the control parameters are given in the next section. Please note that, in contrast to [7], [8], the introduced actuating signal is continuous everywhere due to the fact that the aforementioned switch lies in the same channel as the discontinuous element of the algorithm is. Furthermore, in contrast with [5], the term $k_2 \lceil z \rfloor^{0}$ exists in (5) for any value of u, which prevents high frequency switching on the saturation bounds.

It is worth mentioning that, comparing to [9], the control signal generated here does not remain within the bounds $\pm\rho$ if the initial value of z is very large. However, the proposed control scheme enjoys the advantages that the windup effect is also alleviated and the standard STA is recovered close to the origin. The latter contributes significantly to an enlargement of the class of addressed disturbances in the sense that the Lipschitz constant L_a does not need to be a portion of the limit ρ. It is noted that, in comparison with [7], [8], the implementation of control law (5) is relatively simpler since β switches if the ~~control input absolute value~~ is out of the bound ρ and there is no need to define a neighborhood of the origin. Moreover, without using a disturbance estimator, perturbations with the maximum possible bound a_M that is close to $b_m \rho$ can be handled here.

IV. Stability Analysis

Global finite-time stability properties of the closed-loop are established in this section.

Proposition 1. *Suppose that the inequalities in* (3) *are satisfied. For system* (1) *under control law* (5)*, the origin* $z = 0$ *is globally finite-time stable if the control parameters are selected such that*

$$k_1 > 2\sqrt{\frac{k_2 \rho}{b_m \rho - a_M}} \,, \qquad k_2 > \frac{L_a + L_b a_M}{b_m^2} \,, \qquad \frac{k_2}{k_3} \leq \rho \tag{6}$$

~~*hold.*~~

Proof. In the case that $|u| > \rho$, the closed-loop dynamics reads as

$$\frac{dz}{dt} = \rho b(t) \lceil u \rfloor^{0} + a(t) \,, \tag{7a}$$

$$\frac{d\nu}{dt} = -k_2 \lceil z \rfloor^{0} - k_3 \nu \,. \tag{7b}$$

As it is proved in [9, Lemma 1], $|\nu|$ is bounded here by a calculable constant since (7b) is a linear differential equation with the state variable ν and the bounded input $\lceil z \rfloor^{0}$. Having chosen ν_0 and the control gains as given in (5b) and (6) respectively, $|\nu| \leq \rho$ holds as long as $\beta = 1$. Thus, in this phase ~~that~~ the actuator is saturated, either $-k_1 \lceil z \rfloor^{\frac{1}{2}}$ and ν are in the same sign or they have different signs with $k_1 |z|^{\frac{1}{2}} > |\nu| + \rho$. Therefore,

$$\lceil u \rfloor^{0} = \left\lceil -k_1 \lceil z \rfloor^{\frac{1}{2}} \right\rfloor^{0} = -\lceil z \rfloor^{0}$$

is fulfilled ~~here~~, which implies that $|z|$ is decreasing while ν is bounded. After the finite time T, $|z|^{\frac{1}{2}} \leq \frac{2\rho}{k_1}$ is satisfied, which leads to $|u| = \rho$ for the first time and a switch to $\beta = 0$. In the following, it is shown that the control signal remains within the limits afterwards, i.e. $\big|u(t)\big| \leq \rho \,, \ \forall t > T$.

On the occasion $|u| = \rho$, taking the time derivative of $|u|$ along the trajectory of system (1) yields

$$\begin{aligned} \frac{d|u|}{dt} &= \frac{du}{dt} \lceil u \rfloor^{0} = \left(-\frac{k_1}{2} |z|^{-\frac{1}{2}} \frac{dz}{dt} + \frac{d\nu}{dt} \right) \lceil u \rfloor^{0} \\ &= -\frac{k_1}{2} |z|^{-\frac{1}{2}} \left(b(t)|u| + a(t) \lceil u \rfloor^{0} \right) - k_2 \lceil z \rfloor^{0} \lceil u \rfloor^{0} \\ &\leq -\frac{k_1^2 \left(b_m \rho - a_M \right)}{4\rho} + k_2 \,. \end{aligned} \tag{8}$$

If the sufficient conditions imposed in (6) are met, then $\frac{d|u|}{dt} < 0$ holds, which implies that $|u|$ cannot increase. In order to guarantee the globally boundedness of $|u|$ by ρ in the case that the control signal is within the limits at the very beginning, it needs to be indicated that $|z|^{\frac{1}{2}}$ does not go beyond $\frac{2\rho}{k_1}$ afterwards. For $|u| \le \rho$, the closed-loop dynamics of the standard super-twisting is obtained as

$$\frac{dz}{d\tau} = -k_1 \lceil z \rfloor^{\frac{1}{2}} + \omega\,, \tag{9a}$$

$$\frac{d\omega}{d\tau} = \frac{1}{b(t)}\left(-k_2 \lceil z \rfloor^{0} + \delta\right), \tag{9b}$$

where time is scaled through $d\tau = b(t)dt$, the auxiliary variable ω is defined as $\omega = \nu + \frac{a(t)}{b(t)}$, and δ is the time derivative of $\frac{a(t)}{b(t)}$. By setting $\frac{dz}{d\tau} = 0$, the maximum value of $|z|^{\frac{1}{2}}$ for this case can be determined. It becomes evident that ν and z are in the same sign when $|z|^{\frac{1}{2}}$ has the largest value. Since $\frac{d\nu}{dt}$ has the opposite sign in that case, having selected ν_0 such that $|\nu_0| \le \rho$ is fulfilled, $|\nu|$ cannot be greater than ρ. As it is mentioned in Assumption 1, $|\frac{a(t)}{b(t)}|$ is bounded by ρ. Hence, $\frac{2\rho}{k_1}$ is the maximum value of $|z|^{\frac{1}{2}}$ in the phase that the actuator is not saturated.

In system (9), the asymptotic stability of the state vector $\boldsymbol{\zeta} := \begin{bmatrix} z & \omega \end{bmatrix}^{\mathrm{T}}$ can be ensured by using the Lyapunov function candidate considered in [10] as

$$V(\boldsymbol{\zeta}) = \begin{cases} 2\sqrt{\omega^2 + 3\lambda^2 k_1^2 z} - \omega & \text{for} \quad \boldsymbol{\zeta} \in \mathcal{M}\,, \\ 2\sqrt{\omega^2 - 3\lambda^2 k_1^2 z} + \omega & \text{for} \quad -\boldsymbol{\zeta} \in \mathcal{M}\,, \\ 3\,|\omega| & \text{otherwise.} \end{cases} \tag{10}$$

The positive constant $\lambda < 1$ is chosen such that

$$k_1 > \frac{1}{\lambda}\sqrt{\frac{2k_2}{b_m}} \tag{11}$$

is met, but the set $\mathcal{M}$ is left the same as that one defined in [10] as

$$\mathcal{M} = \left\{\boldsymbol{\zeta} \mid z \ge 0\,,\ \omega \le \lambda k_1 \sqrt{z}\right\}. \tag{12}$$

The time derivative of V in the scenario of $\boldsymbol{\zeta} \in \mathcal{M}$ along the trajectory of system (9) reads as

$$\frac{dV}{d\tau} = \frac{3\lambda^2 k_1^2\left(-k_1\sqrt{z} + \omega\right) - 2\omega\left(\frac{k_2 - \delta}{b(t)}\right)}{\sqrt{\omega^2 + 3\lambda^2 k_1^2 z}} + \frac{k_2 - \delta}{b(t)}. \tag{13}$$

It is noted that, despite the presence of $b(t)$, $\frac{dV}{d\tau}$ is also a homogeneous function (see e.g. [11]) of degree zero with respect to $\sqrt{z}$ and ω. Therefore, as it is done in [10], the performance of function (13) for z and ω such that $\omega^2 + 3\lambda^2 k_1^2 z = 1$ holds is assessed in the following. Having defined the function $g(\omega)$ as

$$\begin{aligned} g(\omega) &= \frac{dV}{d\tau}\Big|_{\omega^2 + 3\lambda^2 k_1^2 z = 1} \\ &= -\lambda k_1^2 \sqrt{3 - 3\omega^2} + 3\lambda^2 k_1^2 \omega + \left(\frac{k_2 - \delta}{b(t)}\right)(1 - 2\omega)\,, \end{aligned} \tag{14}$$

the second derivative of g with respect to ω in the interval $\left[-1, \frac{1}{2}\right]$ is

$$\frac{d^2 g}{d\omega^2} = \frac{\sqrt{3}\lambda k_1^2}{(1 - \omega^2)^{\frac{3}{2}}} \ge 0 \qquad \forall \omega \in \left[-1, \frac{1}{2}\right] \setminus \{-1\}\,. \tag{15}$$

The local maximums of g that are on the border of the aforementioned interval are computed as

$$g(-1) = 3\left(\frac{k_2 - \delta}{b(t)} - \lambda^2 k_1^2\right), \tag{16a}$$

$$g(\frac{1}{2}) = \frac{3\lambda\,(\lambda - 1)\,k_1^2}{2}. \tag{16b}$$

It is noted that if the control constants k_1 and k_2 are selected as given in (6), the condition set on k_1 in (11) as well as $k_2 > |\delta|$ is fulfilled. Thus, $g(-1)$ is less than $3\left(\frac{2k_2}{b_m} - \lambda^2 k_1^2\right)$. From (11) and $0 < \lambda < 1$, it is derived that both $g(-1)$ and $g(\frac{1}{2})$ are negative. As a result of this negativeness, through homogeneity, it can be concluded that $\frac{dV}{d\tau} < 0$ holds in the entire set $\mathcal{M}$. The second scenario in (10), i.e. $-\boldsymbol{\zeta} \in \mathcal{M}$, can be investigated symmetrically. In the third scenario, differentiating V with respect to τ and applying (9b) to that gives

$$\frac{dV}{d\tau} = \frac{3}{b(t)}\left(-k_2 \lceil z\omega \rfloor^{0} + \lceil \omega \rfloor^{0} \delta\right). \tag{17}$$

As it is mentioned above, $|\delta|$ is less than the chosen parameter k_2, which also results in the negative definiteness of $\frac{dV}{d\tau}$ in this scenario. Owing to that the time derivative of V along the trajectory of system (9) is upper bounded by a negative constant almost everywhere, the finite-time convergence of $\boldsymbol{\zeta}$ is realized. This completes the global finite-time stability proof of the origin $z = 0$. □

Remark 1. According to (6), it is achieved that

$$a_M < b_m \rho - \frac{4k_2\rho}{k_1^2}\,. \tag{18}$$

It can be seen that having chosen $k_2 > |\delta|$, by assigning a larger value to k_1, perturbations with a larger bound a_M close to $b_m\rho$ can be addressed. Please note that k_3 does not impose any constraint on the permissible bound and class of disturbances.

V. Simulation Examples

In this section, it is demonstrated in simulation how the proposed control scheme comparing to the control strategies recorded in the literature is able to deal with three different problem settings. Since a multiplicative unknown is not considered in [7], [8], [9], in the first two cases, it is assumed that $b(t) = 1$. However, the bound of perturbations is enlarged from $a_M < \frac{\rho}{2}$ in the first case to a_M that is close to ρ in the second case. In the third case, the system is subject to both unknowns and disturbances , i.e. b is a time-varying function. In all the cases, it is supposed that the actuating signal is saturated with $\rho = 5$. It is aimed to drive the system output z to zero in a finite time applying the continuous control input u.

In the first and second cases, the results obtained through the aforementioned versions of saturated STA are compared with the achieved closed-loop performance of the proposed technique in this paper. The saturated version presented in [7], [8] is implemented as

$$\begin{bmatrix} u \\ \frac{d\nu}{dt} \end{bmatrix} = \begin{cases} \begin{bmatrix} -\rho \lceil z \rfloor^0 \\ 0 \end{bmatrix}, & \nu_0 = 0 \quad \text{if } s = 0\,, \\ \begin{bmatrix} -k_1 \lceil z \rfloor^{\frac{1}{2}} + \nu \\ -k_2 \lceil z \rfloor^0 \end{bmatrix}, & \nu(t = t_1) = \bar{\nu} \quad \text{if } s = 1\,, \end{cases} \tag{19}$$

where $\bar{\nu}$ is set to zero and negative sign of disturbances estimation in [7] and [8] respectively. The value of the binary variable s is determined by a dynamic switching law. The parameters k_1 and k_2 are chosen therein such that

$$k_1 > 0\,, \qquad k_2 > 3L_a + \left(\frac{2L_a}{k_1}\right)^2 \tag{20}$$

are satisfied. The saturated STA considered in [9] is implemented as

$$u = -k_1 \text{sat}_\epsilon \left(\lceil z \rfloor^{\frac{1}{2}}\right) + \nu\,, \tag{21a}$$

$$\frac{d\nu}{dt} = -k_2 \lceil z \rfloor^0 - k_3 \nu\,, \qquad |\nu_0| \leq \frac{k_2}{k_3}, \tag{21b}$$

where the sat_ϵ function is defined as given in (2). It is guaranteed that the control signal u remains within the bounds $\pm\rho$ if the positive gains k_1, ϵ, k_2, and k_3 are selected such that

$$k_1 \epsilon + \frac{k_2}{k_3} \leq \rho \tag{22}$$

is fulfilled. Having carried out the stability analysis of the closed-loop system in the presence of perturbations, fairly restrictive conditions on the parameters are imposed therein. However, in the simulation example presented in [9], it is shown that controller (21) may be tuned satisfying (22) and the necessary condition

$$k_2 > k_3 a_M + L_a\,. \tag{23}$$

Furthermore, in order to make a comparison, the system under the conventional super-twisting control law given in (4) is simulated in all the cases. The sufficient conditions set in [4] as

$$k_1 > 1.8\sqrt{\frac{k_2 + \Gamma}{b_m}}\,, \qquad k_2 > \Gamma = \frac{L_a + L_b a_M}{b_m^2} \tag{24}$$

are met here for selecting the control constants.

A. First Case

In this simulation case, perturbations are represented as

$$a(t) = 1 + 0.6\sin(t) + 0.8\sin(5t)\,. \tag{25}$$

As mentioned above, $b(t) = 1$ ~~is known~~ in this case. Therefore, in the conditions of the proposed scheme as well as the conventional STA given in (6), (24), k_2 just needs to be greater than $L_a = 4.6$. The selected control constants are listed in Table I. Since disturbances are bounded with $a_M = 2.4 < \frac{\rho}{2}$, the saturated STA considered in [7], in which the disturbance estimator is not incorporated, is employed in this case and its gains are set fulfilling (20). For the saturated version introduced in [9], having met (22) and (23), the parameters values are provided in Table I. For all the algorithms, the numerical simulation is carried out through MATLAB/Simulink with a sampling step size of 1 ms and the initial values $z(t = 0) = z_0 = 30$ and $\nu_0 = 0$. Their performance is shown in Fig. 1. It is revealed in the upper plot that the same rate of convergence is achieved by applying the approaches proposed in this paper and in [7].

TABLE I
PARAMETERS OF ~~DIFFERENT~~ CONTROLLERS

Controller	Parameter	Simulation Case		
		1st	2nd	3rd
Proposed Scheme	k_1	6.1	14	21.8
	k_2	4.7	4.7	14.2
	k_3	1	1	3
Conventional STA	k_1	5.5	5.5	12.4
	k_2	4.7	4.7	14.2
Saturated STA in [7], [8]	k_1	4	4	—
	k_2	16.5	16.5	—
Saturated STA in [9]	k_1	4	4	—
	k_2	10.7	10.7	—
	k_3	2.5	2.5	—
	ϵ	0.18	0.18	—

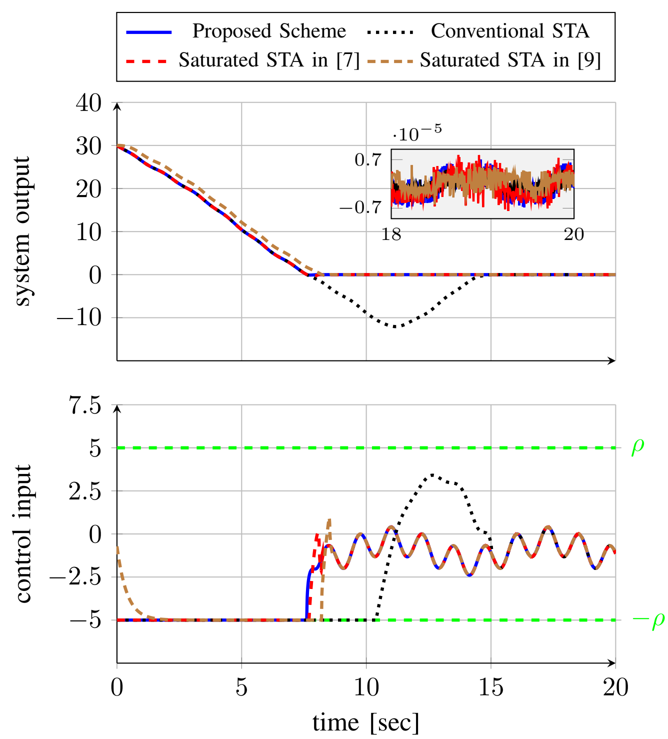

Fig. 1. Simulation response curves of the first case obtained through the system under four different control laws (5), (4), (19) with $\bar{\nu} = 0$, and (21), which are labeled respectively with the Proposed Scheme, Conventional STA, Saturated STA in [7], and Saturated STA in [9].

The output of the system under the control law designed in [9] ~~drives~~ also similarly to the origin. Please note that the implementation of the strategy introduced in [7] is not as simple as the implementation of others. As it is illustrated in the zoomed ~~portion of the~~ plot, similar precision is obtained through all the algorithms. The evolution of the control signals introduced to the system through the saturating actuator is depicted in the lower plot. It is noted that in contrast to the bounded signals produced through control laws (19) and (21), the signals generated by controllers (5) and (4) are not bounded by the saturation limits. Although, this results in a large overshoot and long settling time in the performance of the conventional STA, the proposed scheme contributes significantly to the counteraction of the windup effect.

B. Second Case

In this scenario, $b(t)$ ~~$=1$~~ remains known, but the bound of disturbances given in (25) is increased as

$$a(t) = 3.1 + 0.6\sin(t) + 0.8\sin(5t)\,. \tag{26}$$

Since L_a remains the same as last case, some of the control gains ~~values are left the same~~, see Table I. However, $a_M = 4.5$ is close to the saturation bound ρ. Hence, having well-tuned the perturbation estimator constants such that the estimation error converges in a finite time faster than the convergence of the system output, information of the estimator is exploited in implementation of control law (19). As it is indicated in Fig. 2, this saturated version of STA with the estimator as well as the proposed technique produces a similar satisfactory performance. It is worth mentioning that the simulation is initialized in this case with $z_0 = -30$ and $\nu_0 = 0$, and disturbances estimation is not incorporated into the design of control law (5). In the zoomed ~~portion of the~~ upper plot, it is demonstrated that the output of the system under control law (21) does not converge with the same accuracy as those obtained through the other algorithms. This is due to the fact that in this approach, L_a needs to be a portion of the limit ρ and therefore there is no chance both inequalities (22) and (23) are satisfied in the case a_M is close to ρ. The values given in Table I for this controller is the second simulation case are assigned such that the absolute value of the control signal is confined to ρ.

C. Third Case

In this case, in addition to perturbations $a(t)$ given in (25), a multiplicative unknown is taken into consideration as

$$b(t) = 0.8 + 0.2\sin(t)\,. \tag{27}$$

It is noted that $b(t)$ is lower bounded by $b_m = 0.6$ and therefore disturbances need to be bounded by $a_M < 3$ (according to Assumption 1). As mentioned above, this problem setting is not dealt ~~with~~ in [7], [8], [9]. Thus, simulation response curves obtained through the system under control laws (5), (4) are compared in this scenario, see Fig. 3. It can be clearly seen that the windup effect is also mitigated in this performance of the proposed approach, in which the precision is satisfactory comparing the result of the conventional STA.

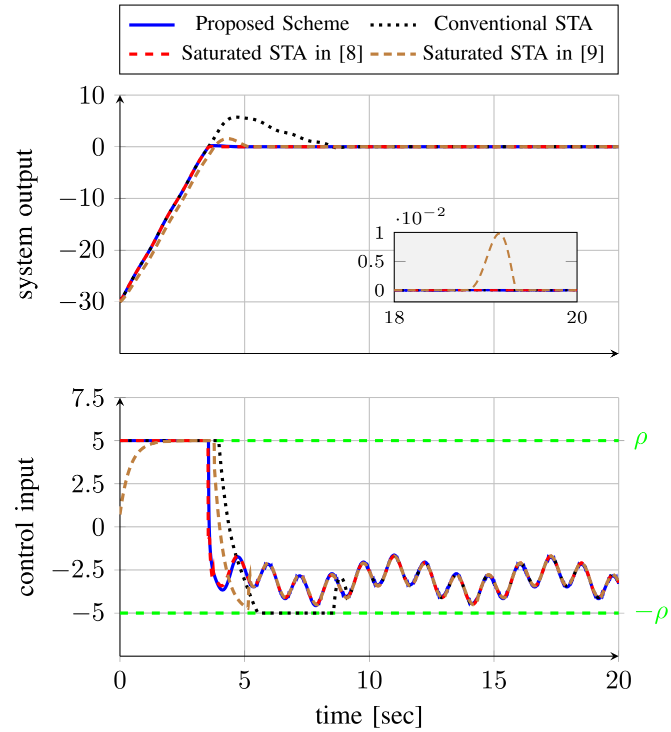

Fig. 2. Simulation results of the second case comparing the proposed approach with the conventional STA, the saturated STA applying the disturbance estimator as presented in [8], as well as the saturated version of STA introduced in [9].

VI. Conclusion

This article presents an anti-windup strategy for the super-twisting algorithm. It is shown that the generated control signal is continuous everywhere. Having applied this control technique to a first-order system with a saturating actuator, disturbances, and unknowns, the finite-time convergence of the closed-loop system states is guaranteed by means of ~~the~~ Lyapunov function. Since the standard super-twisting algorithm is recovered after a finite time, the class of addressed perturbations is not restricted by the saturation limit. Furthermore, without using ~~the~~ disturbance estimator, perturbations with the maximum permissible bound (by the actuator) are tackled. For different problem settings, the performance of the proposed scheme is assessed and compared in simulation with the results of the saturated and conventional versions of the super-twisting algorithm.

References

[1] Y. Shtessel, C. Edwards, L. Fridman, and A. Levant, *Sliding mode control and observation*. Springer, 2014.

[2] S. Li, X. Yu, L. Fridman, Z. Man, and X. Wang, *Advances in Variable Structure Systems and Sliding Mode Control—Theory and Applications*. Springer, 2017, vol. 115.

[3] U. Pérez-Ventura and L. Fridman, "When is it reasonable to implement the discontinuous sliding-mode controllers instead of the continuous ones? frequency domain criteria," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 3, pp. 810–828, 2019.

[4] A. K. Behera, A. Chalanga, and B. Bandyopadhyay, "A new geometric proof of super-twisting control with actuator saturation," *Automatica*, vol. 87, pp. 437 – 441, 2018.

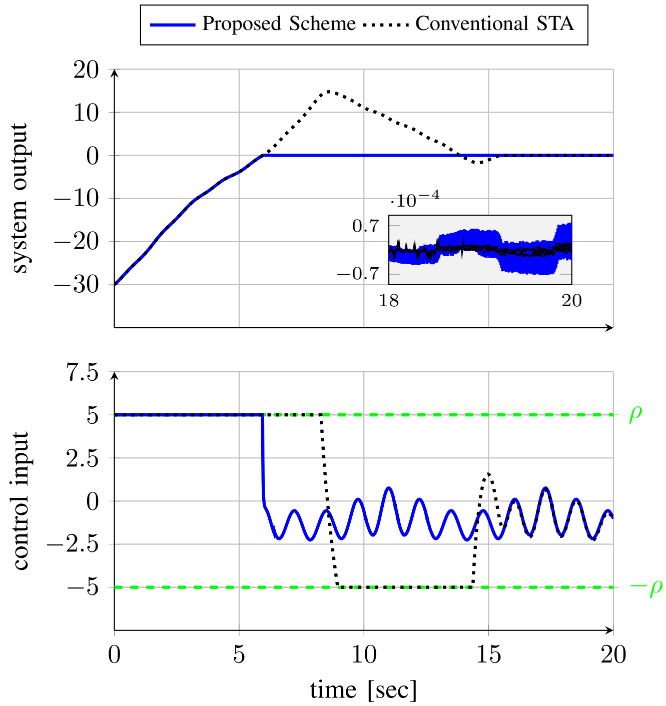

Fig. 3. The system output z and control input u simulated in the third case, where the system also affected by a time-varying multiplicative unknown. The results obtained through the proposed approach and the conventional STA are compared since this unknown is not addressed in [7], [8], [9].

[5] A. Levant, "Sliding order and sliding accuracy in sliding mode control," *International journal of control*, vol. 58, no. 6, pp. 1247–1263, 1993.

[6] A. Ferrara and M. Rubagotti, "A sub-optimal second order sliding mode controller for systems with saturating actuators," *IEEE Transactions on Automatic Control*, vol. 54, no. 5, pp. 1082–1087, May 2009.

[7] I. Castillo, M. Steinberger, L. Fridman, J. A. Moreno, and M. Horn, "Saturated super-twisting algorithm: Lyapunov based approach," in *14th International Workshop on Variable Structure Systems (VSS)*, June 2016, pp. 269–273.

[8] I. Castillo, M. Steinberger, L. Fridman, J. Moreno, and M. Horn, "Saturated super-twisting algorithm based on perturbation estimator," in *IEEE 55th Conference on Decision and Control (CDC)*, Dec 2016, pp. 7325–7328.

[9] M. A. Golkani, S. Koch, M. Reichhartinger, and M. Horn, "A novel saturated super-twisting algorithm," *Systems & Control Letters*, vol. 119, pp. 52 – 56, 2018.

[10] R. Seeber and M. Horn, "Stability proof for a well-established super-twisting parameter setting," *Automatica*, vol. 84, pp. 241 – 243, 2017.

[11] S. P. Bhat and D. S. Bernstein, "Geometric homogeneity with applications to finite-time stability," *Mathematics of Control, Signals and Systems*, vol. 17, no. 2, pp. 101–127, Jun 2005.