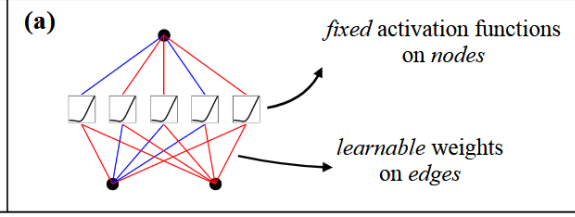
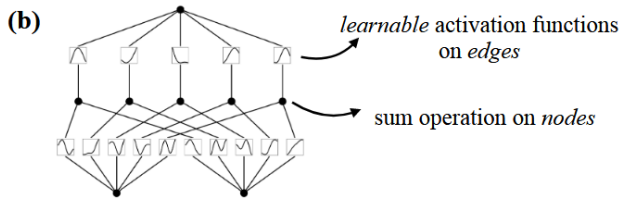
Alternative Representation Theorems for KANs

Model	Multi-Layer Perceptron (MLP)	Kolmogorov-Arnold Network (KAN)
Theorem	**Universal Approximation Theorem**	**Kolmogorov-Arnold Representation Theorem**
Formula (Shallow)	$f(\mathbf{x}) \approx \sum_{i=1}^{N(\epsilon)} a_i \sigma(\mathbf{w}_i \cdot \mathbf{x} + b_i)$	$f(\mathbf{x}) = \sum_{q=1}^{2n+1} \Phi_q \left(\sum_{p=1}^{n} \phi_{q,p}(x_p) \right)$
Model (Shallow)		

Deep learning is fundamentally about function approximation. According to universal approximation theorems, an MLP approximates arbitrary multivariate functions using weighted sums combined with a single nonlinear monovariate activation function. Very recently, Kolmogorov–Arnold Networks (KANs) [1] have emerged as a promising alternative, with potential improvements in generalization, accuracy, and interpretability. Instead of learning summation weights, KANs learn monovariate activation functions on each edge. This also guarantees (under certain assumptions) universal representation, according to the Kolmogorov-Arnold-Theorem. Equivalent formulations of this theorem exist [2], some however remain unexplored. The goal of this project is to compare alternative formulations, implement them as models, and evaluate their respective advantages while identifying connections between them and other Deep Learning models.

[1] Z. Liu et al., 'KAN: Kolmogorov-Arnold Networks', Feb. 09, 2025
[2] L. F. Guilhoto and P. Perdikaris, 'Deep Learning Alternatives of the Kolmogorov Superposition Theorem', Jan. 30, 2025

Goals & Tasks

- Implement alternative representation theorems [2] as models and review performance and scaling.
- Interpret models and find a connecting framework or connections.

Qualifications

- Experience with the Python based deep learning frameworks JAX or PyTorch.
- Interest in theory.
- Registered to one of the following:
  - ☐ **Bachelor Thesis**
  - ✓ **Seminar Project**
  - ✓ **Master Thesis**

Contact

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