The ocean tide model EOT11a in spherical harmonics representation

Technical Note

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1 The model EOT11a

Thirteen tidal constituents have been derived from altimetry data [1]. To complement the tidal spectrum, six long periodic waves are additionally included, which are either taken from FES2004 [2], or computed from the HW95 Tide Generating Potential catalogue [3] in a gravitational self-consistent way according to the sea-level equation. In this spherical harmonics representation, the S1 tide is excluded since it is mainly a radiational tide and its influence on ocean mass redistribution is not clarified yet. The eighteen provided tidal waves are summarized in the following table.

<table>
<thead>
<tr>
<th>Darwin notation</th>
<th>Doodson number</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long period waves</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega_1$</td>
<td>055.565</td>
<td>HW95</td>
</tr>
<tr>
<td>$\Omega_2$</td>
<td>055.575</td>
<td>HW95</td>
</tr>
<tr>
<td>$S_a$</td>
<td>056.554</td>
<td>HW95</td>
</tr>
<tr>
<td>$S_{sa}$</td>
<td>057.555</td>
<td>HW95</td>
</tr>
<tr>
<td>$M_m$</td>
<td>065.455</td>
<td>EOT11a</td>
</tr>
<tr>
<td>$M_f$</td>
<td>075.555</td>
<td>EOT11a</td>
</tr>
<tr>
<td>$M_{tm}$</td>
<td>086.455</td>
<td>FES2004</td>
</tr>
<tr>
<td>$M_{sqm}$</td>
<td>093.555</td>
<td>FES2004</td>
</tr>
<tr>
<td>Diurnal waves</td>
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<td></td>
</tr>
<tr>
<td>$Q_1$</td>
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<td>EOT11a</td>
</tr>
<tr>
<td>$O_1$</td>
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<td>EOT11a</td>
</tr>
<tr>
<td>$P_1$</td>
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</tr>
<tr>
<td>$K_1$</td>
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<td>EOT11a</td>
</tr>
<tr>
<td>Semi-diurnal waves</td>
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</tr>
<tr>
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</tr>
<tr>
<td>$S_2$</td>
<td>273.555</td>
<td>EOT11a</td>
</tr>
<tr>
<td>$K_2$</td>
<td>275.555</td>
<td>EOT11a</td>
</tr>
<tr>
<td>Quarter-diurnal waves</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_4$</td>
<td>455.555</td>
<td>EOT11a</td>
</tr>
</tbody>
</table>

2 Matlab tide prediction software

For the convenience of the users, Matlab routines are supplied to perform tidal analysis with EOT11a. The formalism for tidal analysis and prediction is summarized in Appendix A.

Routine main.m:

The computations are performed in the routine main.m, where following relevant settings at the beginning of the file can be changed:

<table>
<thead>
<tr>
<th>maxDegree</th>
<th>Maximum degree and order of the series expansion to be evaluated</th>
</tr>
</thead>
<tbody>
<tr>
<td>mjd</td>
<td>Time of evaluation given as Modified Julian Date</td>
</tr>
</tbody>
</table>
To assure a fast computation, for each tidal wave two sets of corresponding potential coefficients \( cnmCos, snmCos \) and \( cnmSin, snmSin \) (including the loading potential) up to maximum degree and order 120 are provided in the ICGEM format for Earth Gravity Field Models \([4]\) in the subfolder \( eot11a/ \). These files are read by the subfunction \( readPotentialCoefficients.m \). The stokes coefficients for the use in equation \((7)\) of Appendix A are obtained by

\[
\begin{align*}
\Delta c_{nm,s} &= cnmCos(\Theta_s) + cnmSin(\Theta_s) \\
\Delta s_{nm,s} &= snmCos(\Theta_s) + snmSin(\Theta_s)
\end{align*}
\]

The astronomical argument \( \Theta_s \) for each tidal wave is computed in subfunction \( doodsonArguments.m \). Note that the Doodson-Warburg phase corrections \( \chi_s \) (cf. Appendix A) are already applied to the provided coefficients and therefore do not have to be considered.

For the evaluation of the tidal potential according to equation \((7)\), the necessary associated Legendre functions are retrieved by the subfunction \( legendreFunctions.m \). In case of evaluating tidal heights as performed within these routines in subfunction \( plotWaterHeight.m \), the norming factor \( \frac{GM}{R^2} \) in equation \((7)\) changes to \( \frac{M}{4\pi R^2 \rho_w} \frac{2n+1}{1+k_n} \). The corresponding load Love numbers \([5]\) are read from the \( data/ \) subfolder.

Routine \( mainWithAdmittance.m \):

The previously described routine \( main.m \) computes the tidal potential or water heights only of the main tidal waves given in EOT11a. However, there exist significant influences of additional minor tide constituents that are not included in the tide model, which should not be neglected in satellite geodesy especially for Low Earth Orbiters. Here we follow the theory of admittance and assume that the admittance (i.e. the relation of the tidal height with respect to the amplitude of the corresponding Tide Generating Potential TGP for a specific tidal wave) is a smooth function of the frequency, which is only slowly varying within a frequency band (cf. \([6]\)).

Based on this assumption, minor tide constituents are linearly interpolated for long periodic (from Ssa, Mm, Mf, Mtm, Msqm), diurnal (from Q1, O1, K1) and semi-diurnal (from 2N2, N2, M2, K2) frequency bands separately according to \([7]\) or \([8]\). For the choice of the minor tides to be considered, we introduced an empirical threshold of \( 2 \times 10^{-4} \) m²/s² of TGP amplitude, which leads to 238 additionally included minor tides.

The interpolation coefficients are provided in a simple matrix in the \( data/ \) subfolder. In this way, the routine \( mainWithAdmittance.m \) evaluates the tidal height of altogether 256 tides (18 major tides from EOT11a and 238 interpolated minor tides).

3 Alternative representation

In addition to the provided internal format, the EOT11a is also published according to the IERS conventions 2010 \([8]\). Hence, the coefficients, amplitudes and phases correspond to \( C_{nm,s}^{+}, S_{nm,s}^{+}, C_{nm,s}^{-}, S_{nm,s}^{-} \) of equation \((6)\). Note that coefficients and amplitudes are dimensioned in [cm], while phases are given in [°]. Therefore, the evaluation of either equation \((4)\) or \((5)\) directly leads to tidal heights.
Acknowledgements

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References


Appendix A: Harmonic synthesis of tidal heights

The reaction of the ocean on the Tide Generating Potential is the sum of the frequency-dependent tidal heights $\zeta_s$, which can be represented in terms of amplitude $\xi_s$ and phase $\delta_s$ at a specific point on the Earth $(\lambda, \phi)$ at a distinct time $t$

$$\zeta(\lambda, \phi, t) = \sum_s \xi_s(\lambda, \phi) \cos[\Theta_s(t) + \chi_s - \delta_s(\lambda, \phi)].$$  \hspace{1cm} (1)

This formula is further depending on the astronomical (or Doodson) argument $\Theta_s$ and the Doodson-Warburg phase correction $\chi_s$ as defined in the IERS conventions 2010, Table 6.6 [8]. In a further step the tidal heights can be developed in terms of spherical harmonics [9]. For this, the tidal height of a particular tide in equation (1) is written as

$$\zeta_s = \xi_s \cos(\delta_s) \cos(\Theta_s + \chi_s) + \xi_s \sin(\delta_s) \sin(\Theta_s + \chi_s)$$  \hspace{1cm} (2)

with the in-phase and quadrature expressions that can be developed in spherical harmonics expansions

$$\xi_s \cos(\delta_s) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} (a_{nm,s} \cos m\lambda + b_{nm,s} \sin m\lambda) P_{nm}(\sin \phi)$$

$$\xi_s \sin(\delta_s) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} (c_{nm,s} \cos m\lambda + d_{nm,s} \sin m\lambda) P_{nm}(\sin \phi).$$  \hspace{1cm} (3)

Here, $a_{nm,s}, b_{nm,s}, c_{nm,s}, d_{nm,s}$ are spherical harmonics coefficients, while $P_{nm}$ are Legendre polynomials in dependency of degree and order $n, m$. Inserting equation (3) in (2) and a re-arrangement applying trigonometric identities leads then to the representation with coefficients $C_{nm,s}^{\pm}, S_{nm,s}^{\pm}$

$$\zeta_s = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{m=0}^{n} \left[C_{nm,s}^{\pm} \cos(\Theta_s + \chi_s \pm m\lambda) + S_{nm,s}^{\pm} \sin(\Theta_s + \chi_s \pm m\lambda)\right] P_{nm}(\sin \phi)$$  \hspace{1cm} (4)

or in terms of (tidal height) amplitudes $\hat{C}_{nm,s}^{\pm}$ and phases $\epsilon_{nm,s}$

$$\zeta_s = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{m=0}^{n} \hat{C}_{nm,s}^{\pm} \sin(\Theta_s + \chi_s \pm m\lambda + \epsilon_{nm,s}) P_{nm}(\sin \phi).$$  \hspace{1cm} (5)

The coefficients, amplitudes and phases can be retrieved from each other by the following relations

$$C_{nm,s}^{\pm} = \hat{C}_{nm,s}^{\pm} \sin(\epsilon_{nm,s}) = \frac{1}{2}(a_{nm,s} \mp b_{nm,s})$$

$$S_{nm,s}^{\pm} = \hat{C}_{nm,s}^{\pm} \cos(\epsilon_{nm,s}) = \frac{1}{2}(c_{nm,s} \pm d_{nm,s}).$$  \hspace{1cm} (6)

The mass redistribution effect of ocean tides on the Earth’s gravitational potential is described by

$$\Delta V_s^O = \frac{GM}{R} \sum_{n=0}^{\infty} \left(\frac{R}{r}\right)^{n+1} \sum_{m=0}^{n} [\Delta c_{nm,s} \cos(m\lambda) + \Delta s_{nm,s} \sin(m\lambda)] P_{nm}(\sin \phi).$$  \hspace{1cm} (7)

$G$ is denoting the gravitational constant, $M$ the mass of the Earth, $R$ the Earth’s radius and $\Delta c_{nm,s}, \Delta s_{nm,s}$ the dimensionless potential coefficients that can be retrieved by

$$\Delta c_{nm,s} = \frac{4\pi R^2 \rho_w}{M} \frac{1 + k_n'}{2n + 1} \left[(C_{nm,s}^{+} + C_{nm,s}^{-}) \cos(\Theta_s + \chi_s) + (S_{nm,s}^{+} + S_{nm,s}^{-}) \sin(\Theta_s + \chi_s)\right]$$

$$\Delta s_{nm,s} = \frac{4\pi R^2 \rho_w}{M} \frac{1 + k_n'}{2n + 1} \left[(S_{nm,s}^{+} - S_{nm,s}^{-}) \cos(\Theta_s + \chi_s) - (C_{nm,s}^{+} - C_{nm,s}^{-}) \sin(\Theta_s + \chi_s)\right]$$  \hspace{1cm} (8)

with the water density $\rho_w$ and the load love numbers $k_n'$. Introducing the factor $1 + k_n'$ therefore accounts for both, the mass effect of ocean tides, as well as the effect of the deformation of the solid Earth as a consequence of ocean mass redistribution.