

# The Mathematics of Energy Economics and Carbon Neutrality

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02.02.2022

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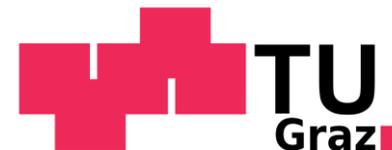


## Background

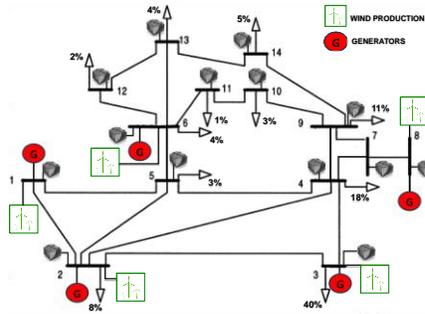
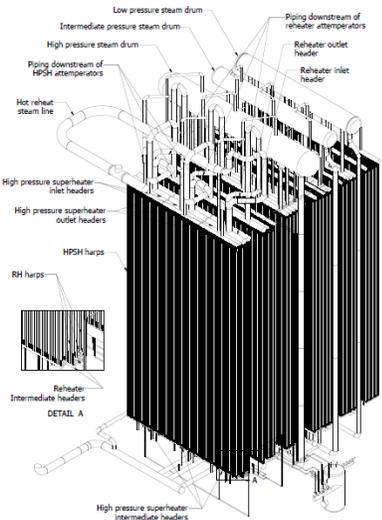
- **PhD in Power Systems (2013)**  
Comillas Pontifical University, Spain
- **MSc in Computation for Design and Optimization (2008)**  
Massachusetts Institute of Technology, USA
- **Dipl.-Ing. Technical Mathematics (2008)**  
Graz University of Technology, Austria



**COMILLAS**  
UNIVERSIDAD PONTIFICIA  
ICA1 ICADE CIHS



**Energy System Modeling:**  
 Transmission/Generation Expansion  
 Unit Commitment  
 Optimal Power Flow  
 Bilevel Programming



$$\forall i \begin{cases} \max_{x_i, q_i} & t(p(q_i, q_{-i}) - \delta)q_i - \beta x_i \\ \text{s.t.} & \text{Second stage} \end{cases}$$



$$\forall i \begin{cases} \max_{x_i, q_i} & t(p(q_i, q_{-i}) - \delta)q_i \\ \text{s.t.} & 0 \leq q_i \leq x_i \end{cases}$$

$$d = q_i + q_{-i}, d = D_0 - \alpha p$$



**Techno-Economic Analyses:**  
 Energy Transition  
 Profitability in Electricity Markets



**Bundesministerium**  
 Klimaschutz, Umwelt,  
 Energie, Mobilität,  
 Innovation und Technologie



**Data Science  
 & Artificial Intelligence:**  
 Fault detection in heat pumps  
 Prediction of cycle life



**Energy Storage:**  
 Allocation & Investment



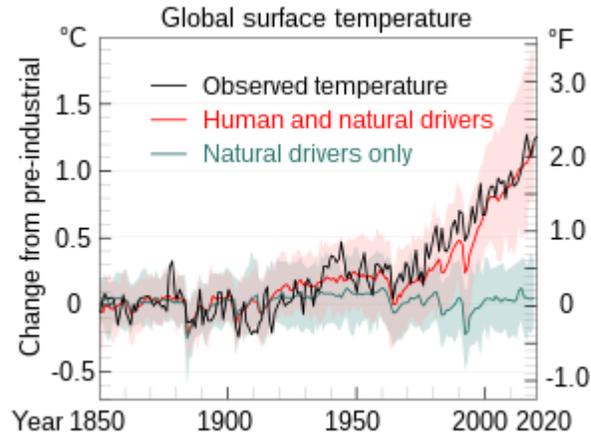
# Agenda

- Motivation. Why do we care about energy system planning?
- Mathematical modeling and optimization as decision support tools
- Strategic generation expansion planning in liberalized electricity markets
- Conclusions



# Why do we care about energy system planning?

There is this thing called **climate change**.



Source: [https://en.wikipedia.org/wiki/Climate\\_change#/media/File:Global\\_Temperature\\_And\\_Forces\\_With\\_Fahrenheit.svg](https://en.wikipedia.org/wiki/Climate_change#/media/File:Global_Temperature_And_Forces_With_Fahrenheit.svg)



Source: <https://www.dw.com/en/greece-wildfirehousands-flee-island-of-evia-as-blazes-continue-to-ravage-lands-a-58794368>



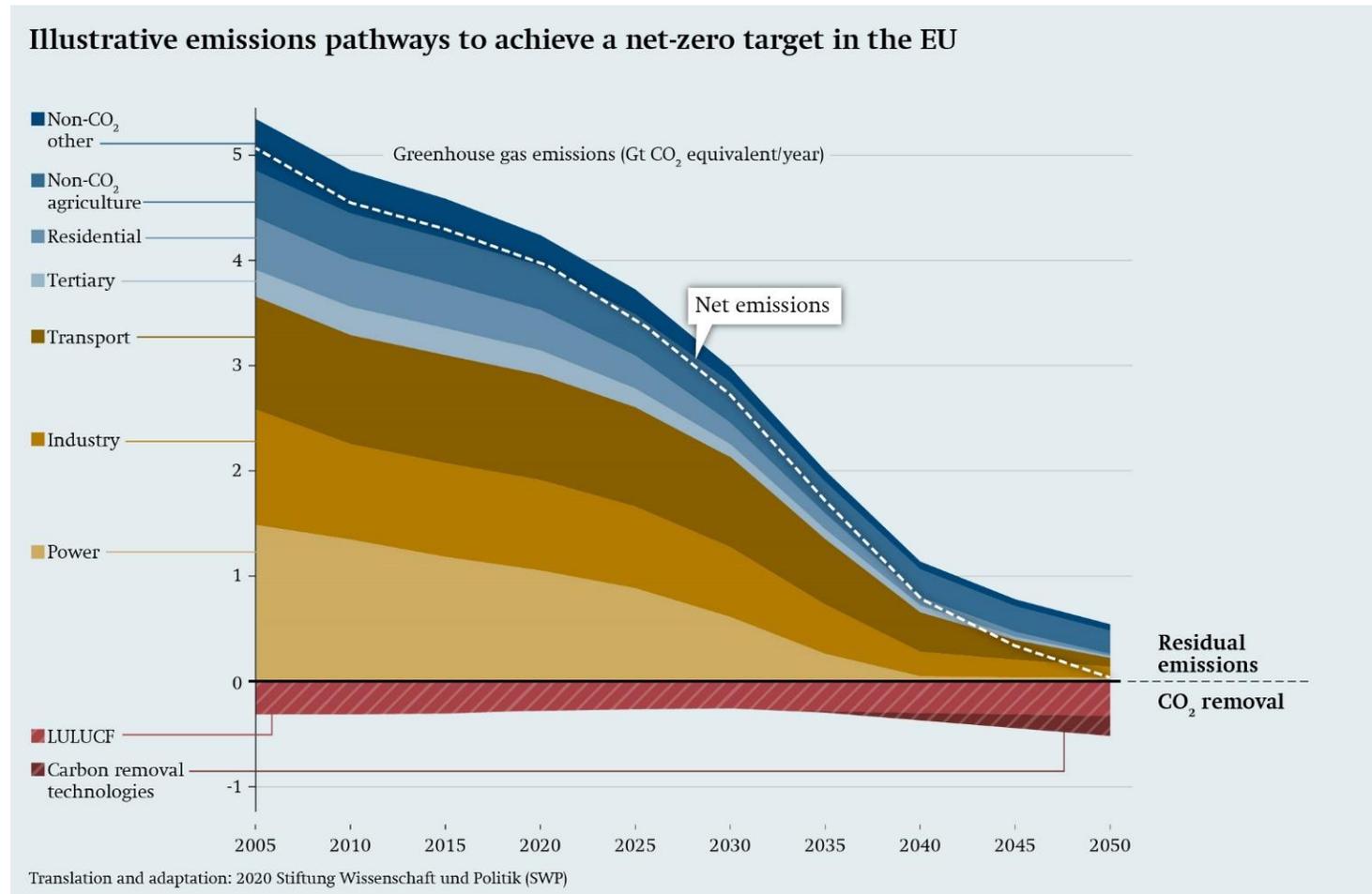
Source: <https://www.premiumpress.com/news/top-news/473966-over-80-dead-dozens-missing-as-floods-hit-germany.html>



Source: <https://www.greenpeace.org/usa/warsaw-climate-talks-so-bad-us-looked-good/melting-ice-polar-bear-on-206311-jpg/>

# Decarbonization in Europe

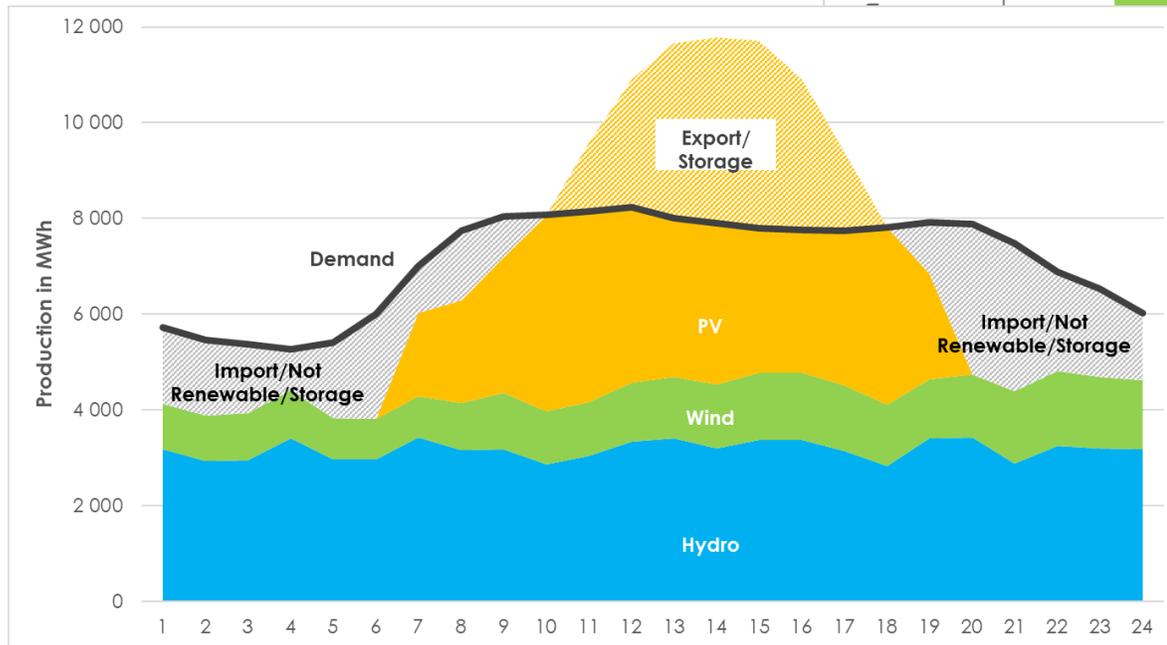
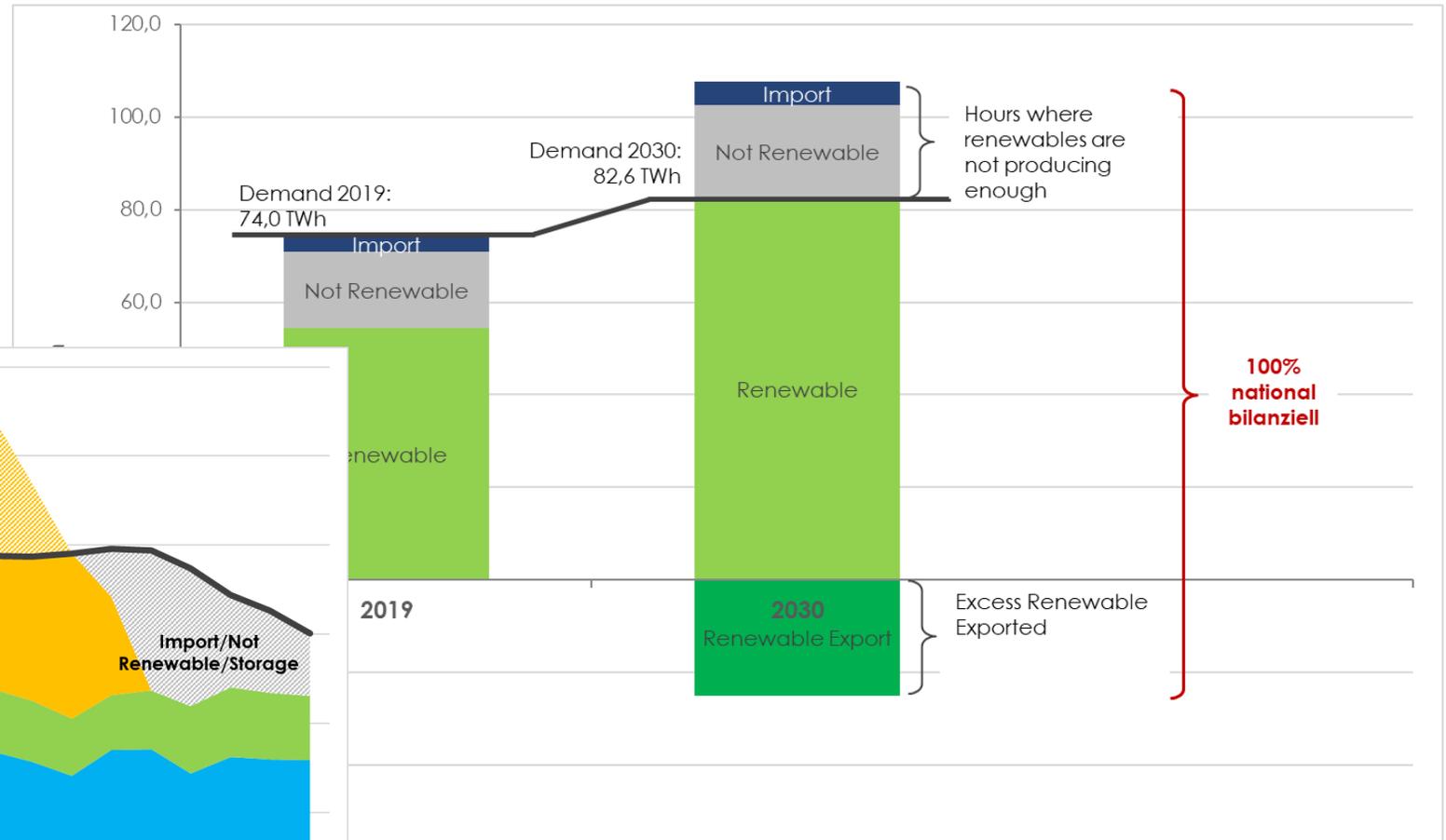
- How do we define **climate neutrality** in Europe.
- We want to achieve it **until 2050** (European Commission).
- Over sectors: Electricity, Industry, Transport, etc.



Source: <https://www.swp-berlin.org/en/publication/eu-climate-policy-unconventional-mitigation>

# Climate neutrality in Austria: Renewable Expansion Law

- In 2030 total system demand has to be 100% **net national** produced by renewables.

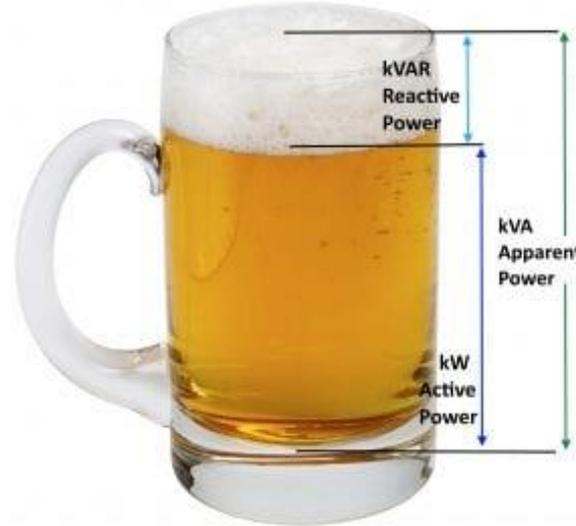


# Challenges of decarbonization



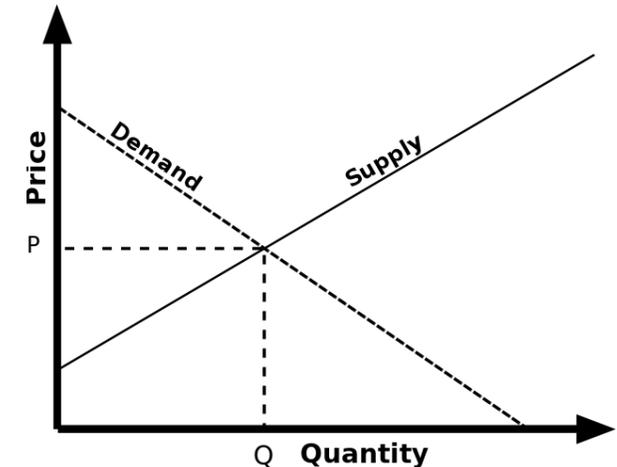
Source: <https://wolfhill.blog/2014/02/24/bitter-cold-anti-wynewind-turbine-protest-at-queens-park/>

- **Social** (e.g. not in my backyard)



Source: <https://blog.se.com/energy-management-energy-efficiency/2018/01/22/can-learn-power-pint-beer/>

- **Technical** (e.g. reactive power, system inertia)



Source: [https://commons.wikimedia.org/wiki/File:Simple\\_supply\\_and\\_demand.svg](https://commons.wikimedia.org/wiki/File:Simple_supply_and_demand.svg)

- **Economic** (e.g. profitability)

**Modeling and optimization** can serve as **decision support tool**.





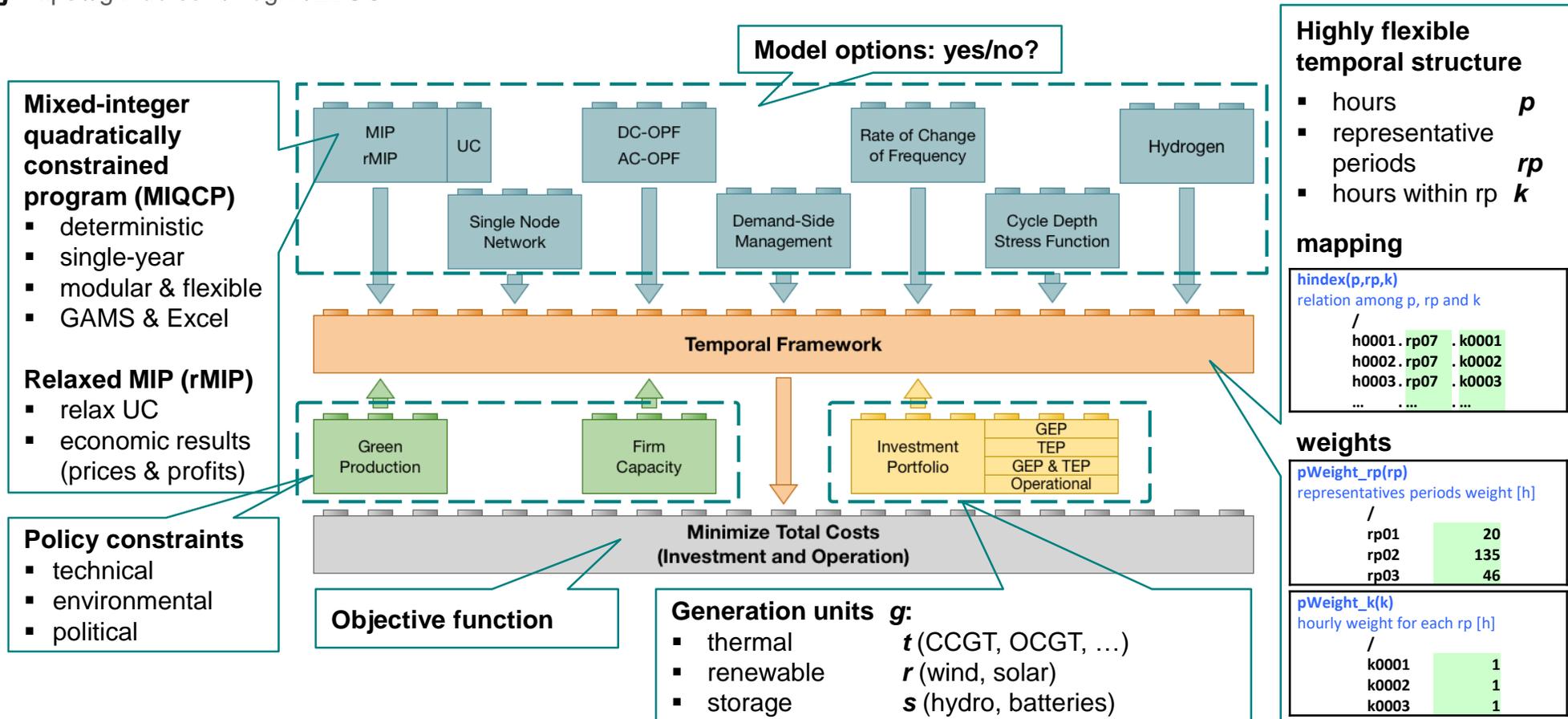
# Open-source tool for Low-carbon Expansion and Generation Optimization (LEGO)



Source: Wogrin, S., Tejada-Arango, D., Delikaraoglou, S. and Botterud, A., 2020. Assessing the impact of inertia and reactive power constraints in generation expansion planning. Applied Energy, 280, p.115925.



<https://github.com/wogrin/LEGO>



# Mathematical Formulation ...

$$\begin{aligned} \min \sum_{rp,k} W_{rp}^{RP} W_k^K & \left( \sum_t (C_t^{SU} y_{rp,k,t} + C_t^{UP} u_{rp,k,t} + C_t^{VAR} p_{rp,k,t}) \right. \\ & \left. + \sum_r C_r^{OM} p_{rp,k,r} + \sum_s C_s^{OM} p_{rp,k,s} + \sum_i C_i^{ENS} pns_{rp,k,i} \right) \\ + \sum_{rp,k} W_{rp}^{RP} W_k^K & \left( \sum_t (C_t^{VAR} C^{RES+} res_{rp,k,t}^+ + C_t^{VAR} C^{RES-} res_{rp,k,t}^-) \right. \\ & \left. + \sum_s (C_s^{OM} C^{RES+} res_{rp,k,s}^+ + C_s^{OM} C^{RES-} res_{rp,k,s}^-) \right) \\ & + \sum_g C_g^{INV} x_g \end{aligned}$$

## Objective function



And so on and so on and so on.

## Power flow

## Hydrogen production

$$\begin{aligned} 0 \leq p_{rp,k,h2g}^{H2} & \leq \bar{P}_{h2g}^E W_k^K HPE_{h2g} (x_{h2g}^{H2} + EU_{h2g}^{H2}) \quad \forall rp, k, h2g \\ cs_{rp,k,h2g}^E W_k^K HPE_{h2g} & = p_{rp,k,h2g}^{H2} \quad \forall rp, k, h2g \\ \sum_{h2gh2i(h2g,h2i)} p_{rp,k,h2g}^{H2} + h2ns_{rp,k,h2i} & = \sum_{h2sec} D_{rp,k,h2i,h2sec}^{H2} \quad \forall rp, k, h2i \\ 0 \leq h2ns_{rp,k,h2i} & \leq \sum_{h2sec} D_{rp,k,h2i,h2sec}^{H2} \quad \forall rp, k, h2i \\ x_{h2g}^{H2} \in \mathbb{Z}^{+,0}, x_{h2g}^{H2} & \leq \bar{X}_{h2g}^{H2} \quad \forall h2g \end{aligned}$$

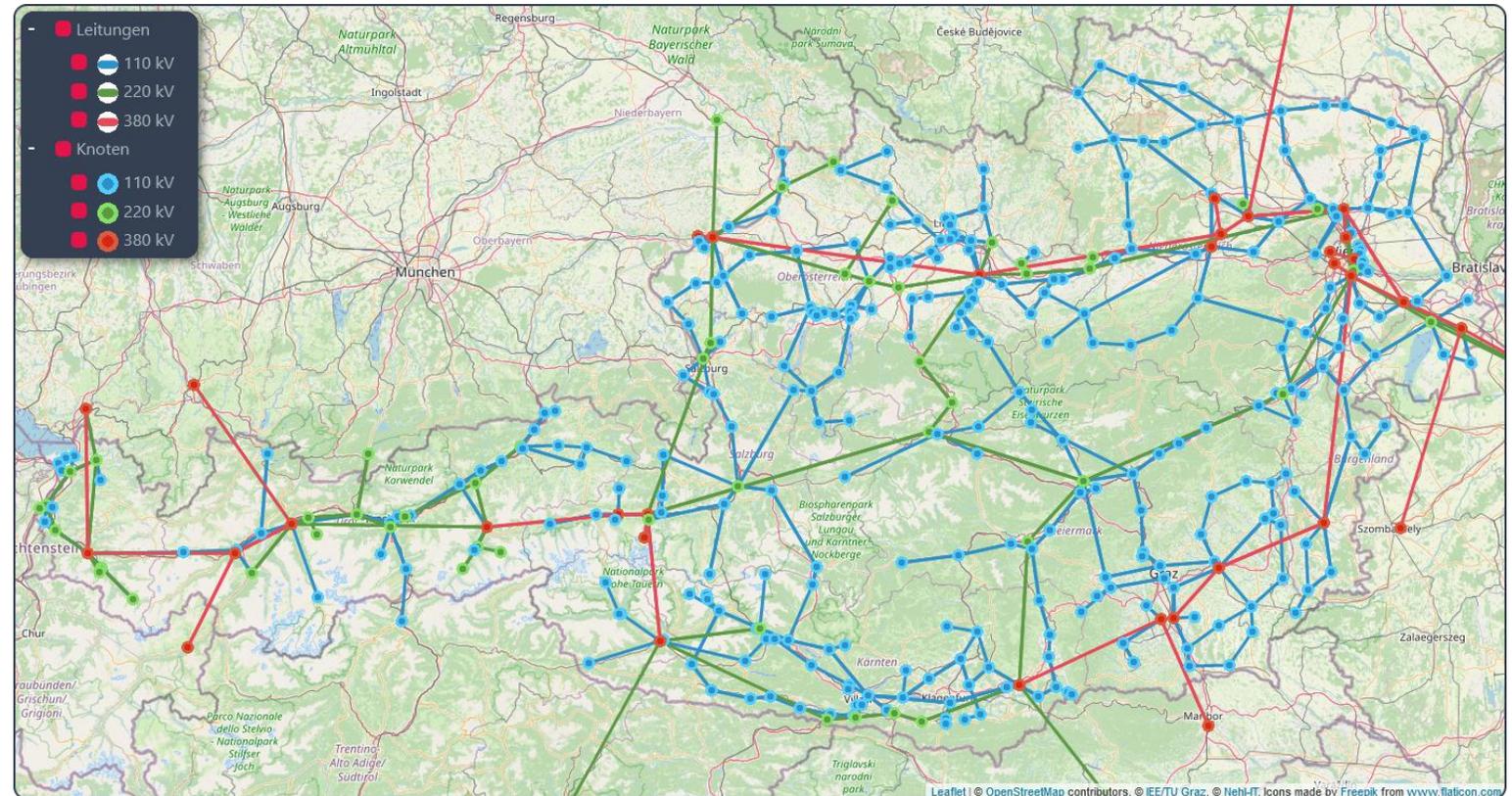
## Unit commitment

$$\begin{aligned} \sum_t res_{rp,k,t}^+ + \sum_s res_{rp,k,s}^+ & \geq RES^+ \sum_i D_{rp,k,i}^P \quad \forall rp, k \\ \sum_t res_{rp,k,t}^- + \sum_s res_{rp,k,s}^- & \geq RES^- \sum_i D_{rp,k,i}^P \quad \forall rp, k \\ p_{rp,k,t} & = u_{rp,k,t} \underline{P}_t + \hat{p}_{rp,k,t} \quad \forall rp, k, t \\ \hat{p}_{rp,k,t} + res_{rp,k,t}^+ & \leq (\bar{P}_t - \underline{P}_t)(u_{rp,k,t} - y_{rp,k,t}) \quad \forall rp, k, t \\ \hat{p}_{rp,k,t} + res_{rp,k,t}^+ & \leq (\bar{P}_t - \underline{P}_t)(u_{rp,k,t} - z_{rp,k,t+1}) \quad \forall rp, k, t \\ \hat{p}_{rp,k,t} & \geq res_{rp,k,t}^- \quad \forall rp, k, t \\ u_{rp,k,t} - u_{rp,k-1,t} & = y_{rp,k,t} - z_{rp,k,t} \quad \forall rp, k, t \\ u_{rp,k,t} & \leq x_t + EU_t \quad \forall rp, k, t \\ \hat{p}_{rp,k,t} - \hat{p}_{rp,k-1,t} + res_{rp,k,t}^+ & \leq u_{rp,k,t} RU_t \quad \forall rp, k, t \\ \hat{p}_{rp,k,t} - \hat{p}_{rp,k-1,t} - res_{rp,k,t}^- & \geq -u_{rp,k-1,t} RD_t \quad \forall rp, k, t \\ 0 \leq p_{rp,k,t} & \leq \bar{P}_t (x_t + EU_t) \quad \forall rp, k, t \\ 0 \leq \hat{p}_{rp,k,t}, res_{rp,k,t}^-, res_{rp,k,t}^+ & \leq (\bar{P}_t - \underline{P}_t)(x_t + EU_t) \quad \forall rp, k, t \\ u_{rp,k,t}, y_{rp,k,t}, z_{rp,k,t} & \in \{0, 1\} \quad \forall rp, k, t \end{aligned}$$

Source: Wogrin, S., Tejada-Arango, D., Delikaraoglou, S. and Botterud, A., 2020. Assessing the impact of inertia and reactive power constraints in generation expansion planning. Applied Energy, 280, p.115925.

# Case study: LEGO Austria

- Austria's electricity infrastructure:
  - 1,304 generators
  - 468 nodes
  - 1,097 power lines (110, 220, 380 kV)
  
- Model summary
  - 900,000 variables
  - 800,000 equations



## START2030

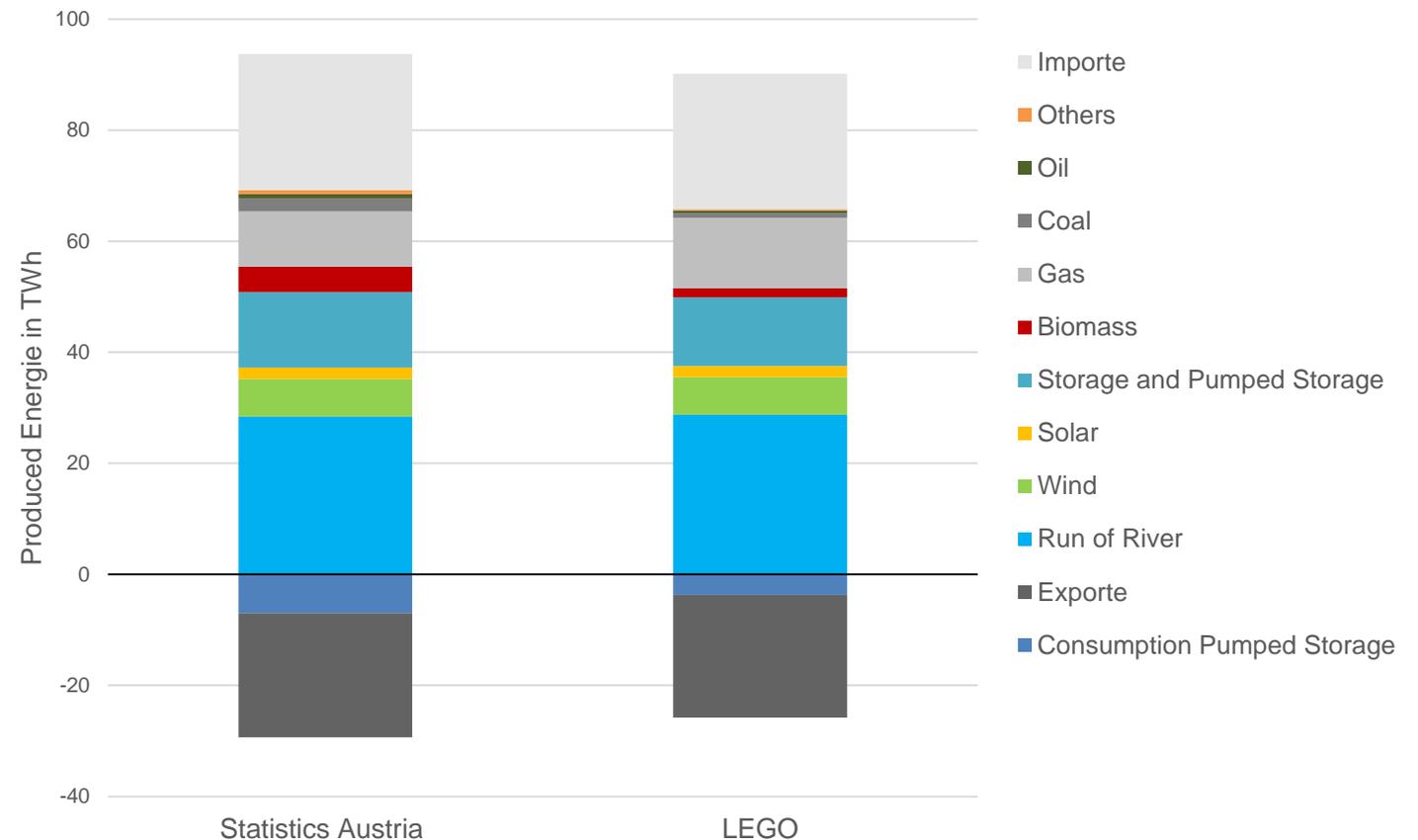


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# Case study: LEGO Austria

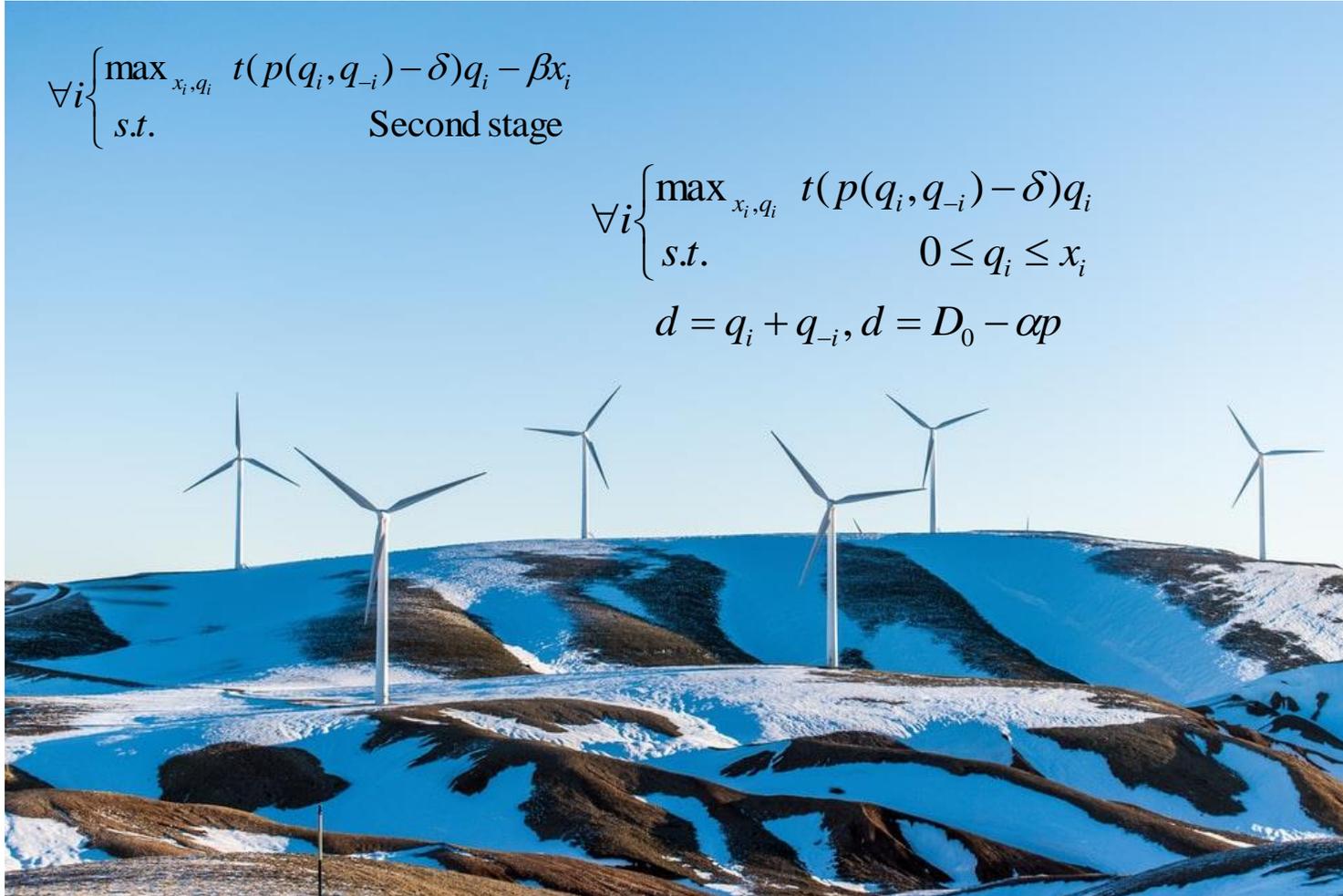
- Optimization to achieve goals of the **EAG by 2030** & **climate neutrality** by 2040.
- LEGO Austria has been validated by comparing produced electricity data for 2020 from Statistics Austria and LEGO Austria model results.

Comparison Statistics Austria with LEGO Results



$$\forall i \begin{cases} \max_{x_i, q_i} & t(p(q_i, q_{-i}) - \delta)q_i - \beta x_i \\ \text{s.t.} & \text{Second stage} \end{cases}$$

$$\forall i \begin{cases} \max_{x_i, q_i} & t(p(q_i, q_{-i}) - \delta)q_i \\ \text{s.t.} & 0 \leq q_i \leq x_i \\ & d = q_i + q_{-i}, d = D_0 - \alpha p \end{cases}$$



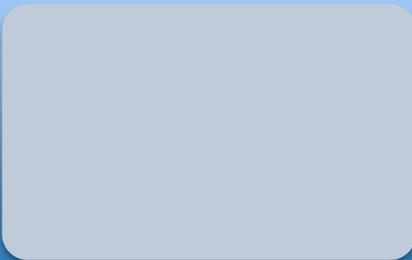
Source: <https://unsplash.com/photos/kufsOr1-F-s>

# Strategic Generation Expansion Planning in Liberalized Electricity Markets

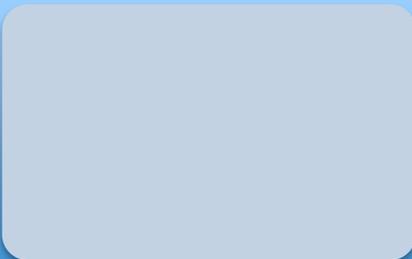
# Liberalization of electricity markets



The liberalization of electricity markets has complicated the organization of the electricity sector.



**Centralized framework:** social welfare maximization; versus **liberalized electricity markets:** private entities maximize profits.



Many decision-making problems in a liberalized power sector can be regarded as a game in search of equilibrium solutions.

# Hierarchical decision making



The sequence in which decisions are taken, can convert simple equilibrium games into complicated hierarchical equilibrium problems.



An application of such hierarchical games in electricity markets: generation expansion planning. Results indicate that the game structure drastically influences outcomes.

Examples:

Wogrin, Sonja, Salvador Pineda, and Diego A. Tejada-Arango. "Applications of bilevel optimization in energy and electricity markets." Bilevel Optimization. Springer, Cham, 2020. 139-168.  
Gabriel, Steven A., et al. Complementarity modeling in energy markets. Vol. 180. Springer Science & Business Media, 2012.

# Generation Expansion Planning Problem Statement

We have two identical firms (e.g. generation companies - GENCOs) with perfectly substitutable products (e.g. electric energy), facing either a one-stage or a two-stage competitive situation.

One-stage  
situation (open  
loop model)

Investment and  
operation decisions  
are made  
simultaneously.

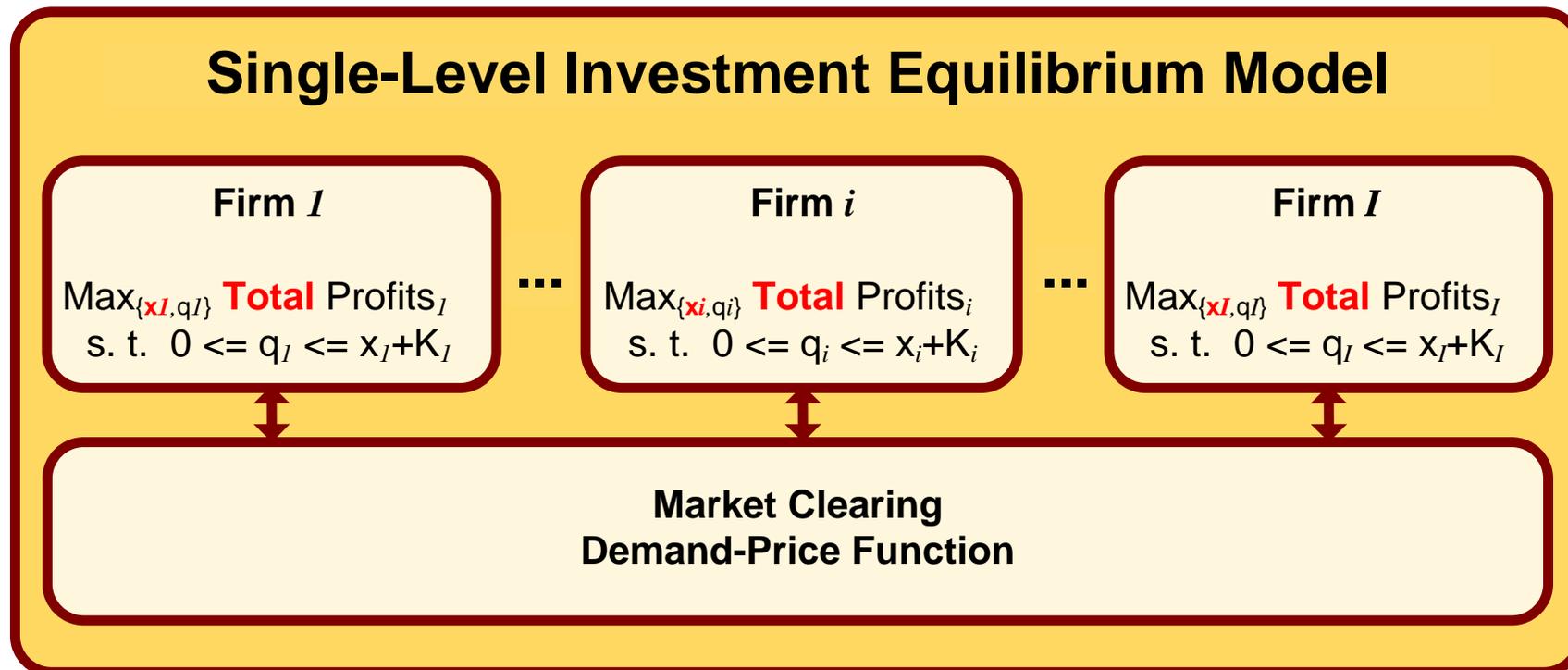
Two-stage situation (closed loop  
model)

First, firms choose  
capacities that  
maximize their profit  
anticipating the second  
stage, where...

...quantities and prices  
are determined by a  
conjectured price  
response market  
equilibrium.

# Single-Level (SL) Investment Equilibrium

- All GENCOs simultaneously maximize their **total** profits (market revenues minus **investment costs** minus production costs) subject to lower and upper bounds on production and a demand balance.



# Single-Level (SL) Investment Formulation

Concept:

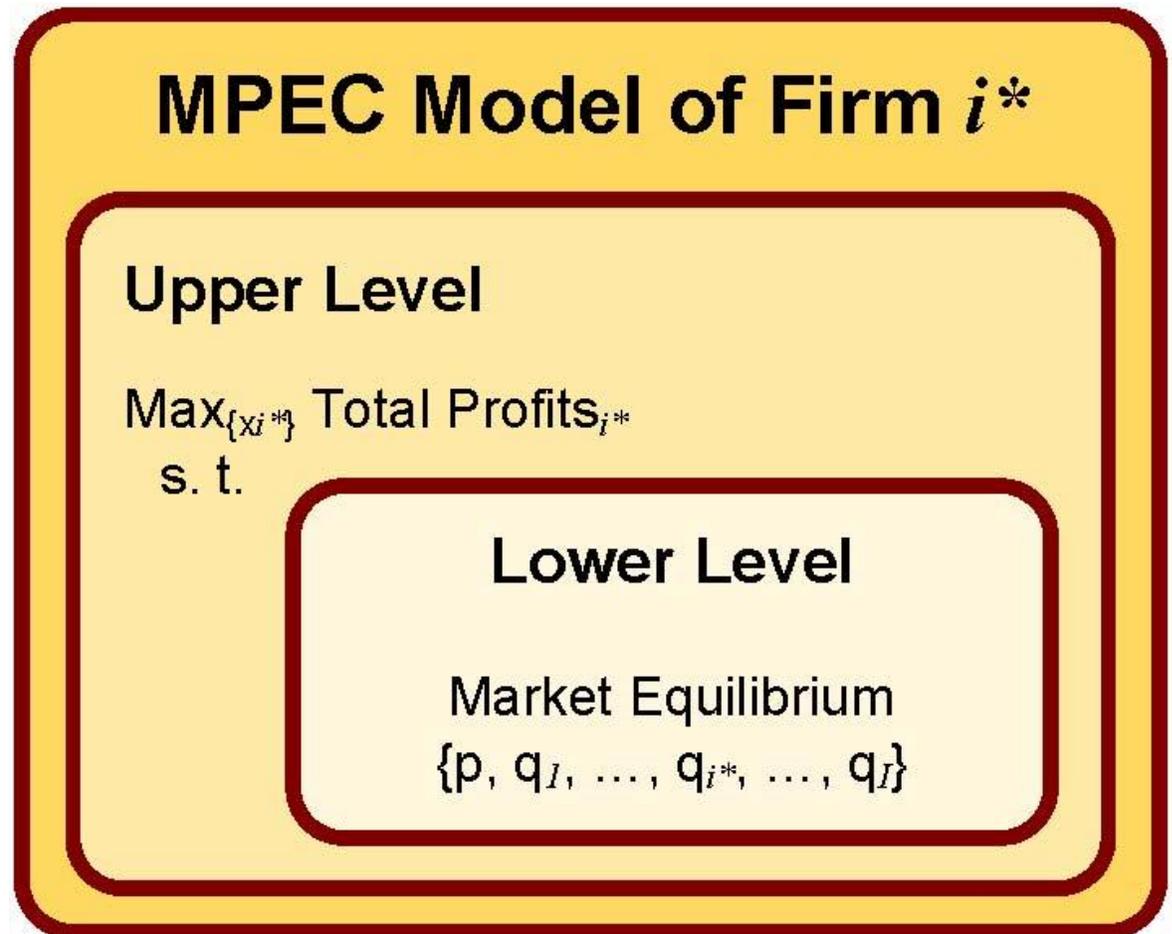
$$\forall i \begin{cases} \max_{x_i, q_i} & t(p(q_i, q_{-i}) - \delta)q_i - \beta x_i \\ \text{s.t.} & q_i \leq x_i \\ & d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}) \end{cases}$$

KKT-conditions:

$$\forall i \begin{cases} \frac{\partial \mathcal{L}_i}{\partial q_i} = tp(q_i, q_{-i}) - t\theta q_i - t\delta - \lambda_i = 0 \\ \frac{\partial \mathcal{L}_i}{\partial x_i} = \beta - \lambda_i = 0 \\ q_i \leq x_i \\ \lambda_i \geq 0 \\ \lambda_i(x_i - q_i) = 0 \\ d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}) \end{cases}$$

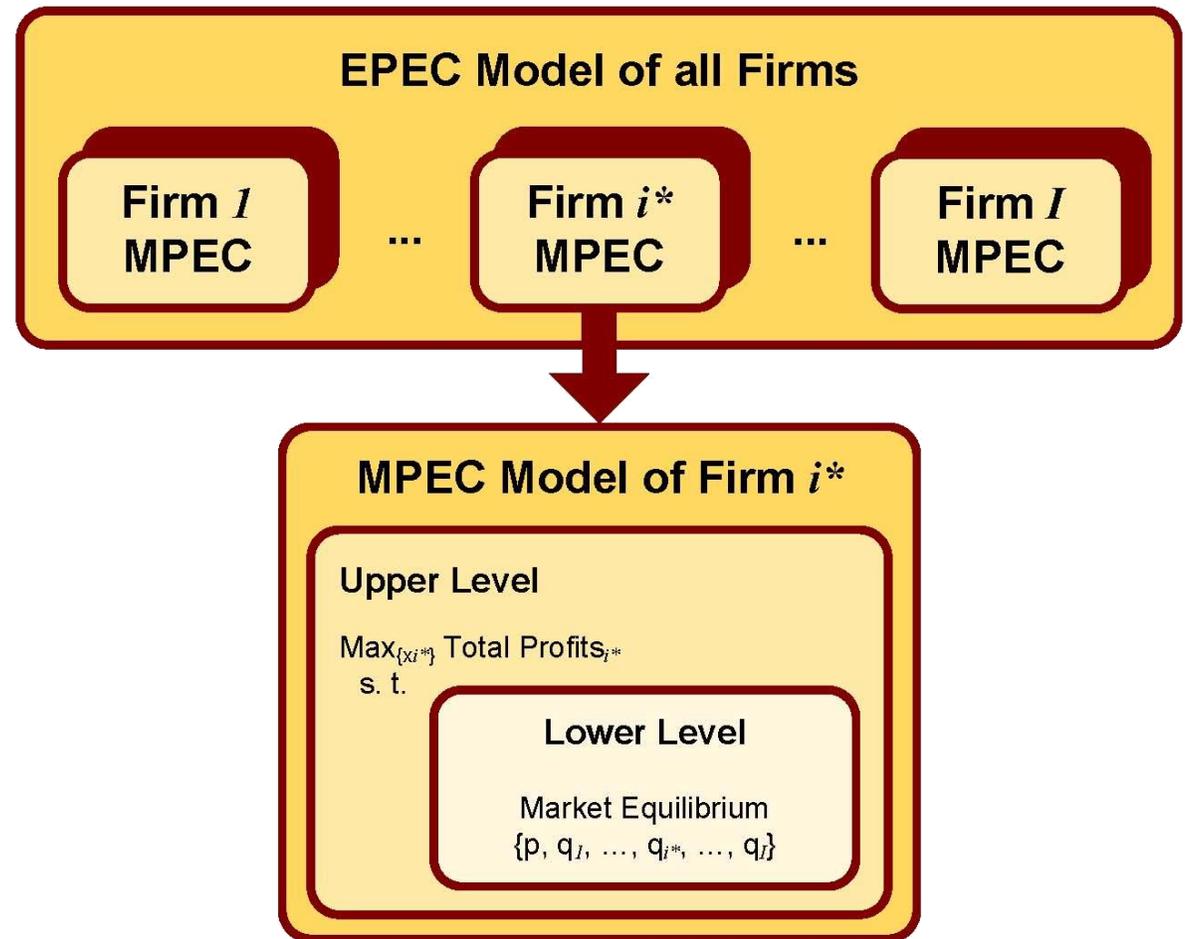
# Bilevel (BL) Investment Optimization

- This model assists **one GENCO** in taking capacity decisions while considering the competitors' investments as fixed.
- This model is a Mathematical Problem with Equilibrium Constraints (**MPEC**).
- In the upper level investment decisions of firm  $i^*$  are taken.



# Bilevel (BL) Investment Equilibrium

- This model assists **ALL** GENCOs in taking capacity decisions.
- This problem is an Equilibrium Problem with Equilibrium Constraints (**EPEC**): all GENCOs simultaneously face an MPEC.



# Bilevel (BL) Investment Formulation

First Stage (Investment):

$$\forall i \begin{cases} \max_{x_i} & t(p(q_i, q_{-i}) - \delta)q_i - \beta x_i \\ \text{s.t.} & \text{Second Stage} \end{cases}$$

Second Stage (Production):

$$\forall i \begin{cases} \max_{q_i} & t(p(q_i, q_{-i}) - \delta)q_i \\ \text{s.t.} & q_i \leq x_i \end{cases}$$

$$d = q_i + q_{-i}, \quad d = D_0 - \alpha p(q_i, q_{-i}),$$

# Comparison Single- and Bilevel Equilibria

Source: Wogrin, Sonja, et al. "Open versus closed loop capacity equilibria in electricity markets under perfect and oligopolistic competition." *Mathematical Programming* 140.2 (2013): 295-322.

We compare (Wogrin et al, 2013) two generation expansion models:

- A **single-level** model where investment and production decisions are considered to be taken **simultaneously**.
- A **bilevel** model where first investment decisions are taken and then **sequentially** production decisions are decided in the market.
- The intensity of competition among producers in the energy market is represented using **conjectural variations**.
- For simplicity, in each of the models we consider two identical generation companies, a one-year time horizon and investment in one technology.

# Comparison Single- and Bilevel Equilibria

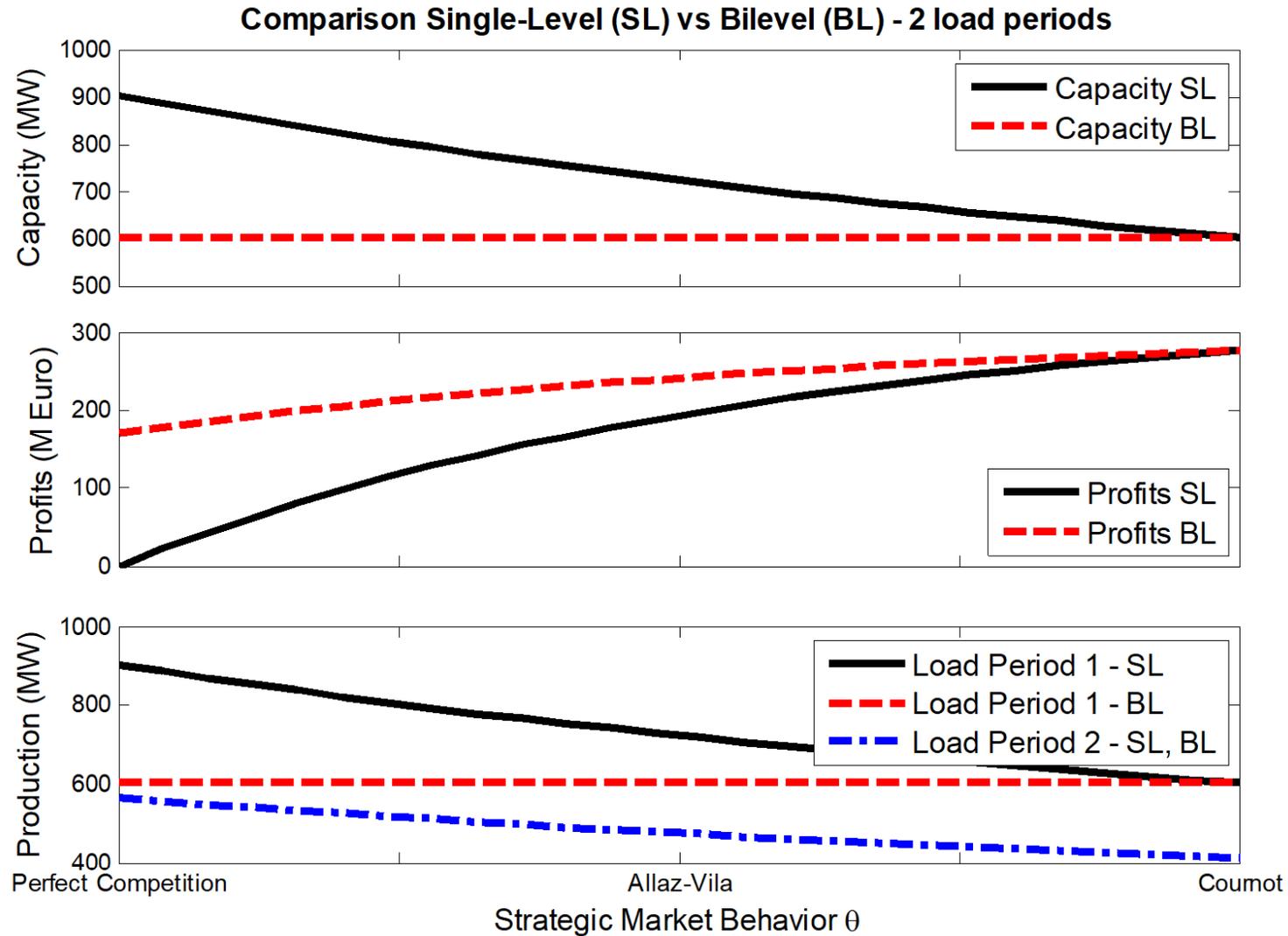
## Theorem

*Let there be two identical firms with perfectly substitutable products and one load period and let the affine price  $p(d)$  and the parameters be as previously defined. When comparing the open and closed loop competitive equilibria for two firms, we find the following:*

**The open loop Cournot solution, is a solution to the closed loop conjectured price response equilibrium for any choice of the conjectured price response parameter  $\theta$  from perfect competition to Cournot competition.**

- The result extends to multiple load periods and – under certain circumstances – to asymmetric firms.
- This is an extension of (Kreps and Scheinkman, 1983).

# Numerical Example: 2 Load Periods



# Summary of Results and Policy Implications

- 1** The **bilevel** model always yields **Cournot capacities** independent of strategic spot market behavior.
- 2** This makes them **more realistic** than single-level models whose capacity decisions vary with market behavior.
- 3** Therefore bilevel models are very useful to study realistic generation expansion decisions and to evaluate the effect of alternative **market designs** for mitigating **market power**.
- 4** Under certain circumstances (Cournot market behavior) both **single-level and bilevel results can coincide**.
- 5** In bilevel models, **more competition can lead to less consumer surplus and less overall market efficiency**, depending on the model parameters.

# Final Takeaways

- **Climate neutrality** poses a complex challenge
- **Mathematical modeling** and **optimization** can help via techno-economic analyses



Source: dba



- **We need young people in/for science!**

# Thank you. Questions?

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