Robot Vision:
Camera calibration

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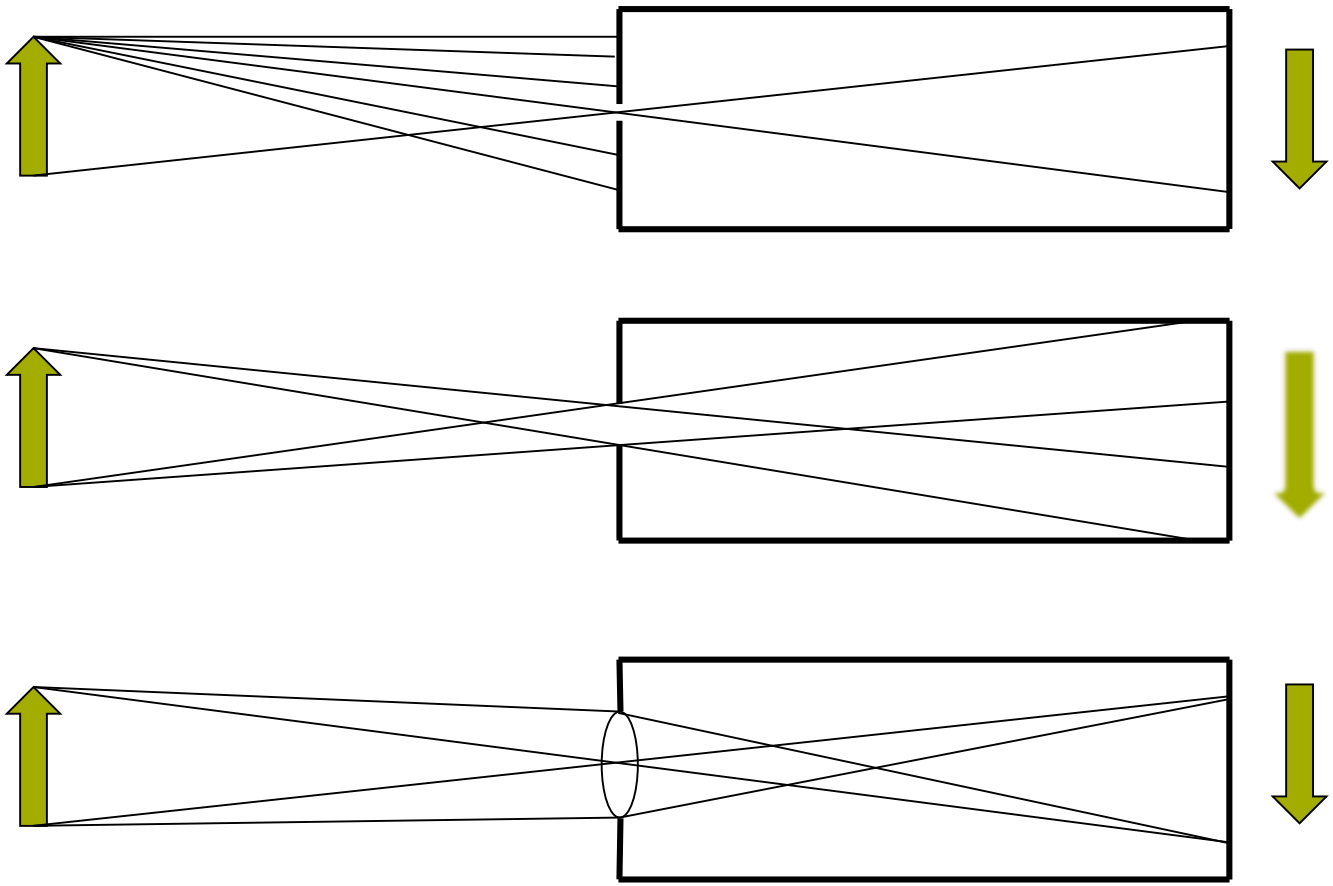
Outline

- Camera calibration
  - Cameras with lenses
  - Properties of real lenses (distortions, focal length, field-of-view)
  - Calibration algorithm using planar targets (Zhang)

Pinhole camera and aperture

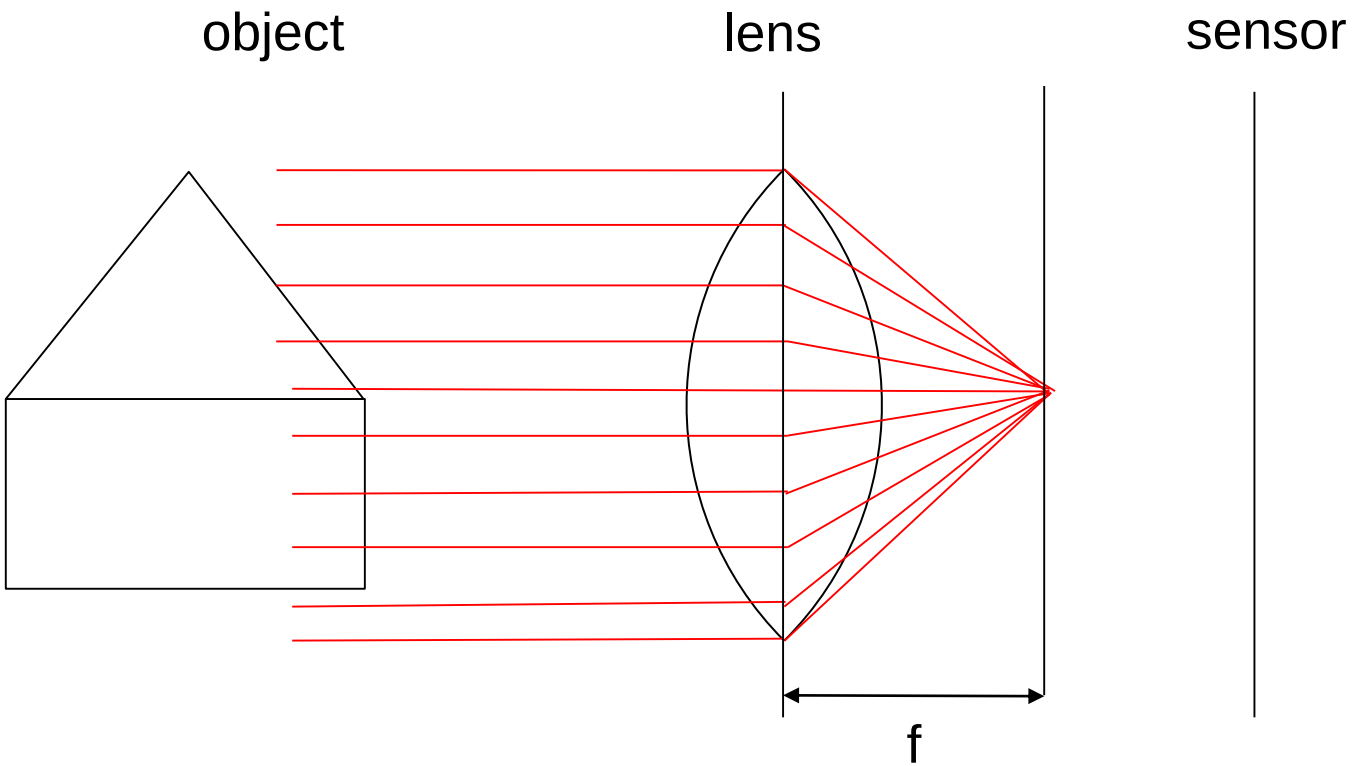


Pinhole

Larger aperture

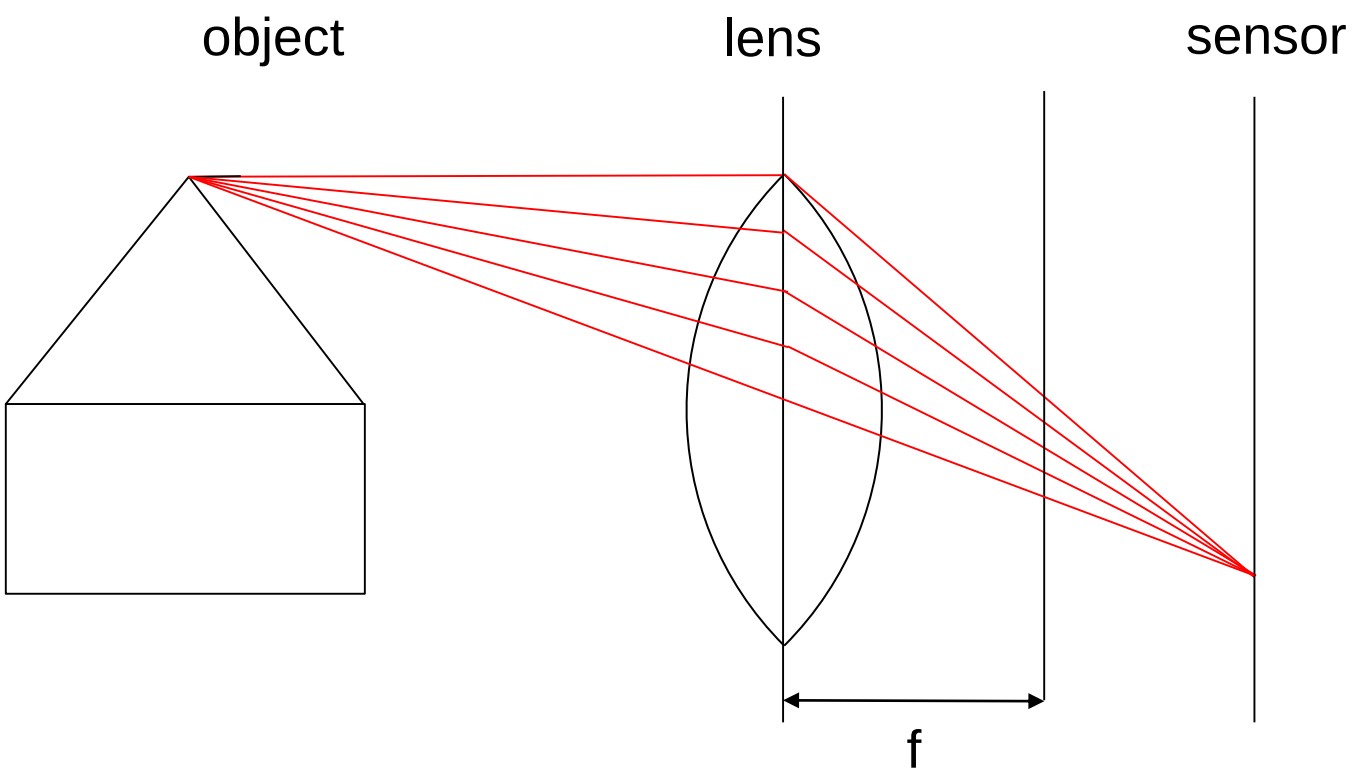
Lens

Effect of a lens



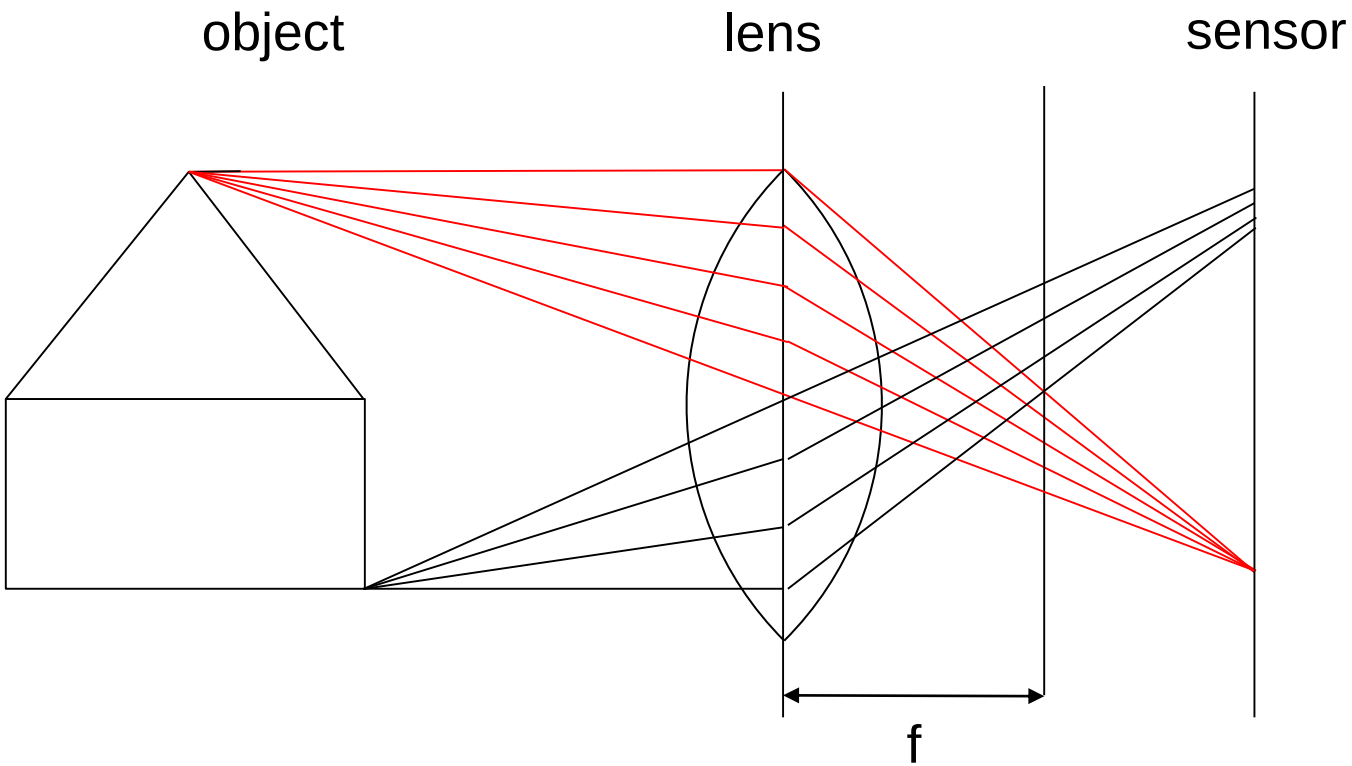
- Thin lens model:
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point at distance f

Effect of a lens



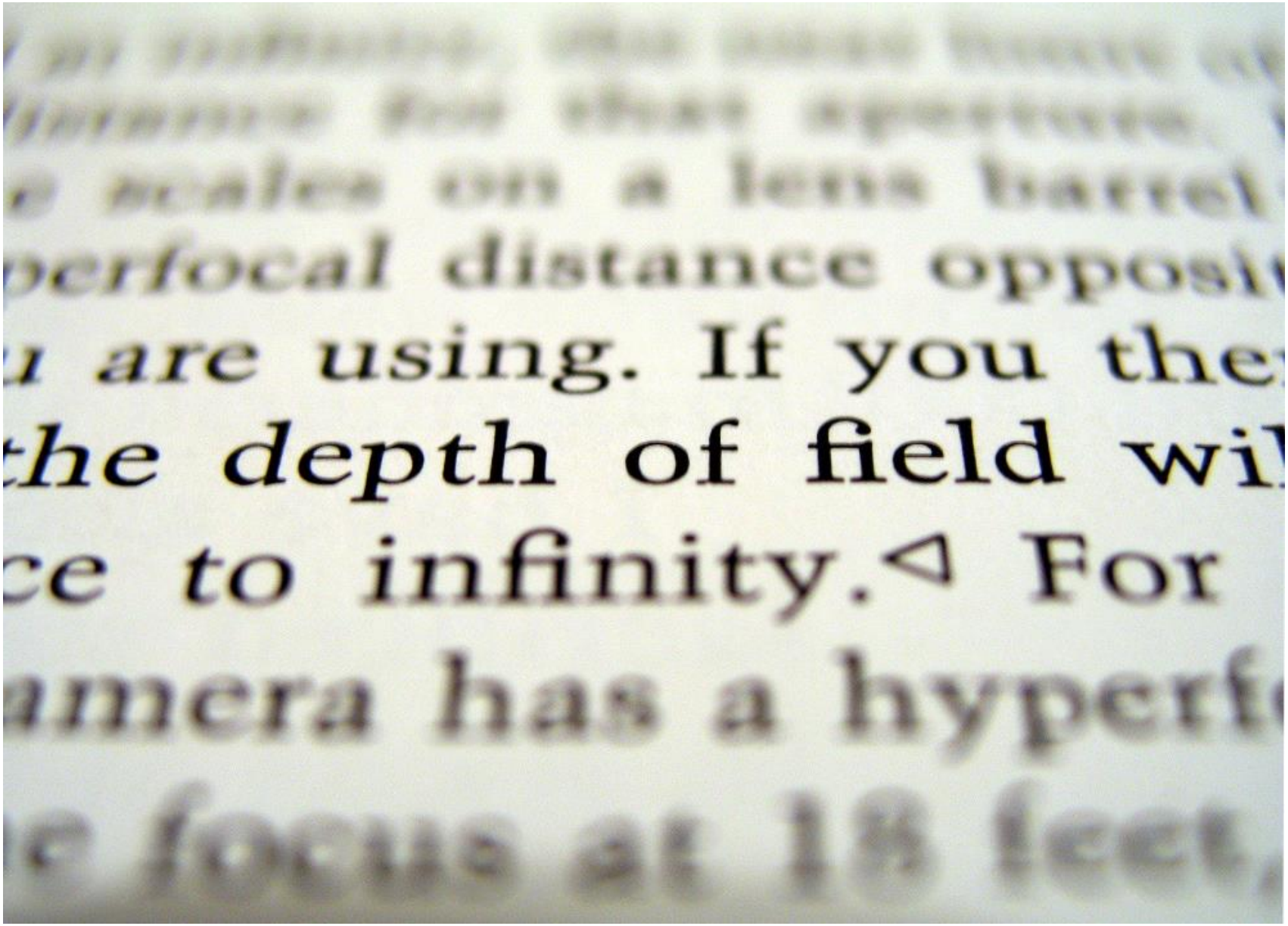
- Thin lens model:
  - Rays passing through the center are not deviated, equivalent to pinhole model

Effect of a lens

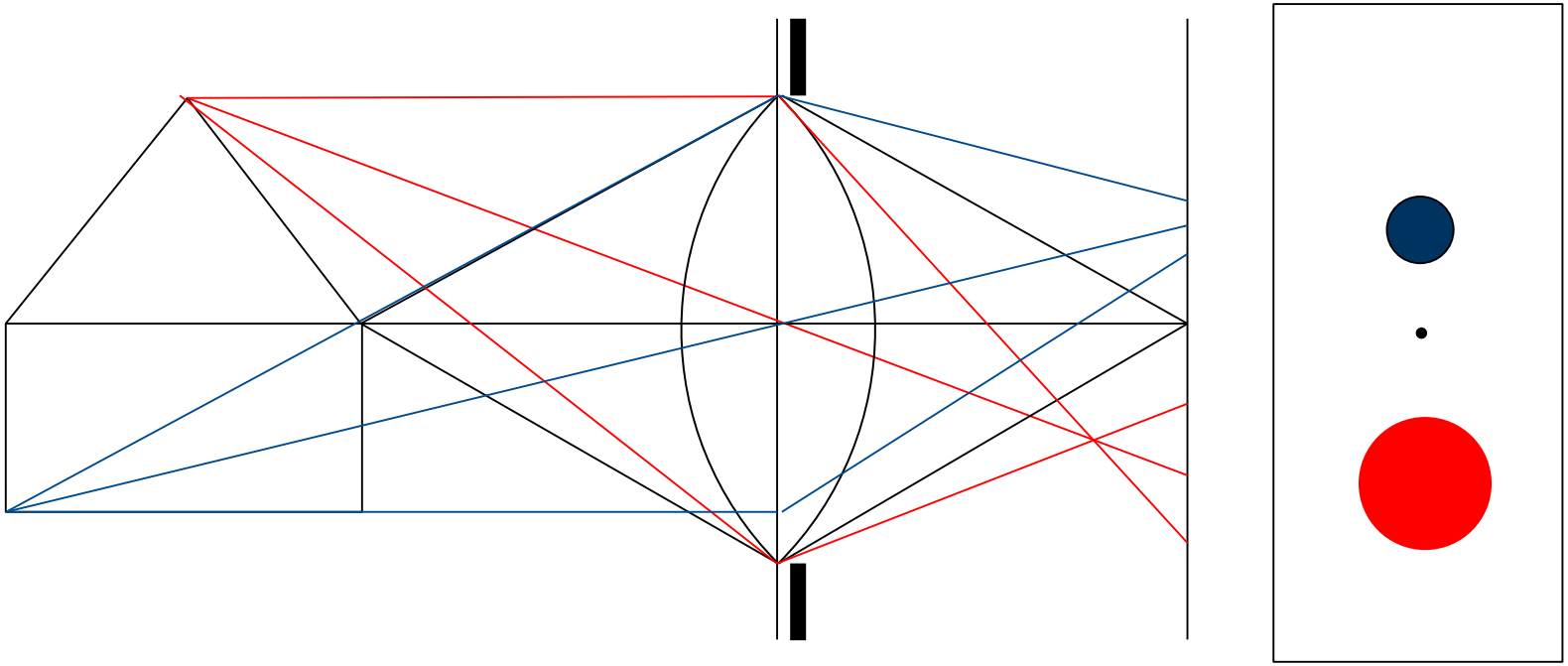


- Thin lens model:
  - Light rays originating from different depths meet at different locations, only images from specific distances are in focus

Depth of Field/Focal depth



Aperture controlls depth of field



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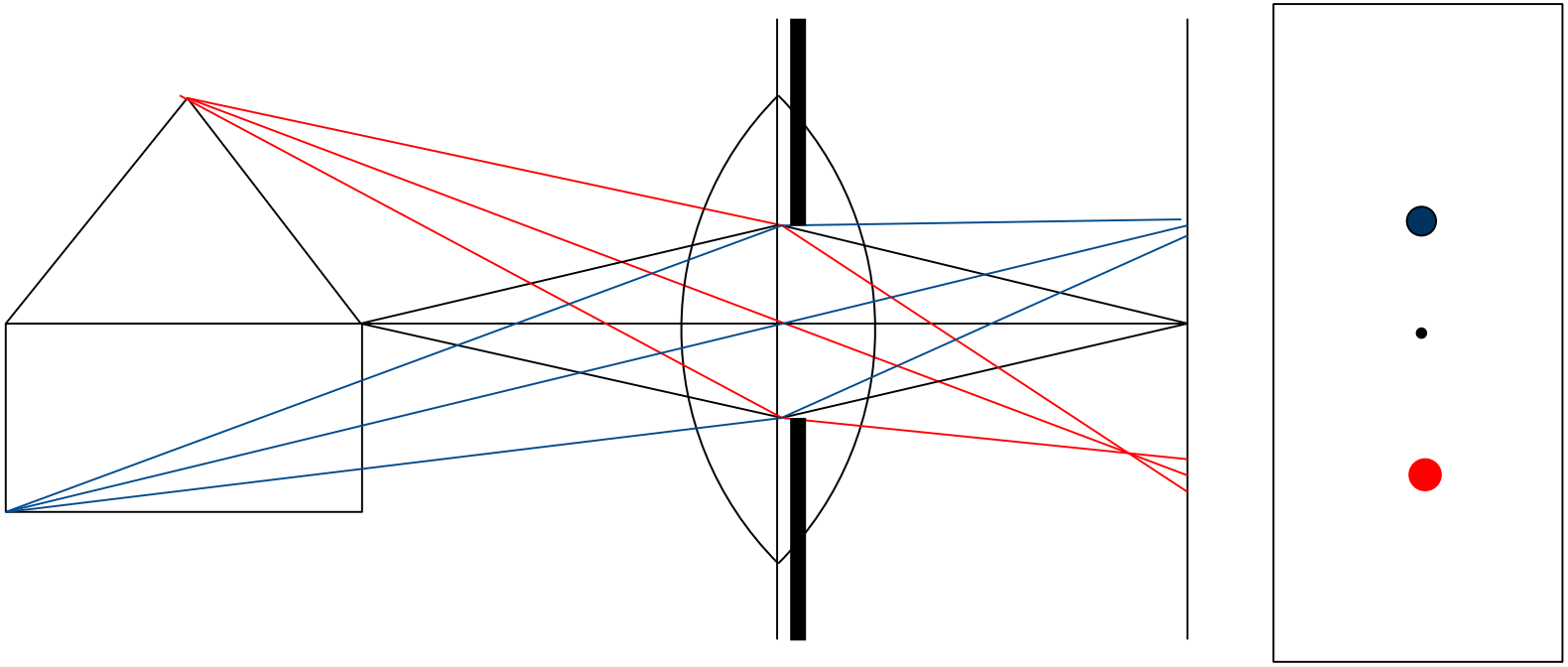


- A smaller aperture increases the range in which the object is approximately in focus
- But a small aperture reduces the amount of light that reaches the sensor (larger exposure time needed)

Field of view/Angle of view

84 deg 63 deg

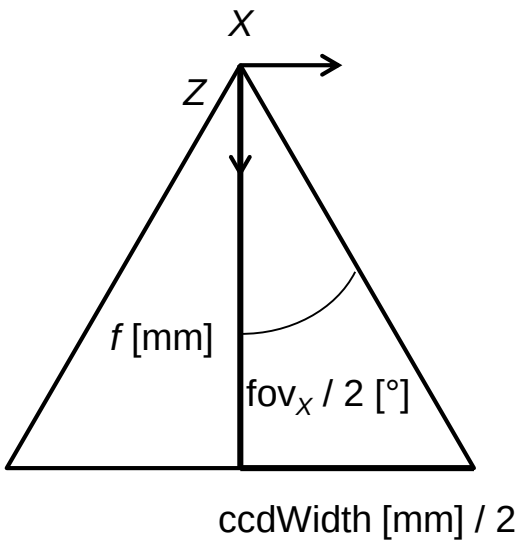
47 deg 34 deg

Source: Wikipedia (Public domain)

Field of view (FOV) calculation

- FOV depends on focal length and chip size

fov_X [rad]
focal length *f* [mm]
ccdWidth [mm]

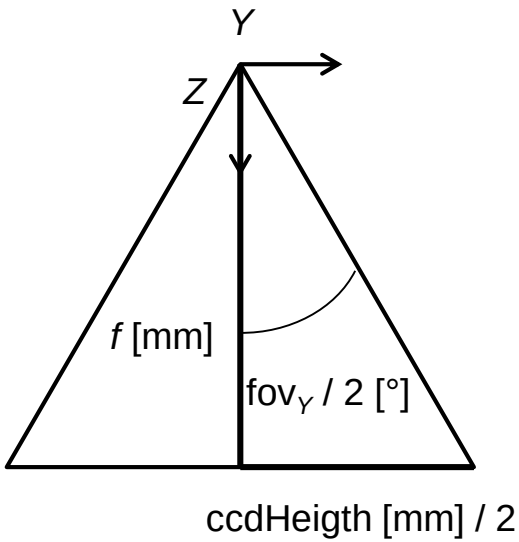


$$fov_X = 2 * \tan^{-1}((ccdWidth / 2) / f)$$

Field-of-view (FOV)

- CCD chip is not quadratic, FOV is different in x/y direction

fov_Y [rad]
focal length *f* [mm]
ccdHeight [mm]



$$fov_Y = 2 * \tan^{-1}((ccdHeigth / 2) / f)$$

Lens distortions - Radial and tangential

Lens distortions – Radial and tangential

Strong radial distortion

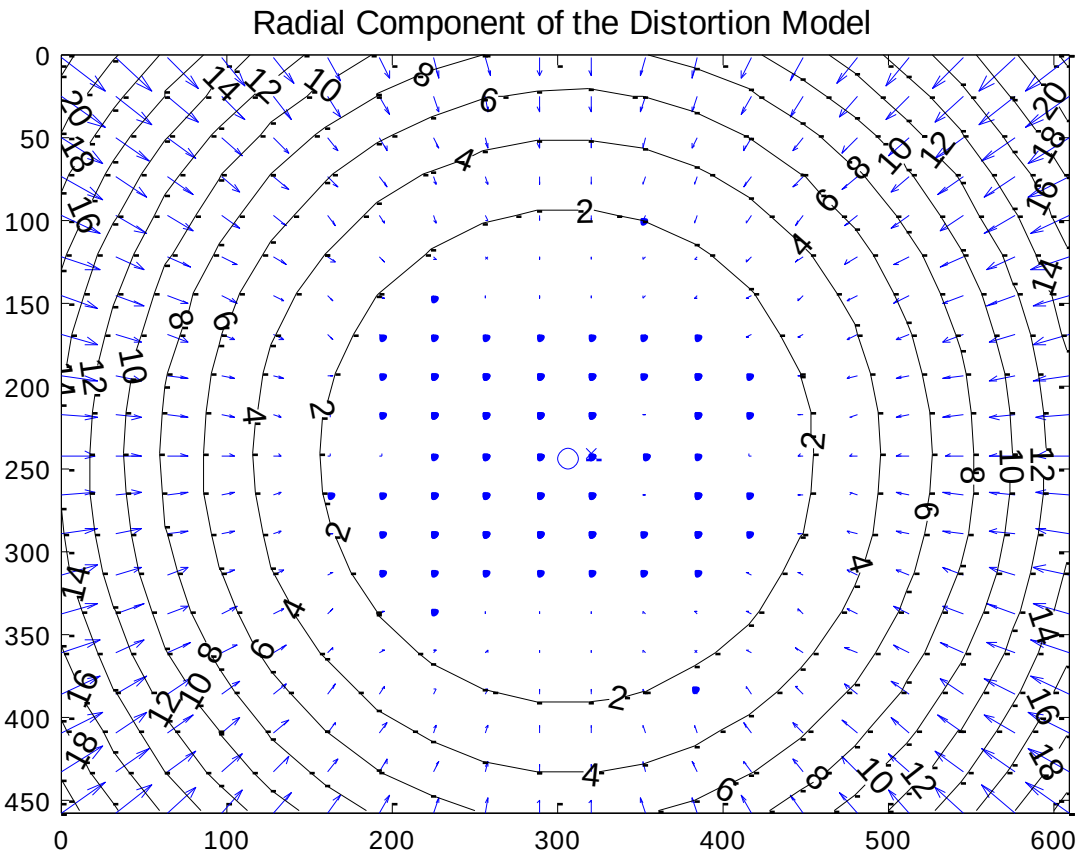
Mathematical model for radial distortion

$$x_{corrected} = x(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$

$$y_{corrected} = y(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$

- $x_{corrected}$, $y_{corrected}$ … undistorted normalized image coordinate
- x,y … normalized measured image coordinate (distorted coordinate)

Example values



Pixel error	= [0.5906, 0.4218]	
Focal Length	= (662.495, 664.678)	+/- [1.434, 1.543]
Principal Point	= (306.513, 241.751)	+/- [2.835, 2.608]
Skew	= 0	+/- 0
Radial coefficients	= (-0.2791, 0.3203, 0)	+/- [0.01144, 0.04729, 0]
Tangential coefficients	= (0.0005044, 0.0002783)	+/- [0.0006436, 0.0006694]

Mathematical model for tangential distortion

$$x_{corrected} = x + [2p_1 y + p_2(r^2 + 2x^2)]$$

$$y_{corrected} = y + [p_1(r^2 + 2y^2) + 2p_2 x]$$

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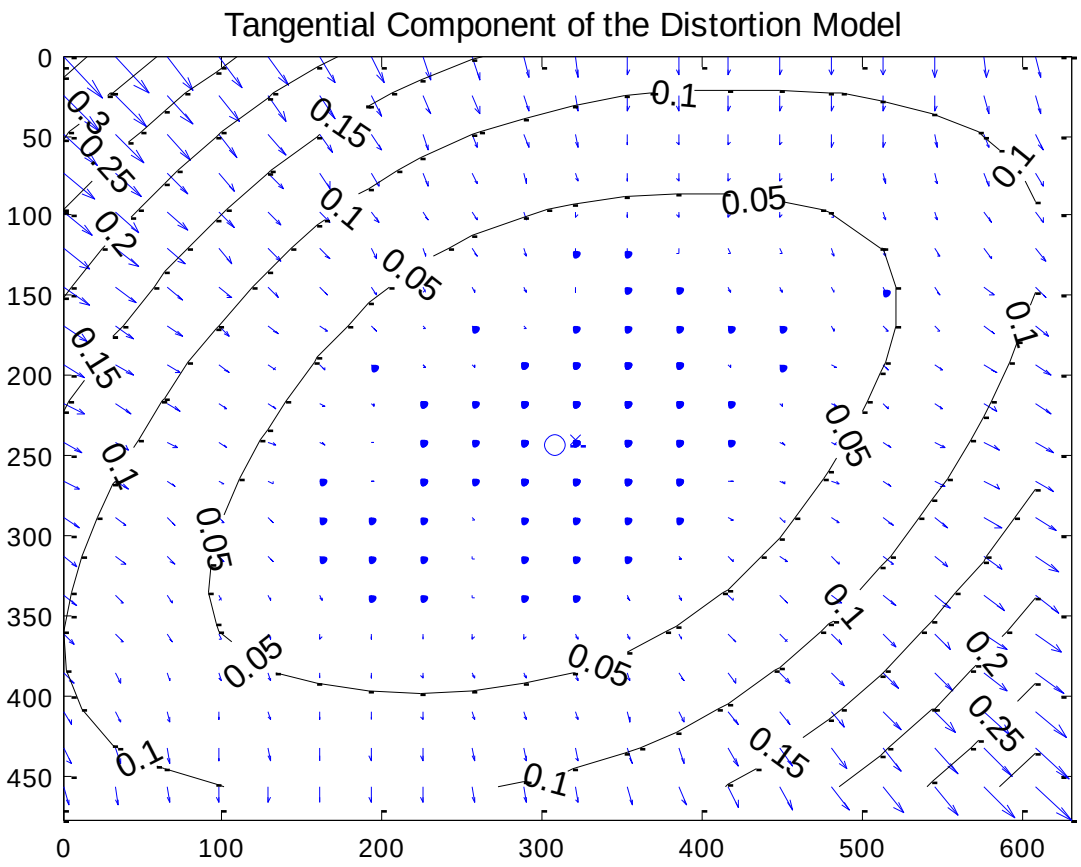
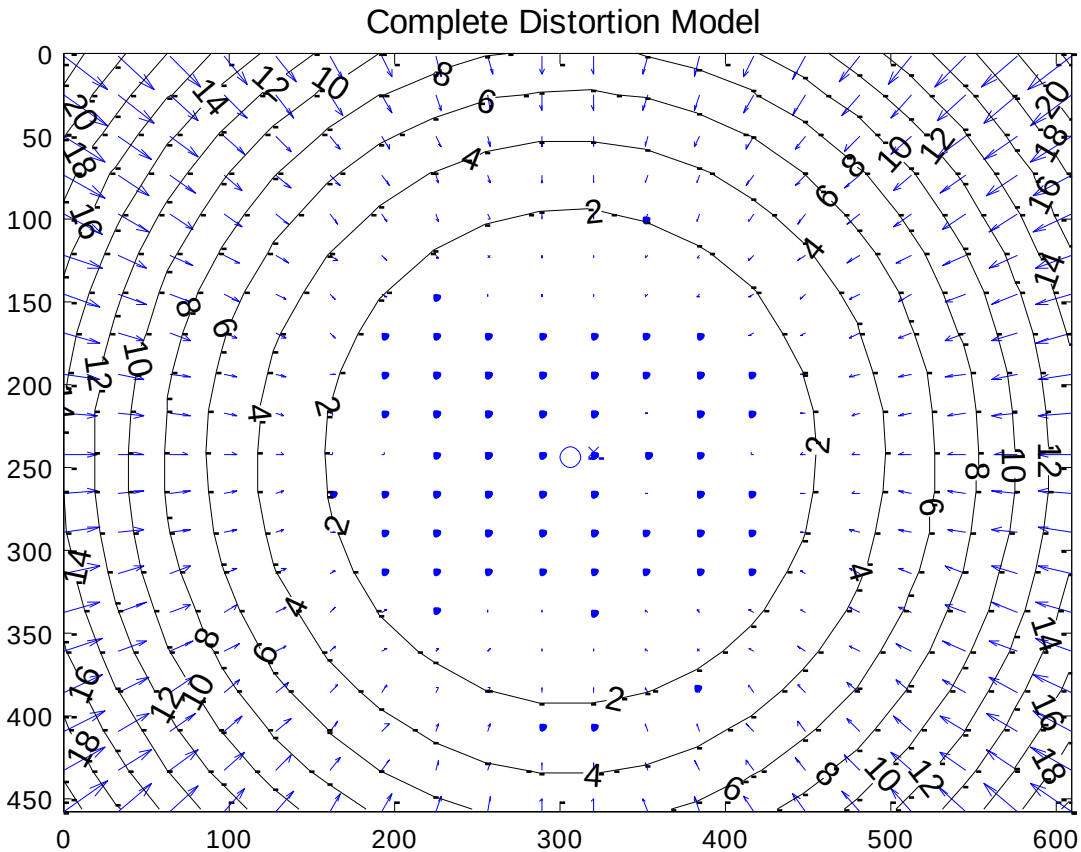
Example values



Example values



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Undistort images

$$\begin{pmatrix} x_{corrected} \\ y_{corrected} \\ 1 \end{pmatrix} = K\, rect\left(K^{-1} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \right)$$

- $x_{corrected}$, $y_{corrected}$ … undistorted normalized image coordinate
- x,y … normalized measured image coordinate (distorted coordinate)

- To render an undistorted image it is best to use target-to-source warping, but this needs the inverse of the undistortion function (does not exist in closed form)
- Many geometric algorithms just undistort a few feature points

The calibration procedure

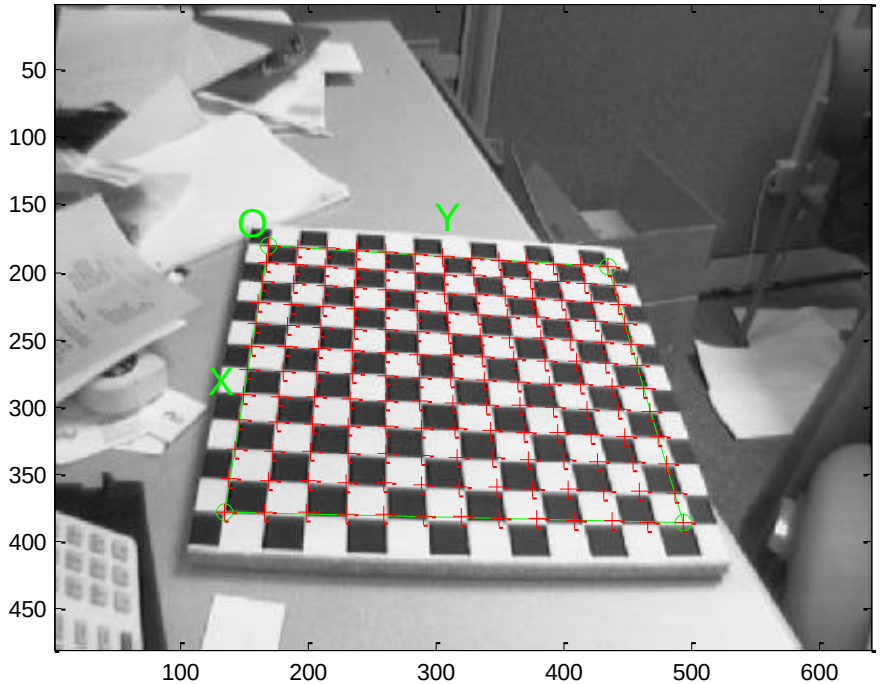
- Calibration needs to estimate intrinsics and distortion paramters
  - Intrinsics (f_x,f_y,c_y,c_y)
  - Distortion parameters (k_1,k_2,k_3,p_1,p_2)

- Method:
  1. Estimate intrinsics first assuming that there are no distortions (does not work for images with strong distortions)
  2. Estimate distortion parameters with fixed values for intrinsic
  3. Refine all estimates at the same time using non-linear optimization

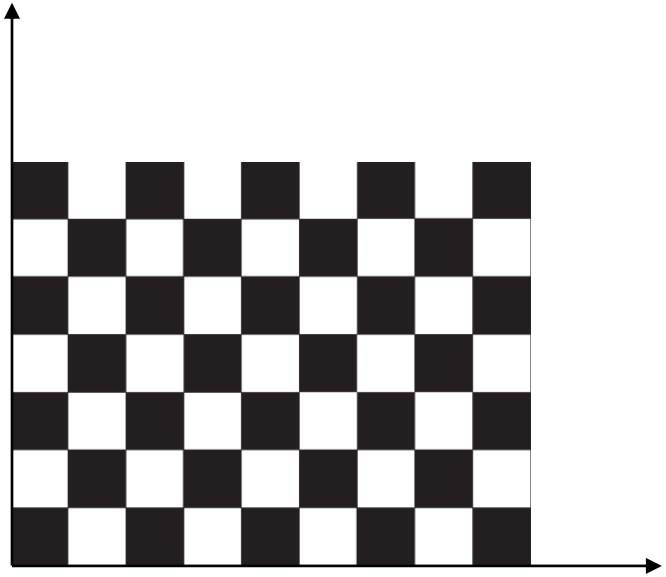
Calibration from planar target

- Method from Zhang 1999, Flexible Calibration by Viewing a Plane From Unknown Orientations

image



object with known dimensions



$$x = K[R\ t]X$$

$$\min_{K,R,t} \sum_{i=1}^{n} \sum_{j=1}^{m} \left\| x_{ij} - P_j X_i \right\|^2$$

Calibration from planar target: Algorithm

1. Find all matches between x and X in all images
2. Compute homographies between x and X
3. Compute initial values for intrinsics from these homographies (see Zhang) by solving a linear equation system ignoring distortion
4. Estimate distortion using non-linear optimization using fixed intrinsics (can start optimization using 0 values for distortion)

$$\min_{p} \sum_{i=1}^{n} \sum_{j=1}^{m} \left\| x_{ij} - rect(P_j X_i; p) \right\|^2$$

5. Re-estimate all parameters (distortion and instrinsics) using non-linear optimization

$$\min_{K,R,t,p} \sum_{i=1}^{n} \sum_{j=1}^{m} \left\| x_{ij} - rect(P_j X_i; p) \right\|^2$$