Robot Vision: Camera calibration

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SS 2024
Outline

- Camera calibration
  - Cameras with lenses
  - Properties of real lenses (distortions, focal length, field-of-view)
  - Calibration algorithm using planar targets (Zhang)
Pinhole camera and aperture

- Pinhole
- Larger aperture
- Lens
Effect of a lens

- **Thin lens model:**
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point at distance $f$
Effect of a lens

- Thin lens model:
  - Rays passing through the center are not deviated, equivalent to pinhole model
Effect of a lens

- Thin lens model:
  - Light rays originating from different depths meet at different locations, only images from specific distances are in focus
Depth of Field/Focal depth

In photography, the depth of field, or depth of focus, is the range of distances over which an object will appear acceptably sharp. The term "focal depth" is also used. It is determined by factors such as the lens aperture and the focal length of the lens. For example, a small aperture (e.g., f/22) will result in a larger depth of field than a large aperture (e.g., f/2). If you use a lens with a hyperfocal distance, the depth of field will extend from the hyperfocal distance to infinity. For a camera with a hyperfocal distance of 15 feet, the depth of field will be from that distance to infinity.
Aperture controls depth of field
Aperture controls depth of field

- A smaller aperture increases the range in which the object is approximately in focus
- But a small aperture reduces the amount of light that reaches the sensor (larger exposure time needed)
Field of view/Angle of view

Source: Wikipedia (Public domain)
Field of view (FOV) calculation

- FOV depends on focal length and chip size

\[ \text{fov}_X \text{ [rad]} \]

focal length \( f \text{ [mm]} \)
ccdWidth [mm]

\[ \text{fov}_X = 2 \times \tan^{-1}\left(\frac{\text{ccdWidth}/2}{f}\right) \]
Field-of-view (FOV)

- CCD chip is not quadratic, FOV is different in x/y direction

\[
\text{fov}_Y \text{ [rad]}
\]

\[
\text{focal length } f \text{ [mm]}
\]

\[
\text{ccdHeight [mm]}
\]

\[
\text{fov}_Y = 2 \times \tan^{-1} \left( \frac{\text{ccdHeight} / 2}{f} \right)
\]
Lens distortions - Radial and tangential
Lens distortions: Radial and tangential
Strong radial distortion
Mathematical model for radial distortion

\[
x_{\text{corrected}} = x(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)
\]
\[
y_{\text{corrected}} = y(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)
\]

- $x_{\text{corrected}}$, $y_{\text{corrected}}$ … undistorted normalized image coordinate
- $x,y$ … normalized measured image coordinate (distorted coordinate)
- $r$ .. distance to image center \( r^2 = x^2 + y^2 \)
- k-values .. distortion coefficients
Example values

Pixel error                      = [0.5906, 0.4218]
Focal Length                 = (662.495, 664.678)
Principal Point               = (306.513, 241.751)
Skew                              = 0
Radial coefficients         = (-0.2791, 0.3203, 0)
Tangential coefficients  = (0.0005044, 0.0002783)  
                             +/- [0.01144, 0.04729, 0]
                             +/- [0.0006436, 0.0006694]
Mathematical model for tangential distortion

\[
x_{\text{corrected}} = x + [2p_1y + p_2(r^2 + 2x^2)] \\
y_{\text{corrected}} = y + [p_1(r^2 + 2y^2) + 2p_2x]
\]

- \( x_{\text{corrected}}, y_{\text{corrected}} \) … undistorted normalized image coordinate
- \( x, y \) … normalized measured image coordinate (distorted coordinate)
- \( r \) … distance to image center \( r^2 = x^2 + y^2 \)
- \( p \)-values … distortion coefficients
Example values

Tangential Component of the Distortion Model

Pixel error: $[0.5906, 0.4218]$
Focal Length: $(662.495, 664.678)$
Principal Point: $(306.513, 241.751)$
Skew: $0$
Radial coefficients: $(-0.2791, 0.3203, 0)$
Tangential coefficients: $(0.0005044, 0.0002783)$

$\pm [1.434, 1.543]$
$\pm [2.835, 2.608]$
$\pm 0$
$\pm [0.01144, 0.04729, 0]$
$\pm [0.0006436, 0.0006694]$
Example values

Pixel error = [0.5906, 0.4218]
Focal Length = (662.495, 664.678)
Principal Point = (306.513, 241.751)
Skew = 0
Radial coefficients = (-0.2791, 0.3203, 0)
Tangential coefficients = (0.0005044, 0.0002783)

Complete Distortion Model
Undistort images

\[
\begin{pmatrix}
  x_{\text{corrected}} \\
  y_{\text{corrected}} \\
  1
\end{pmatrix}
= K \text{ rect} \left( \begin{pmatrix}
  x \\
  y \\
  1
\end{pmatrix}
\right)
\]

- \( x_{\text{corrected}}, y_{\text{corrected}} \) … undistorted normalized image coordinate
- \( x, y \) … measured image coordinate (distorted coordinate)

- To render an undistorted image it is best to use target-to-source warping, but this needs the inverse of the undistortion function (does not exist in closed form)
- Many geometric algorithms just undistort a few feature points
The calibration procedure

- Calibration needs to estimate intrinsics and distortion parameters
  - Intrinsics \((f_x, f_y, c_x, c_y)\)
  - Distortion parameters \((k_1, k_2, k_3, p_1, p_2)\)

- Method:
  1. Estimate intrinsics first assuming that there are no distortions (does not work for images with strong distortions)
  2. Estimate distortion parameters with fixed values for intrinsic
  3. Refine all estimates at the same time using non-linear optimization
Calibration from planar target

- Method from Zhang 1999, Flexible Calibration by Viewing a Plane From Unknown Orientations

\[ x = K[R \ t]X \]

\[ \min_{K, R, t} \sum_{i=1}^{n} \sum_{j=1}^{m} \left\| x_{ij} - P_j X_i \right\|^2 \]
Calibration from planar target: Algorithm

1. Find all matches between x and X in all images
2. Compute homographies between x and X (represent camera poses)
3. Compute initial values for intrinsics from these homographies (see Zhang) by solving a linear equation system ignoring distortion
4. Estimate distortion using non-linear optimization using fixed intrinsics (can start optimization using 0 values for distortion)

\[
\min_p \sum_{i=1}^{n} \sum_{j=1}^{m} \left\| x_{ij} - \text{dist}(P_j X_i; p) \right\|^2
\]

5. Re-estimate all parameters (distortion and intrinsics) using non-linear optimization

\[
\min_{K,R,t,p} \sum_{i=1}^{n} \sum_{j=1}^{m} \left\| x_{ij} - \text{dist}(P_j X_i; p) \right\|^2
\]