Robot Vision:
Structure-from-Motion (SFM)

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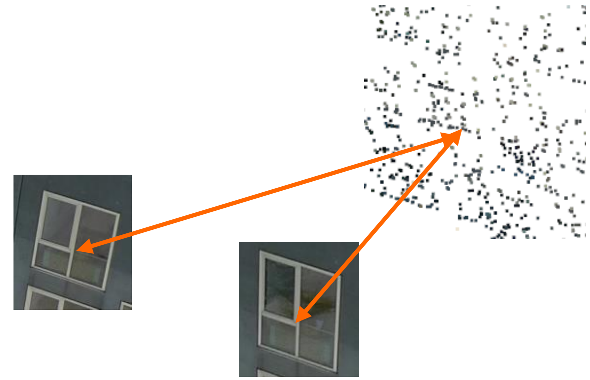
Outline

- SfM concept
- SfM pipeline
- Image similarity using visual words
- Incremental geometry estimation
- Bundle adjustment

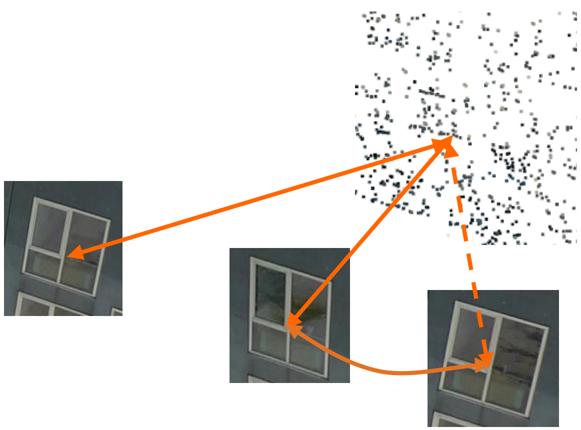
Structure-from-Motion (SfM) concept

Structure-from-Motion (SfM) concept

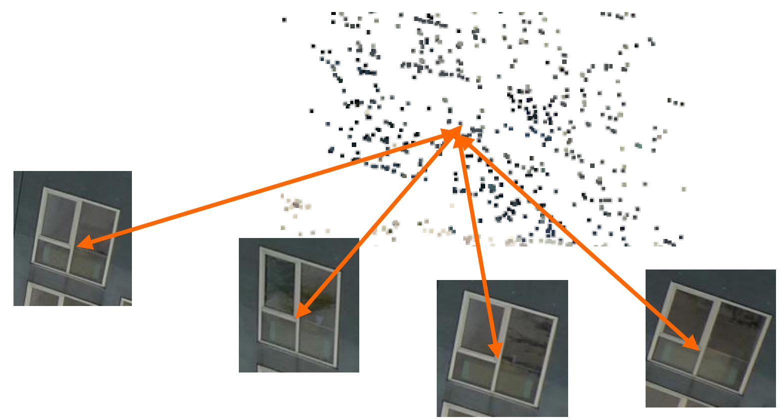
Initialize Motion
(P_1,P_2 compatible with F)



Initialize Structure
(minimize reprojection error)

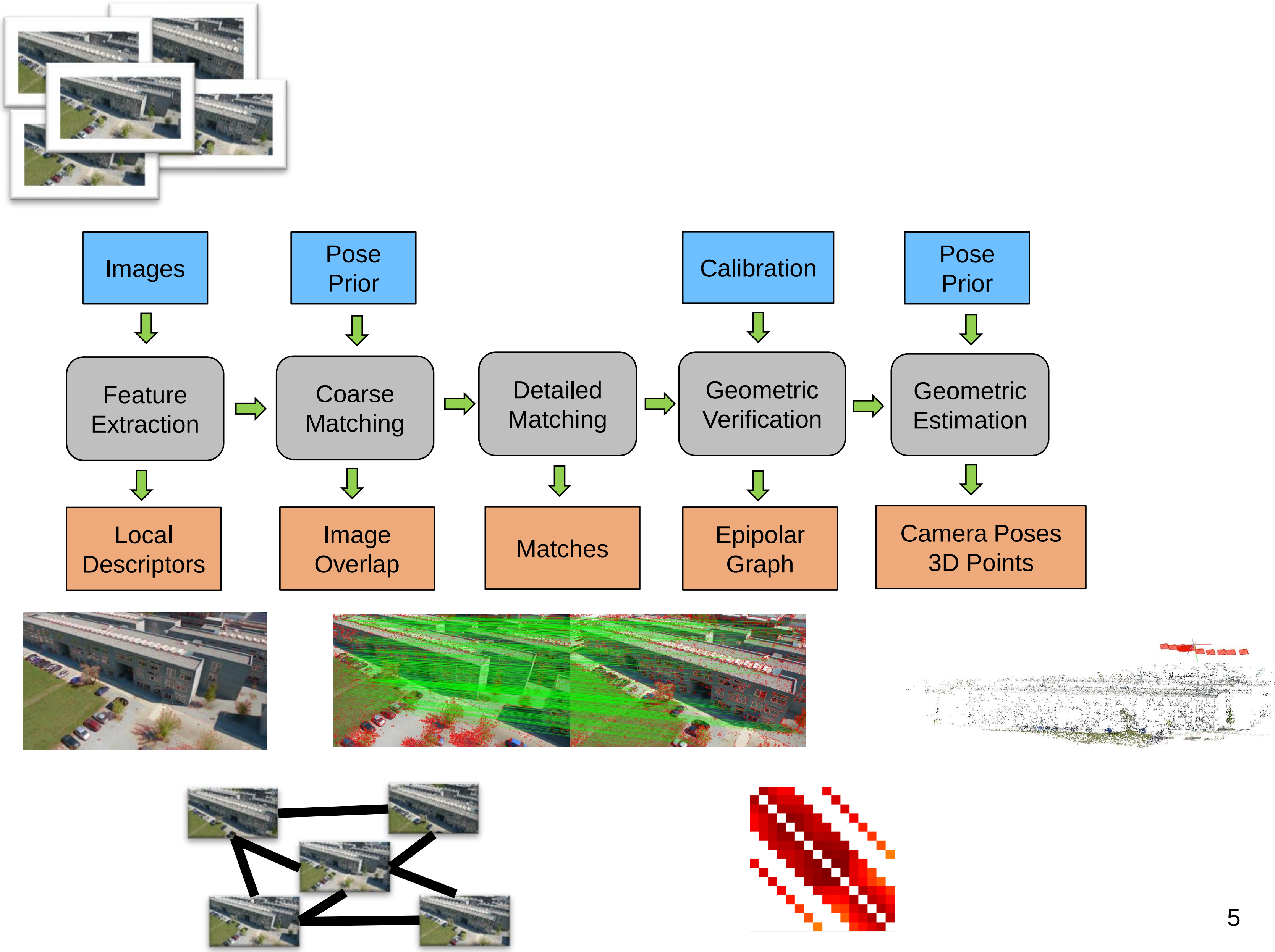


Extend motion
(compute pose through matches
seen in 2 or more previous views)



Extend structure
(Initialize new structure,
refine existing structure)

Structure-from-Motion (SfM) core pipeline

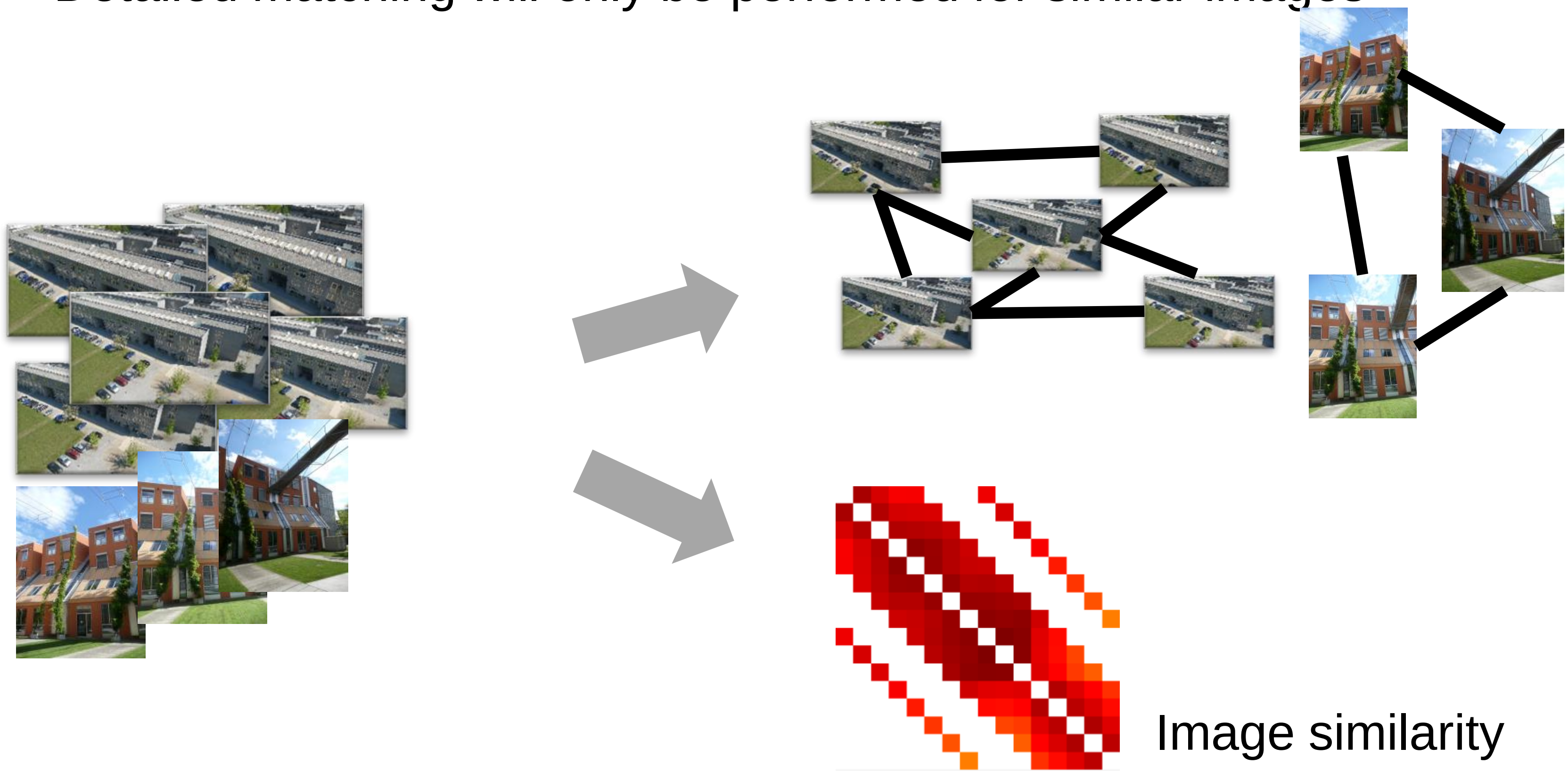


Feature extraction

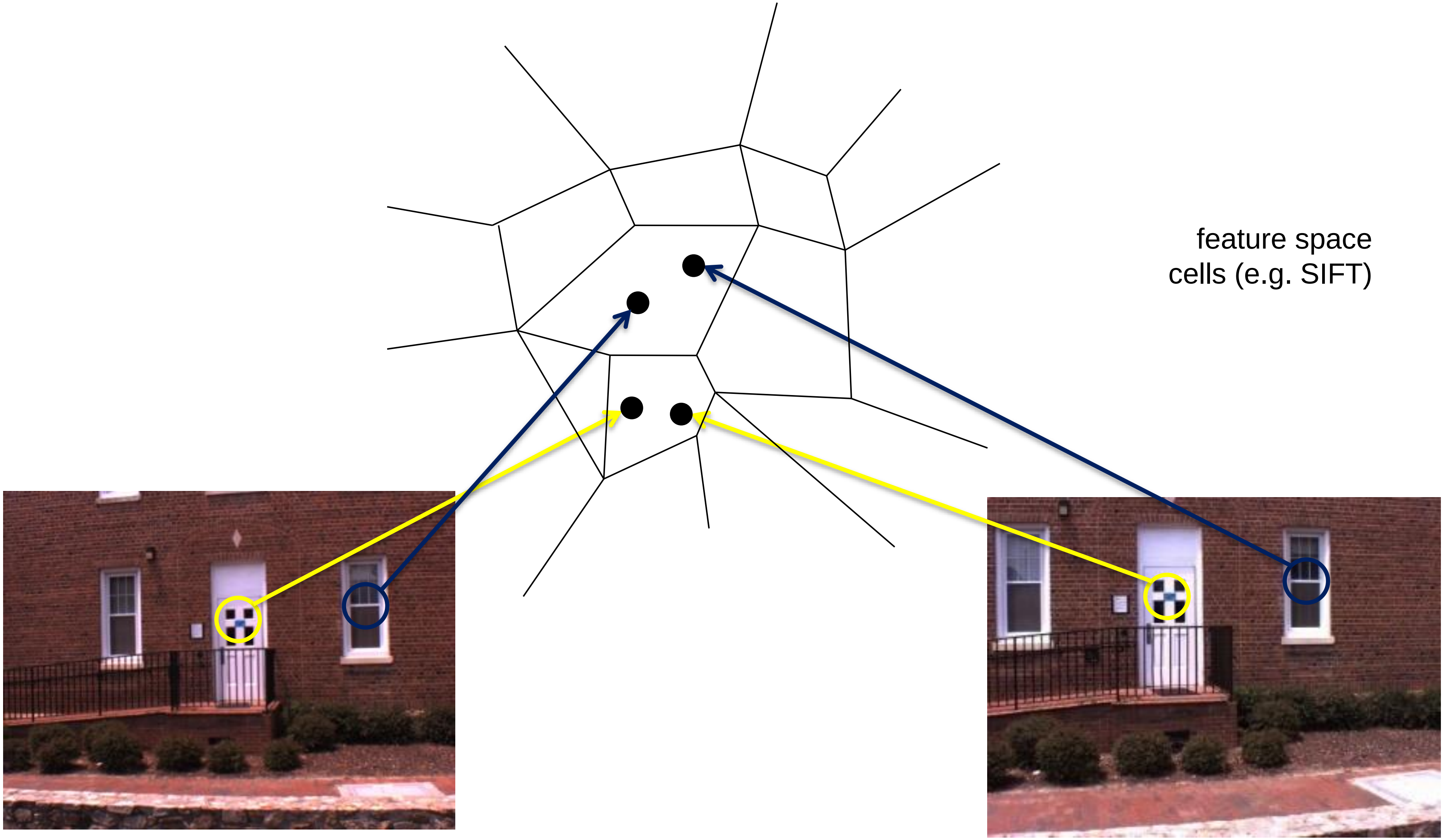
- Extract features (point locations and descriptors) for each of the N images
- SIFT features are recommended (best working features for matching right now)
- GPU accelerated implementations exist

Coarse matching

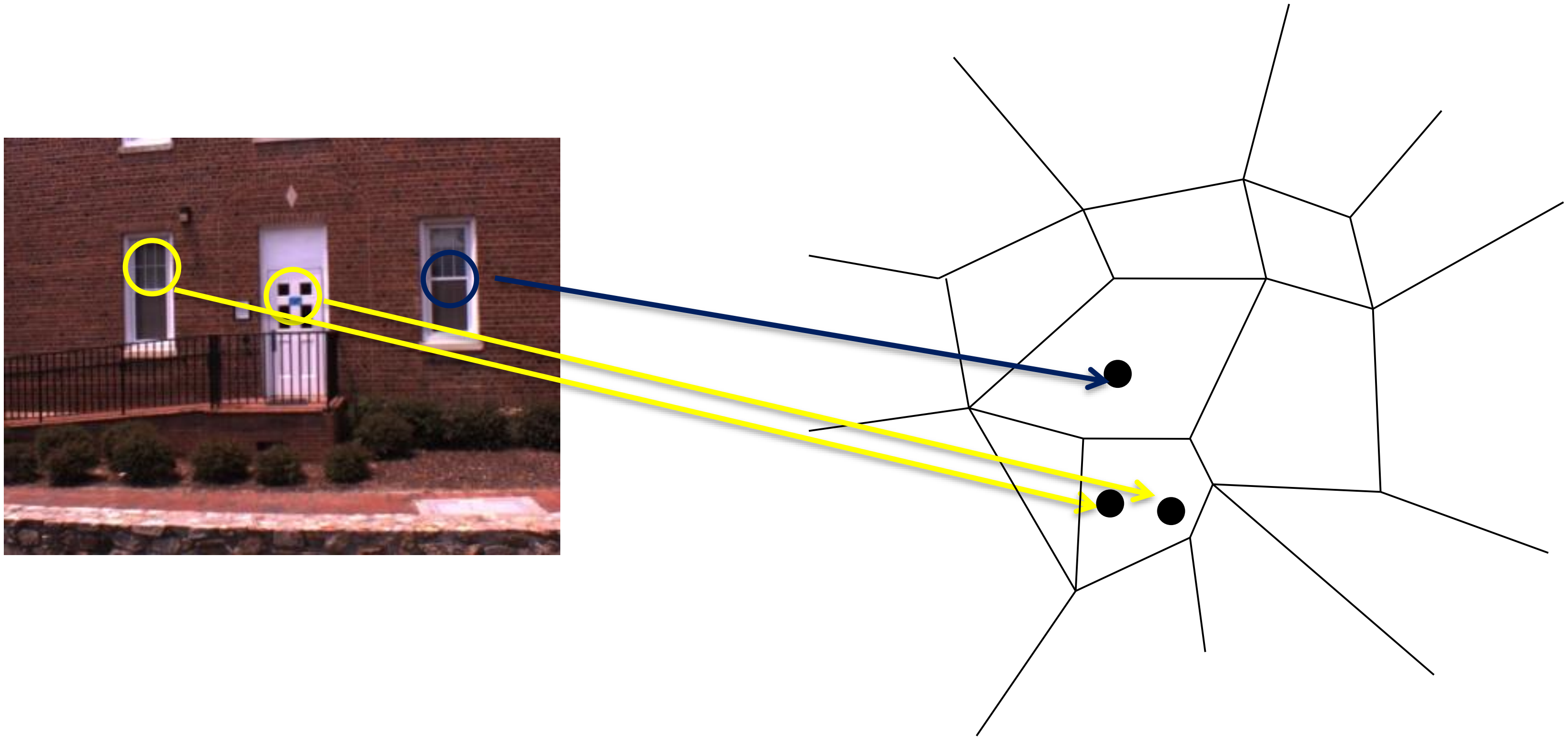
- To avoid NxN feature matching
- Many possible image pairs in the dataset will not have overlap, detailed feature matching will produce no matches for such pairs
- Cluster similar images by similarity using visual words
- Detailed matching will only be performed for similar images



Visual words



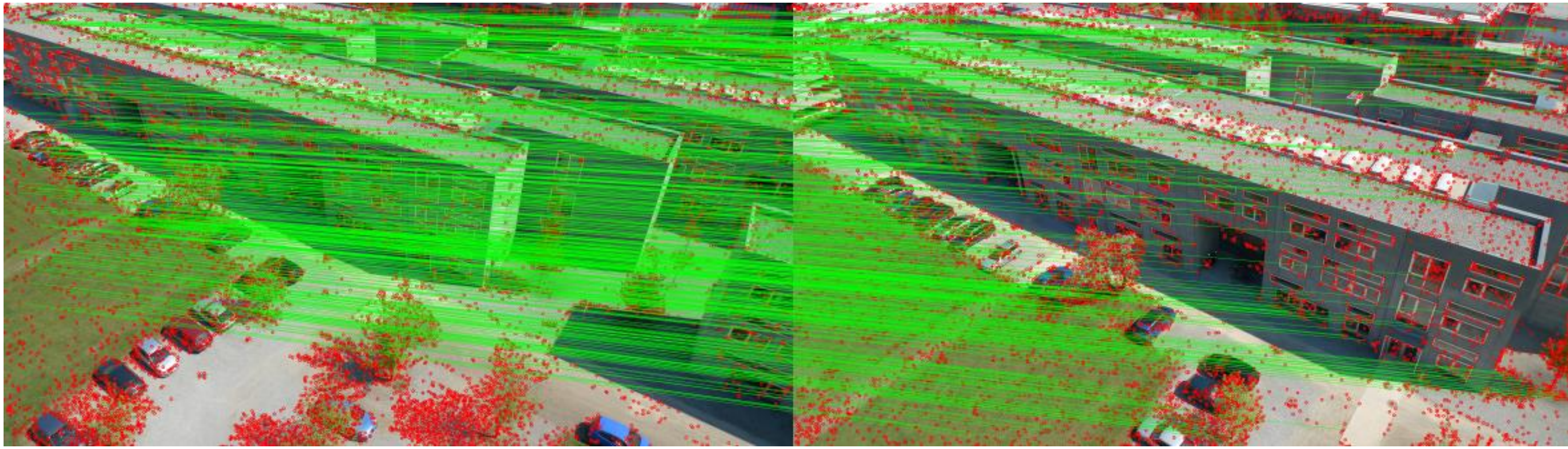
Histogram of visual words (bags of words)

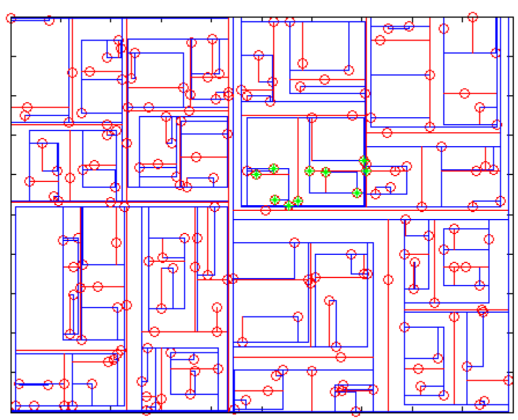


0		100	101	102	103	104	105	106	107	108	109	110		999999
0		0	2	1	0	0	0	0	0	0	0	0		0

Detailed matching

- Typically using an approximated nearest neighbor (ANN) algorithm





Geometric verification and epipolar graph

- Geometric verification of 2-view matches using fundamental matrix or essential matrix computation
- Epipolar graph: Is a plot of the number of geometrically verified 2-view feature matches
- Defines the sequential order for geometry processing

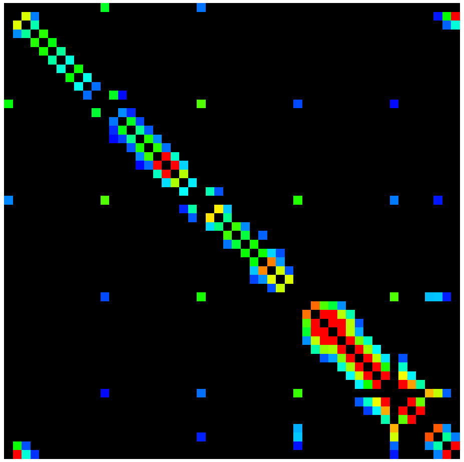
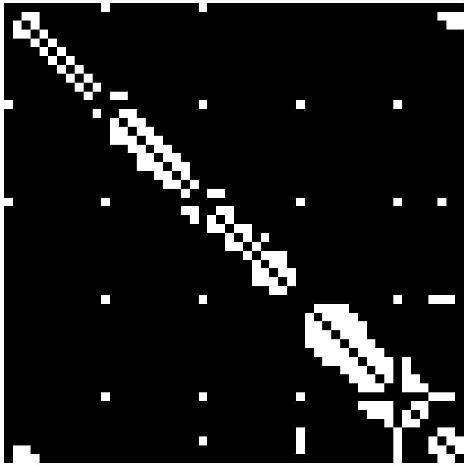


Image similarity



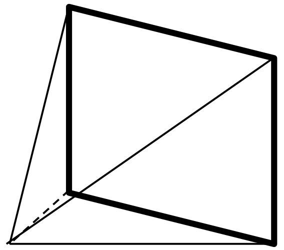
Epipolar graph

Geometry estimation

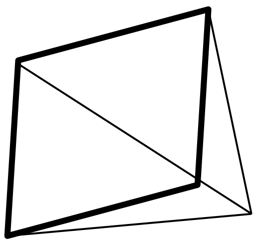
- Following the sequence ordering from the epipolar graph geometry is estimated for all images
- Geometry estimation is an alternating scheme:
  - Estimate camera pose of new images (position, rotation)
  - Triangulate new 3D data points seen in new image
  - Refinement by non-linear optimization (Bundle adjustment)

Geometry estimation steps

- Compute camera poses of the first two images from feature matches



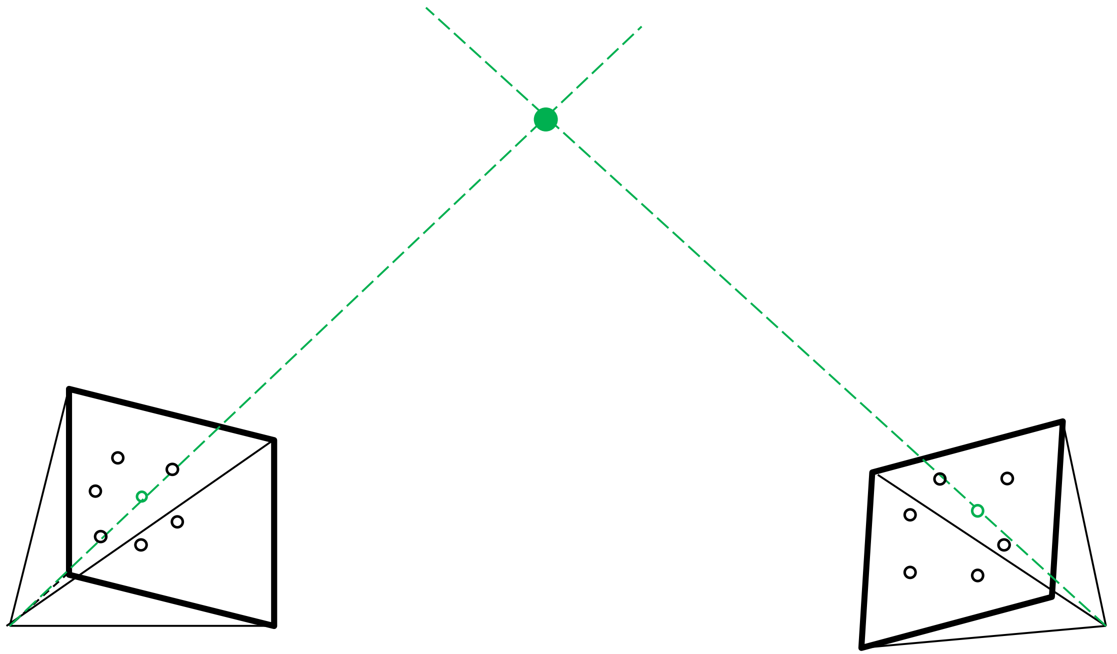
$$P = K[I|0]$$



$$P' = K'[R'|t']$$

Geometry estimation steps

- Computation of first 3D points by triangulation

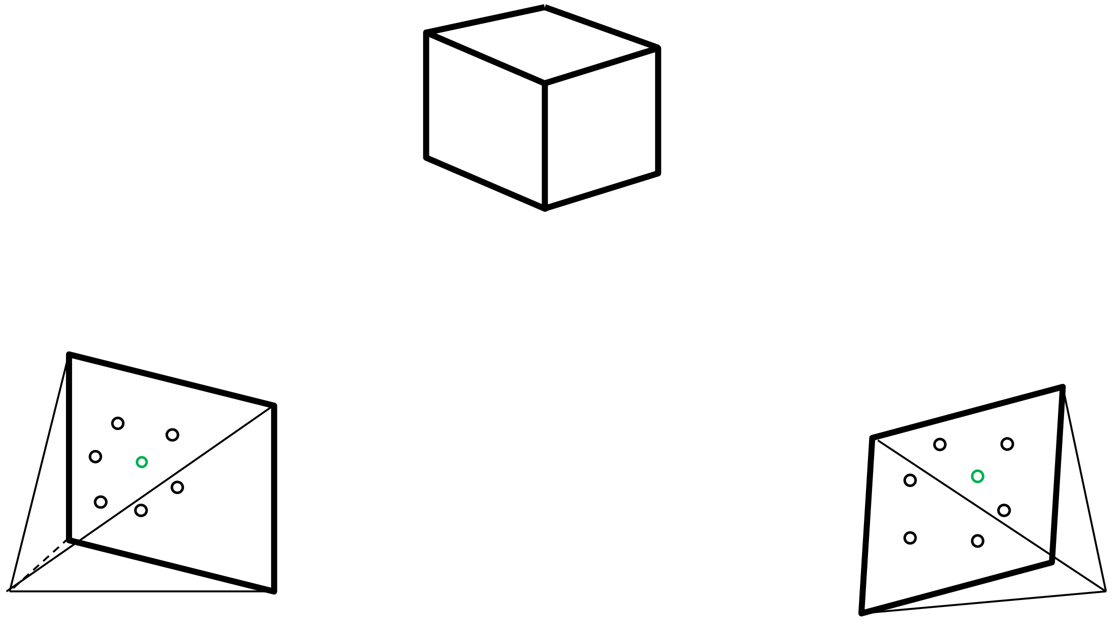


$P = K[I|0]$

$P' = K'[R'|t']$

Geometry estimation steps

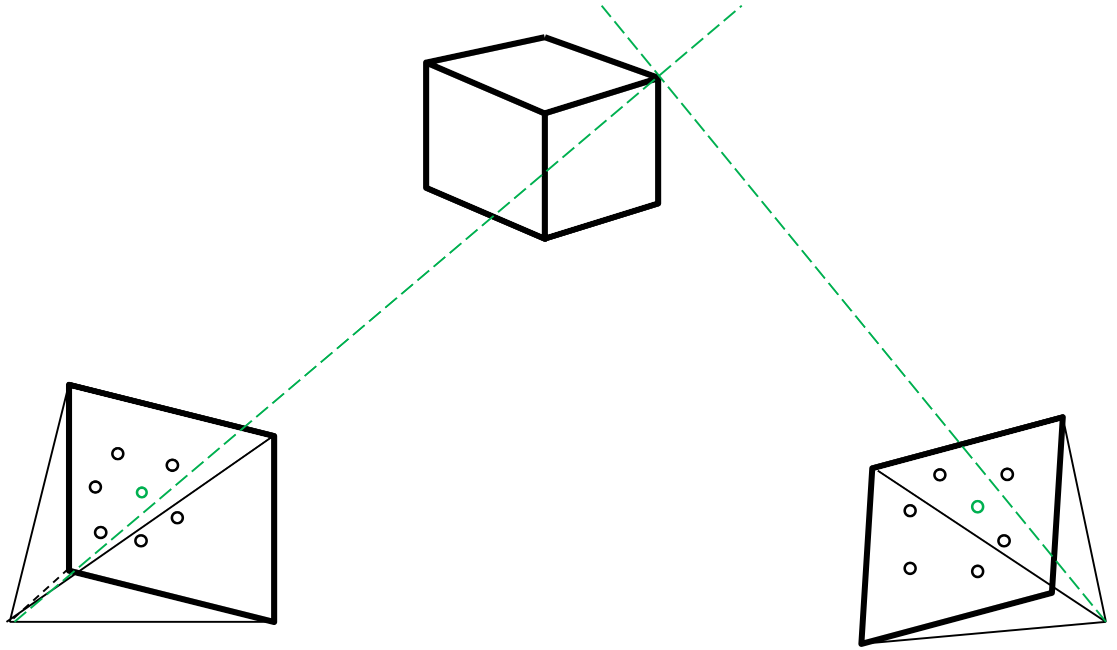
- Triangulate all feature matches of the first images



$P = K[I|0]$ $\qquad$ $P' = K'[R'|t']$

Geometry estimation steps

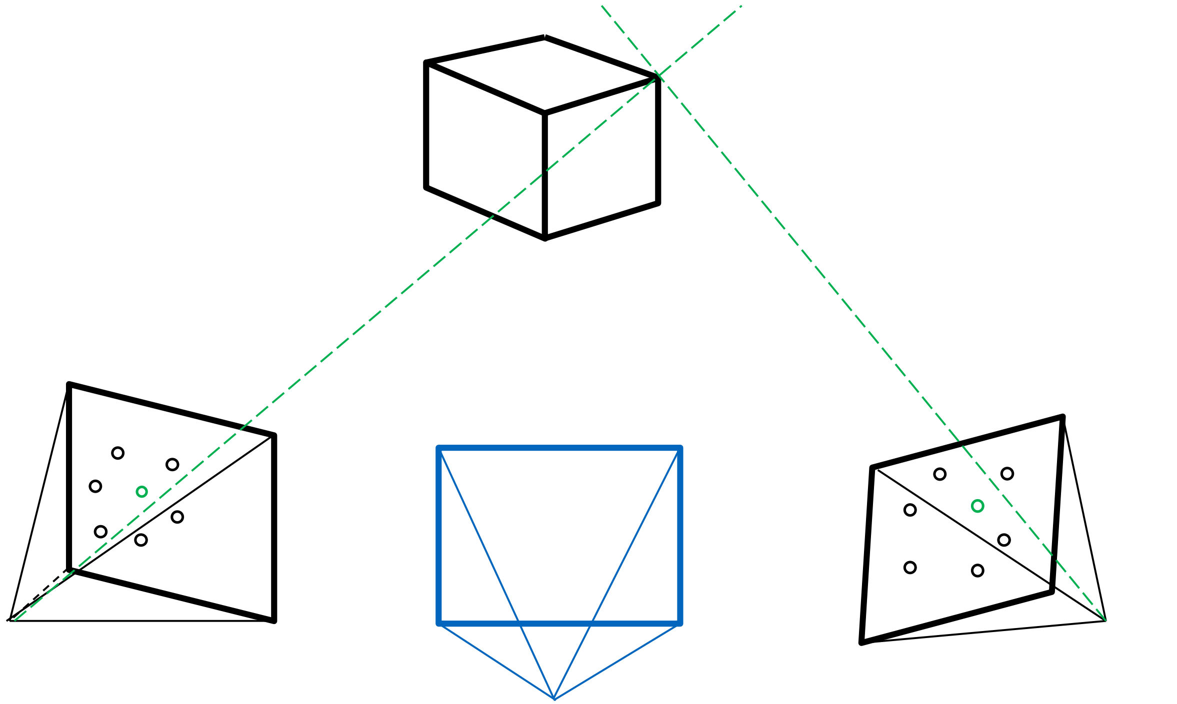
- First refinement of camera poses and 3D points by non-linear estimation of the re-projection error through bundle adjustment



$P = K[I|0]$ $\qquad$ $P' = K'[R'|t']$

Geometry estimation steps

- Start processing the next image



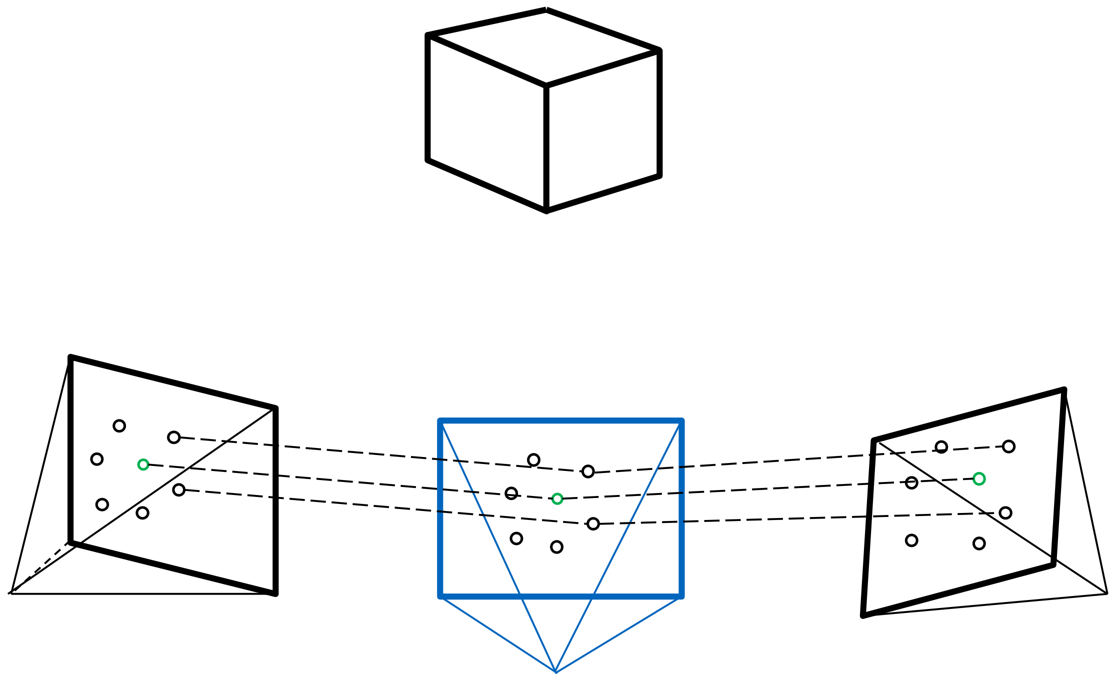
$P = K[I|0]$

$P' = K'[R'|t']$

$P'' = ?$

Geometry estimation steps

- First, create feature matches to all the previous, neighboring images

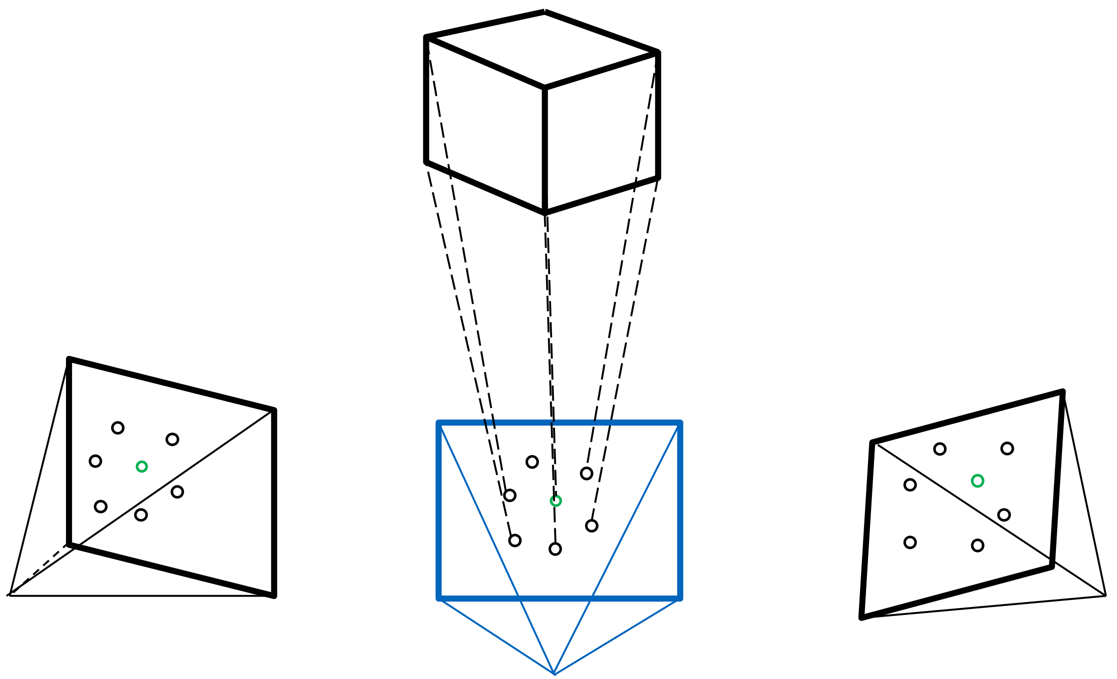


$P = K[I|0]$ $\qquad$ $P' = K'[R'|t']$

$P'' = ?$

Geometry estimation steps

- Feature matches give correspondences to already computed 3D points
- From corresponding 2D and 3D points the pose of the new camera can be computed using the PnP-Algorithm



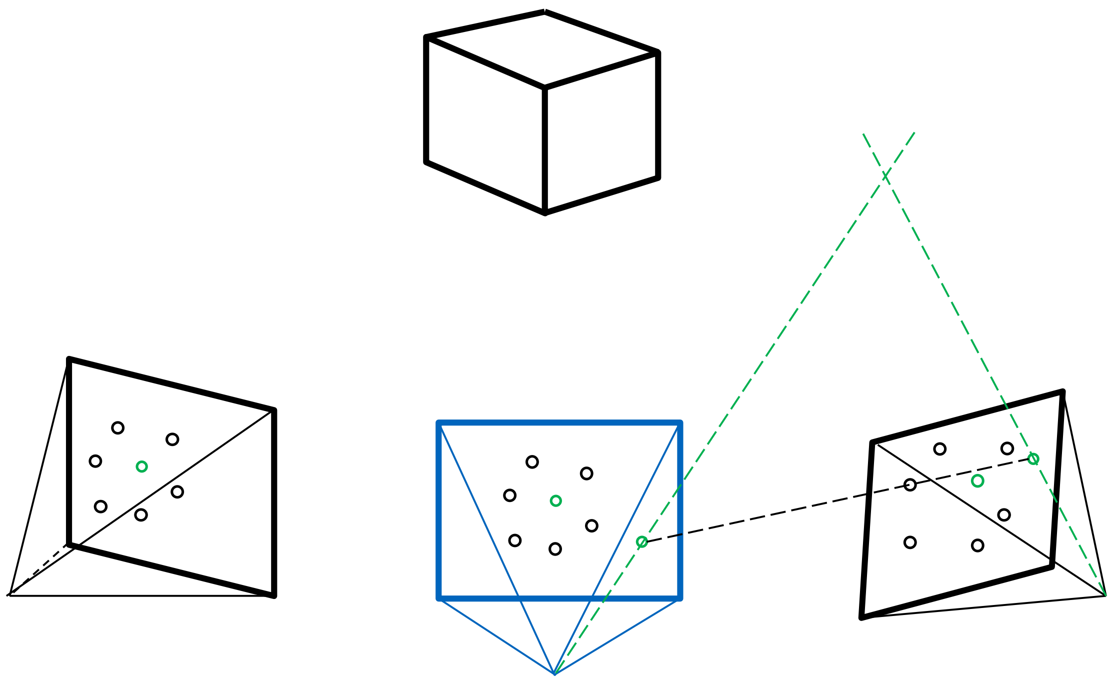
$P = K[I|0]$

$P'' = ?$

$P' = K'[R'|t']$

Geometry estimation steps

- Repeat the process starting again from triangulation of new features



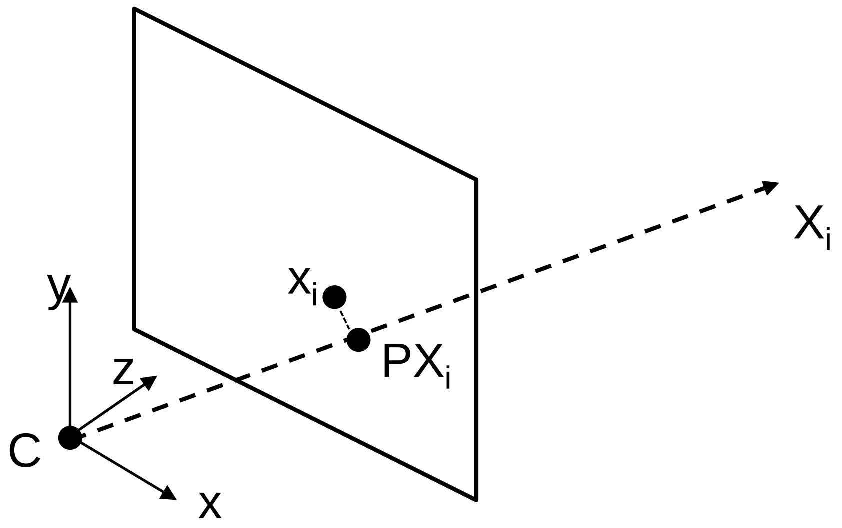
$P = K[I|0]$ $P' = K'[R'|t']$

$P'' = K''[R''|t'']$

Bundle adjustment

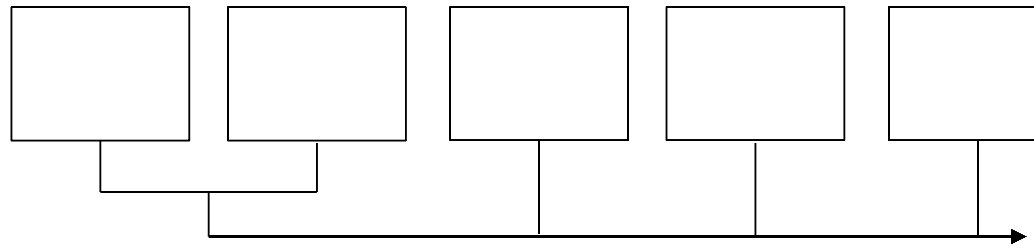
- Levenberg-Marquard optimization of re-projection error
- Parameters are camera poses and all 3D points (millions of parameters to optimize!)

$$\min_{P_j, X_i} \left(\sum_i \sum_j \| x_{i,j} - P_j X_i \| \right)$$

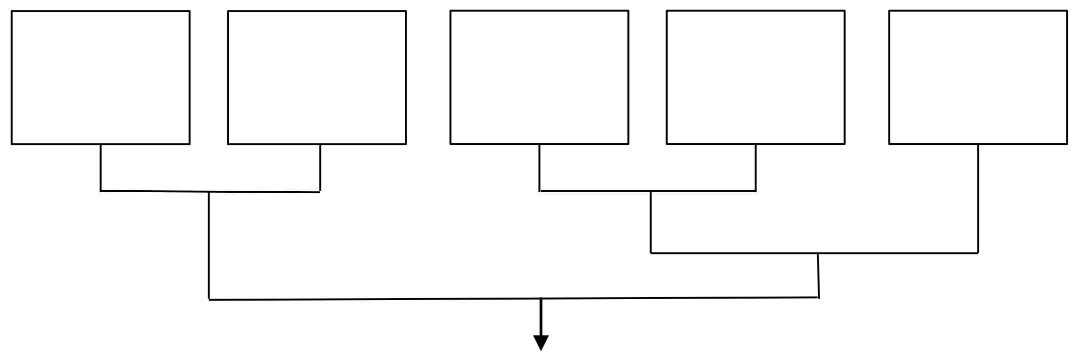


3 paradigms

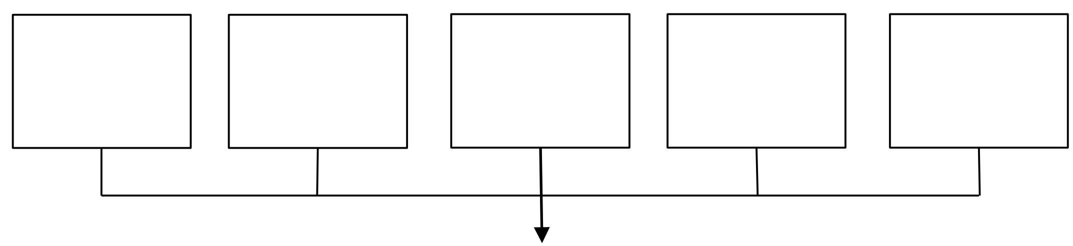
sequential
(incremental)



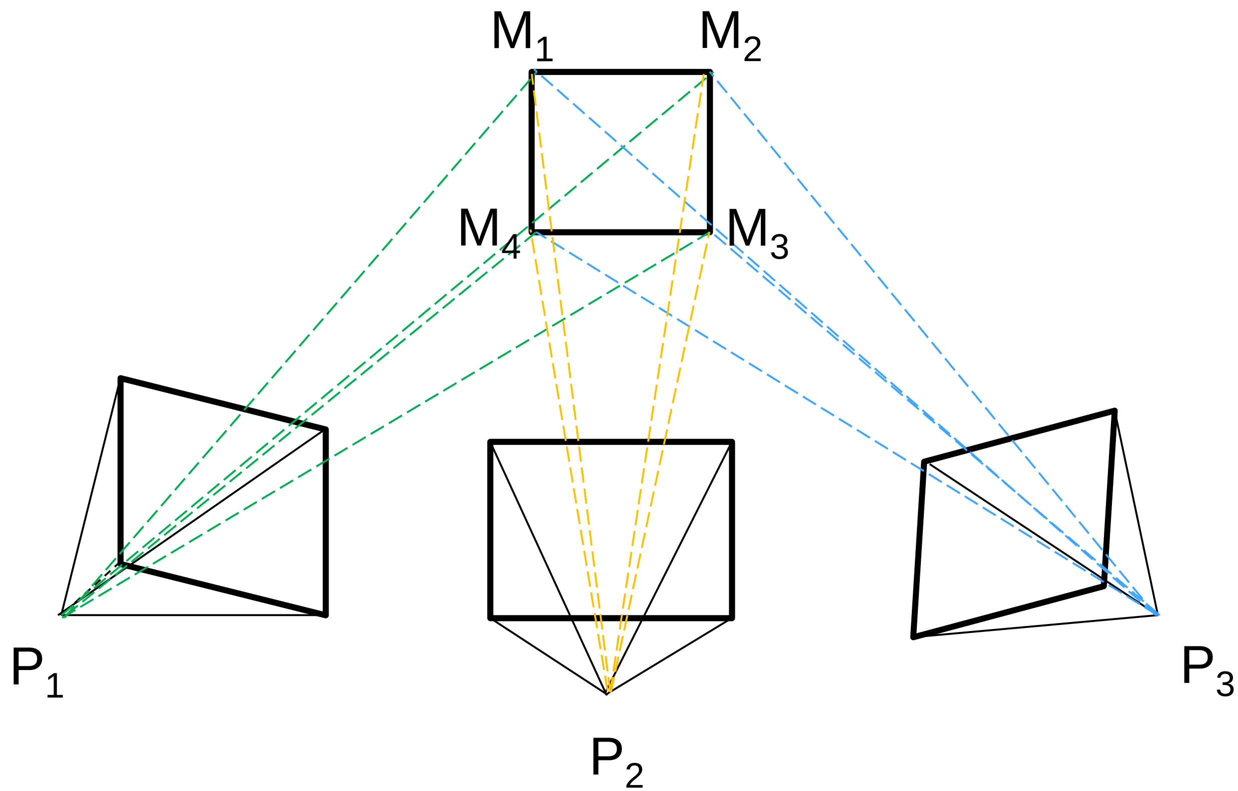
hierarchical



global



Bundle adjustment (BA)



$$\min_{P_j, M_i} \left(\sum_i \sum_j \left\| p_{i,j} - P_j M_i \right\| \right) = \varepsilon = f(x)$$

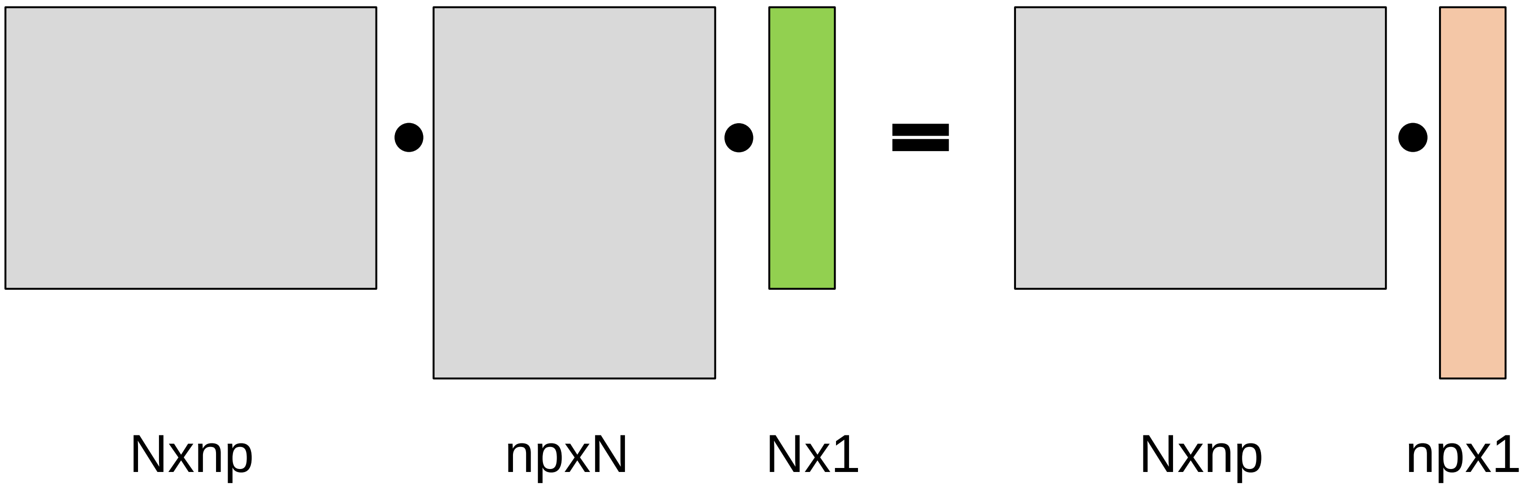
objective function to be minimized

$$x_k = x_{k-1} + d_{k-1}$$
$$J_{k-1}^T J_{k-1} d_{k-1} + J_{k-1}^T \varepsilon_{k-1} = 0$$

Gauss-Newton update equation

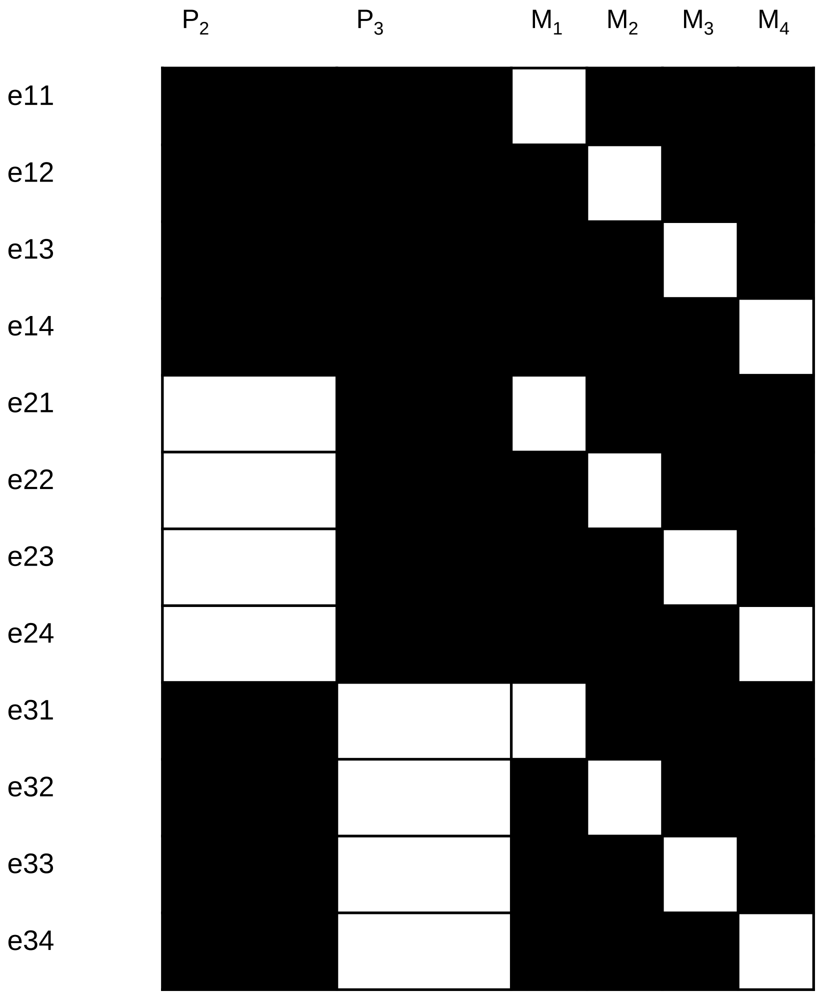
Calculating the update vector d

$$J_{k-1}^T J_{k-1} d_{k-1} = -J_{k-1}^T \varepsilon_{k-1}$$
$$Ax = b$$



- J … npxN matrix (n … #cameras, p … #points, N … #parameters)
- residual vector e is computed from e=||x-PM|| for every iteration
- Then the values for the Jacobian J are computed for every iteration

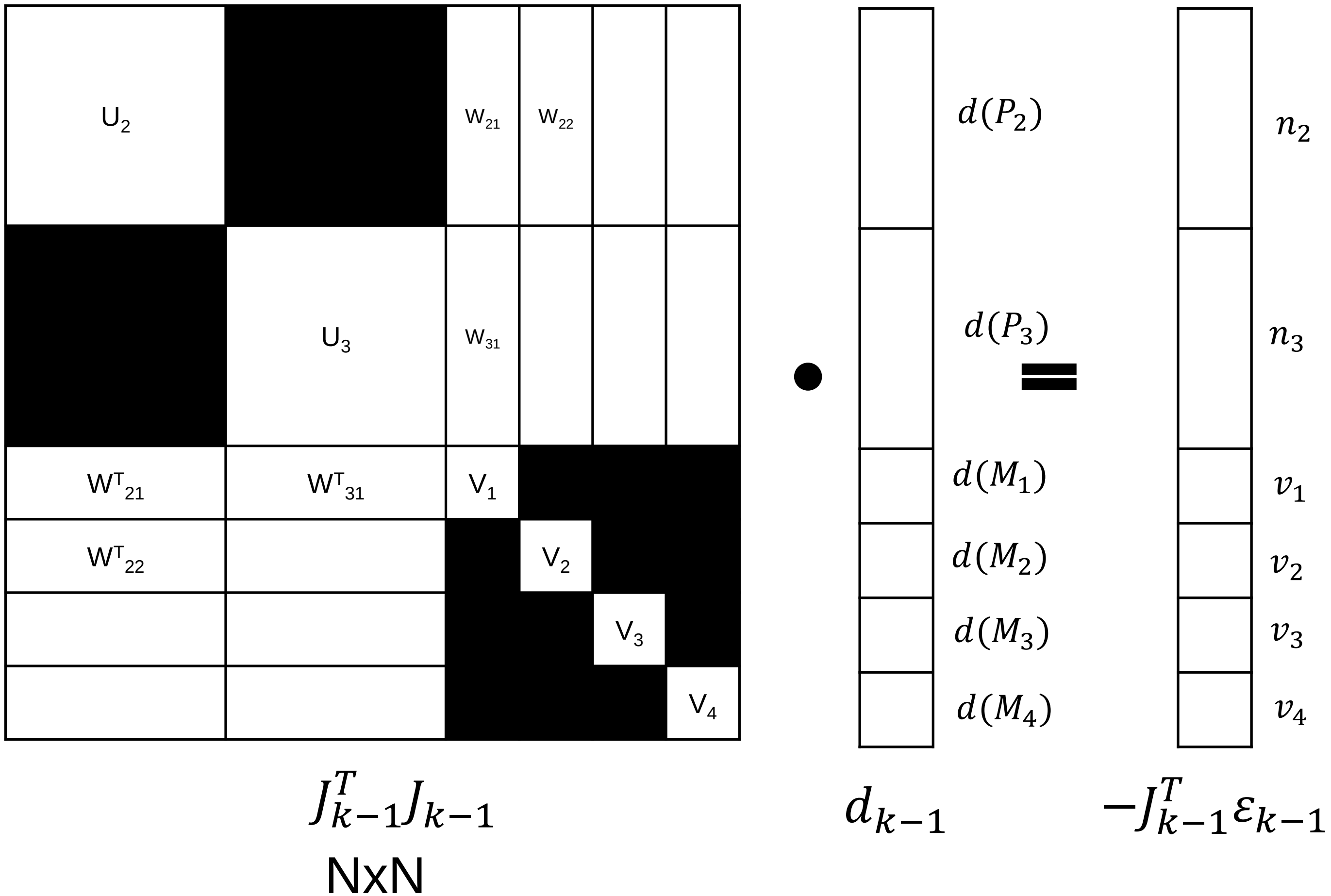
The Jacobian J (example for 3 cameras and 4 3D points)



white blocks are non-zero entries

- J … npxN matrix (n … #cameras, p … #points, N … #parameters
- M .. 1x3 matrix, P … 1x11 matrix

J^TJ



- J^TJ is called the "Hessian Matrix" (symmetric matrix)
- U … 11x11 symmetric matrix, V … 3x3 symmetric matrix
- W ... 11x3 matrix

Schur complement trick/Sparse BA

$$\begin{bmatrix} U & W \\ W^T & V \end{bmatrix} \begin{bmatrix} d(P) \\ d(M) \end{bmatrix} = \begin{bmatrix} n(P) \\ v(M) \end{bmatrix}$$

$$\begin{bmatrix} I_{11(n-1)} & -WV^{-1} \\ 0_{3px11(n-1)} & I_{3p} \end{bmatrix}$$

Multiply the above equation with this line to obtain

$$\begin{bmatrix} U - WV^{-1}W^T & 0_{3p} \\ W^T & V \end{bmatrix} \begin{bmatrix} d(P) \\ d(M) \end{bmatrix} = \begin{bmatrix} n(P) - WV^{-1}v(M) \\ v(M) \end{bmatrix}$$

d(P) and d(M) are separated (first row only contains d(P))

d(P) can be computed solving this equation system of type Ax=b
Only the matrix V needs to be inverted (efficiently possibly because it is block diagonal)

$$(U - WV^{-1}W^T)\ d(P) = n(P) - WV^{-1}v(M)$$

d(M) is computed by back-substitution

$$d(M) = V^{-1}(v(M)\text{-}W^T d(P))$$