Outline

- SfM concept
- SfM pipeline
- Image similarity using visual words
- Incremental geometry estimation
- Bundle adjustment
Structure-from-Motion (SfM) concept
Structure-from-Motion (SfM) concept

- Initialize Motion
  \((P_1, P_2 \text{ compatible with } F)\)

- Initialize Structure
  (minimize reprojection error)

- Extend motion
  (compute pose through matches seen in 2 or more previous views)

- Extend structure
  (Initialize new structure, refine existing structure)
Structure-from-Motion (SfM) core pipeline
Feature extraction

- Extract features (point locations and descriptors) for each of the N images
- SIFT features are recommended (best working features for matching right now)
- GPU accelerated implementations exist
Coarse matching

- To avoid NxN feature matching
- Many possible image pairs in the dataset will not have overlap, detailed feature matching will produce no matches for such pairs
- Cluster similar images by similarity using visual words
- Detailed matching will only be performed for similar images
Visual words

feature space cells (e.g. SIFT)
Histogram of visual words (bags of words)
Detailed matching

- Typically using an approximated nearest neighbor (ANN) algorithm
Geometric verification and epipolar graph

- Geometric verification of 2-view matches using fundamental matrix or essential matrix computation
- Epipolar graph: Is a plot of the number of geometrically verified 2-view feature matches
- Defines the sequential order for geometry processing

Image similarity  Epipolar graph
Geometry estimation

- Following the sequence ordering from the epipolar graph geometry is estimated for all images
- Geometry estimation is an alternating scheme:
  - Estimate camera pose of new images (position, rotation)
  - Triangulate new 3D data points seen in new image
  - Refinement by non-linear optimization (Bundle adjustment)
Geometry estimation steps

- Compute camera poses of the first two images from feature matches

\[ P = K[I|0] \]
\[ P' = K'[R'|t'] \]
Geometry estimation steps

- Computation of first 3D points by triangulation

\[
P = K[I|0] \quad \quad P' = K'[R'|t']
\]
Geometry estimation steps

- Triangulate all feature matches of the first images

\[ P = K[I|0] \]

\[ P' = K'[R'|t'] \]
Geometry estimation steps

- First refinement of camera poses and 3D points by non-linear estimation of the re-projection error through bundle adjustment

\[ P = K[I|0] \]

\[ P' = K'[R'|t'] \]
Geometry estimation steps

- Start processing the next image

\[ P = K[I|0] \]

\[ P' = K'[R'|t'] \]

\[ P'' = ? \]
Geometry estimation steps

- First, create feature matches to all the previous, neighboring images
Geometry estimation steps

- Feature matches give correspondences to already computed 3D points
- From corresponding 2D and 3D points the pose of the new camera can be computed using the PnP-Algorithm

\[ P = K[I|0] \]
\[ P' = K'[R'|t'] \]
\[ P'' =? \]
Geometry estimation steps

- Repeat the process starting again from triangulation of new features

\[ P = K[I|0] \]

\[ P'' = K''[R''|t''] \]

\[ P' = K'[R'|t'] \]
Bundle adjustment

- Levenberg-Marquard optimization of re-projection error
- Parameters are camera poses and all 3D points (millions of parameters to optimize!)

\[
\min_{P_j, X_i} \left( \sum_i \sum_j \| x_{i,j} - P_j X_i \| \right)
\]

\[\begin{align*}
X_i &= P X_i \\
x_{i,j} &= \text{pixel coordinates of point } (X_i, Y_i, Z_i) \\
P &= \text{projection matrix}
\end{align*}\]
3 paradigms

sequential (incremental)

hierarchical

global
Bundle adjustment (BA)

Objective function to be minimized

\[
\min_{P_{j, M_i}} \left( \sum_i \sum_j \| p_{i,j} - P_j M_i \| \right) = \varepsilon = f(x)
\]

Gauss-Newton update equation

\[
x_k = x_{k-1} + d_{k-1}
\]

\[
J_{k-1}^T J_{k-1} d_{k-1} + J_{k-1}^T \varepsilon_{k-1} = 0
\]
Calculating the update vector $d$

$$J^T_{k-1} J_{k-1} d_{k-1} = -J^T_{k-1} e_{k-1}$$
$$Ax = b$$

- $J$ ... npxN matrix (n ... #cameras, p ... #points, N ... #parameters)
- residual vector $e$ is computed from $e = ||x - PM||$ for every iteration
- Then the values for the Jacobian $J$ are computed for every iteration
The Jacobian J (example for 3 cameras and 4 3D points)

- $J$ … npxN matrix (n … #cameras, p … #points, N … #parameters)
- $M$ … 1x3 matrix, $P$ … 1x11 matrix
$J^T J$

- $J^T J$ is called the “Hessian Matrix” (symmetric matrix)
- $U$ … 11x11 symmetric matrix, $V$ … 3x3 symmetric matrix
- $W$ … 11x3 matrix
Schur complement trick/Sparse BA

\[
\begin{bmatrix}
U & W \\
W^T & V
\end{bmatrix}
\begin{bmatrix}
d(P) \\
d(M)
\end{bmatrix} =
\begin{bmatrix}
n(P) \\
v(M)
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_{11(n-1)} & -WV^{-1} \\
0_{3p \times 11(n-1)} & I_{3p}
\end{bmatrix}
\]

Multiply the above equation with this line to obtain

\[
\begin{bmatrix}
U - WV^{-1}W^T & 0_{3p} \\
W^T & V
\end{bmatrix}
\begin{bmatrix}
d(P) \\
d(M)
\end{bmatrix} =
\begin{bmatrix}
n(P) - WV^{-1}v(M) \\
v(M)
\end{bmatrix}
\]

d(P) and d(M) are separated (first row only contains d(P))

d(P) can be computed solving this equation system of type Ax=b

Only the matrix V needs to be inverted (efficiently possibly because it is block diagonal)

\[
(U - WV^{-1}W^T) d(P) = n(P) - WV^{-1}v(M)
\]

d(M) is computed by back-substitution

\[
d(M) = V^{-1}(v(M) - W^T d(P))
\]