Mathematical Principles in Vision and Graphics: Hidden Variable Resultant

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The Hidden Variable Resultant [1]

Let's assume we have:

$$\mathbf{M}\mathbf{x} = 0 \quad (1)$$

with $\mathbf{x}$ a vector of monomials in $x_1, x_2, \ldots, x_N$.

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We can rewrite Eq. (1) in the form:

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where

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IF the number of equations equals the number of monomial terms in $\mathbf{Y}$ (i.e. $\mathbf{N}(x_1)$ is square), then the equation system has non-trivial solutions if and only if:

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Applying the Hidden Variable Resultant to our system

$$\mathbf{M}_{10\times 20}\mathbf{x} = 0$$

with

$$\mathbf{x} = [x^3, y^3, x^2y, xy^2, x^2z, x^2, y^2z, y^2, xyz, xy, xz^2, xz, x, yz^2, yz, y, z^3, z^2, z, 1]^\top$$

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$$\det(\mathbf{N}_{10\times 10}(z)) = 0\,.$$

Expanding the determinant results in a 10^{th}-degree polynomial in z.

References I

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