Learning goals

- Understand the concept of vanishing points and vanishing lines
- Understand the calculation of vanishing points
- Understand the relation between vanishing points and camera orientation and calibration
Outline

- Vanishing points and lines
- Applications of vanishing points
Vanishing points

- Ideal points in the projective 3-space are located at infinity, and have homogeneous coordinates of the form \((x, y, z, 0)\).
- These points are also called points at infinity.
- The image of an ideal point under a projective mapping is called a vanishing point.
- Recall that two parallel lines in the projective 3-space meet at an ideal point.
- Thus the images of two or more parallel world lines converge at a vanishing point.

[Source: Flickr]
Vanishing points

The vanishing point $v$ is the projection of a point at infinity. Think of extending the line on the ground plane further and further into infinity.
Vanishing points

- Any two parallel lines have the same vanishing point $v$
- The vanishing point is the image of the intersection point of the two parallel lines.
- The ray from $C$ through $v$ is parallel to the lines
- An image may have more than one vanishing point
Vanishing lines

- Multiple vanishing points
  - Any set of parallel lines on the plane define a vanishing point
  - Lines at different orientation result in a different vanishing point
  - The union of all the vanishing points from lines on the same plane is the vanishing line
- Different planes define different vanishing lines.
Vanishing lines

- A set of parallel planes that are not parallel to the image plane intersect the image plane at a vanishing line.
- The horizon is a special vanishing line when the set of parallel planes are parallel to the ground reference.
- Anything in the scene that is above the camera will be projected above the horizon in the image.
Computing vanishing points

- Properties
  - $\mathbf{v} = \Pi \mathbf{P}_\infty$
  - $\mathbf{P}_\infty$ is a point at infinity, $\mathbf{v}$ is its projection
  - They depend only on line direction
  - Parallel lines $\mathbf{P}_0 + t\mathbf{D}$, $\mathbf{P}_1 + t\mathbf{D}$ intersect at $\mathbf{P}_\infty$

\[
\mathbf{P}(t) = \mathbf{P}_0 + t\mathbf{D}
\]

\[
\mathbf{P}(t) = \begin{bmatrix} P_X + tD_X \\ P_Y + tD_Y \\ P_Z + tD_Z \\ 1 \end{bmatrix} \approx \begin{bmatrix} P_X / t + D_X \\ P_Y / t + D_Y \\ P_Z / t + D_Z \\ 1/t \end{bmatrix} \quad t \to \infty \quad \mathbf{P}_\infty \equiv \begin{bmatrix} D_X \\ D_Y \\ D_Z \\ 0 \end{bmatrix}
\]
Computing vanishing points (from lines)

- Intersect \( p_1 q_1 \) with \( p_2 q_2 \)

\[
v = (p_1 \times q_1) \times (p_2 \times q_2)
\]
Computing vanishing points by projection

- Let \( P = K[I \ 0] \) be a camera matrix. The vanishing point of lines with direction \( d \) in 3-space is the intersection \( v \) of the image plane with a ray through camera center with direction \( d \). This vanishing point \( v \) is given by \( v = Kd \).

\[
v = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix}
\]

- Example: Computing vanishing points of lines on a XZ plane
- (1) parallel to the Z axis, (2) at 45 deg to the Z axis
- (3) parallel to the X axis
Vanishing points and projection matrix

\[ P = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \end{bmatrix} \]

- \( p_1 = P[1 \ 0 \ 0 \ 0]^T = v_x \) (X vanishing point)
- similarly, \( p_2 = v_y \), \( p_3 = v_z \)
- \( p_4 = P[0 \ 0 \ 0 \ 1]^T = \) projection of world origin O

\[ P = \begin{bmatrix} v_X & v_Y & v_Z & o \end{bmatrix} \]
Real vanishing points

Vanishing point of a line parallel to a plane lies on the vanishing line of the plane.
Image rectification using vanishing points

before rectification

after rectification
Camera rotation from vanishing points

- Two images of a scene obtained by the same camera from different position and orientation.
- The images of the points at infinity, **the vanishing points**, are not affected by the camera translation, but are affected only by the camera rotation.
- Vanishing points $v_i$ and $v_i'$ have the following directions $d_i$, $d_i'$

$$
d_i = K^{-1}v_i/\|K^{-1}v_i\| \quad \quad d_i' = K^{-1}v_i'/\|K^{-1}v_i'\|
$$

- The directions are related by a rotation matrix:

$$
d_i' = Rd_i
$$

- If the directions are known, the rotation matrix can be computed from two directions
Camera rotation from vanishing points

- First camera is aligned with world coordinate system. \(d_i = [0 \ 0 \ 1]\)
- Second camera deviates, \(d_i' = [d_x, d_y, d_z]\) and can be computed from the image coordinates of \(v_i'\)
- The rotation that aligns the second image with the first image can be computed from \(d_i\) and \(d_i'\) and 1 more direction.
Measuring heights using vanishing points

Height column = height of reference object \times \frac{d_2}{d_1}
Camera calibration from orthogonal vanishing points

[Image source: Richard Hartley and Andrew Zisserman]
Recap - Learning goals

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