Mathematical Principles in Visual Computing

Prof. Friedrich Fraundorfer

SS 2024

About me

- Prof. Dr. Friedrich Fraundorfer
- Email: fraundorfer@icg.tugraz.at
- Institute of Computer Graphics and Vision
- Inffeldgasse 16/II
- +43 (316) 873 - **5020**
- Send email to schedule an appointment

Additional lecturers

- Dr. Jörg Müller
- Email: joerg.mueller@icg.tugraz.at
- Institute of Computer Graphics and Vision
- Inffeldgasse 16/II

- Arno Coomans
- Huawei Zürich

Lecture schedule

- 06.03.2024 Fraundorfer
- 13.03.2024 Fraundorfer
- 20.03.2024 Fraundorfer
- 10.04.2024 Fraundorfer
- 17.04.2024 Fraundorfer
- 24.04.2024 Fraundorfer
- 08.05.2024 Müller
- 15.05.2024 Müller
- 22.05.2024 Müller
- 29.05.2024 Müller
- 05.06.2024 Coomans
- 12.06.2024 Coomans
- 19.06.2024 Coomans
- 26.06.2024 Exam

Tutor

- Jun Zhang
- Email: jun.zhang@tugraz.at
- Responsible for questions about classroom assignments
- Q&A slots with tutor
- Q&A in TC forum or e-mail

Course grading

- 3 class room assignments (50% of grade)
  - Math problems
  - Small programming assignments
- Final written exam (50% of grade)

- Written exam at last lecture slot (26 June 2024)
- Submitting the first assignment counts as attempt. A grade will be issued in this case.

Assignments

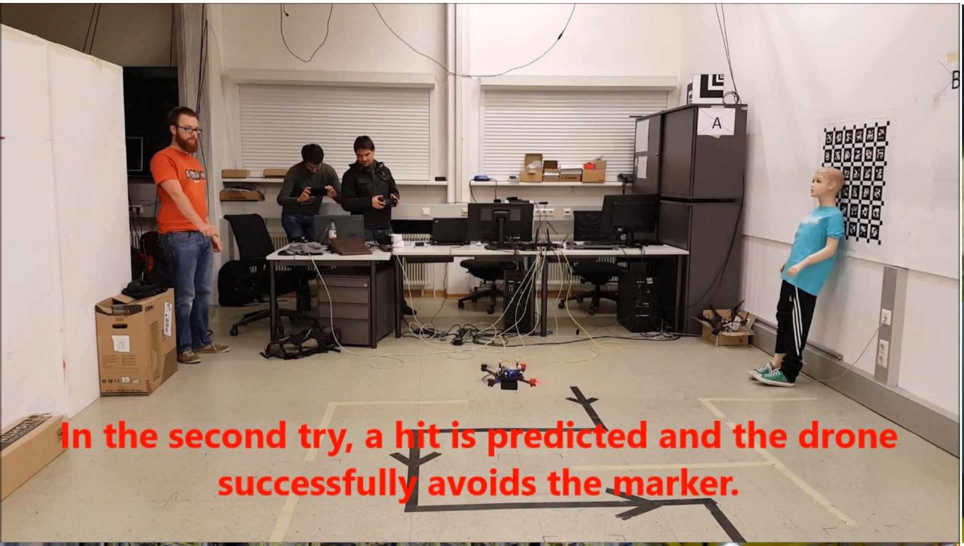
- Individual work, no group work
- Electronic submission using the TeachCenter (Hand-writing and scanning is ok)

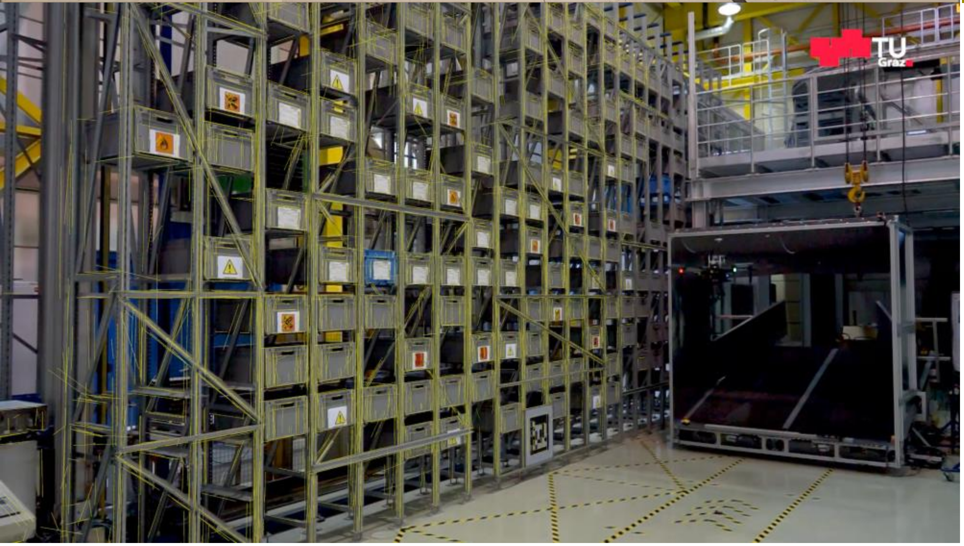
- Schedule:
  - Assignment 1
    - Handout: 20.3.2024
    - Deadline: 30.4.2024
  - Assignment 2
    - Handout: 24.4.2024
    - Deadline: 28.5.2024
  - Assignment 3
    - Handout: 22.5.2024
    - Deadline: 18.6.2024

Lecture material

- Slides are the main material
- Links to relevant publications and book sections will be given
- Lecture recordings from last years are available in the Teach Center

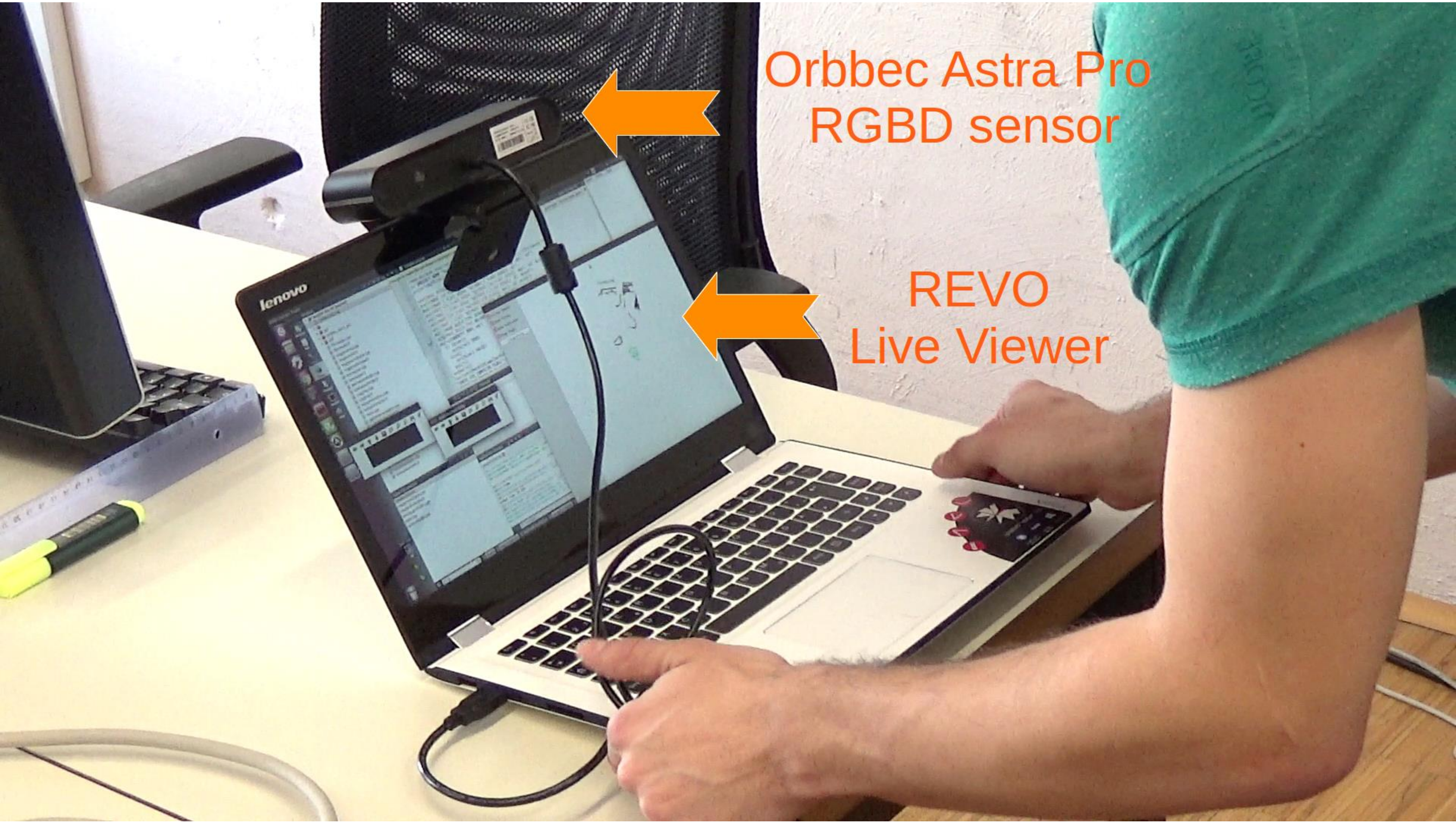
Research areas



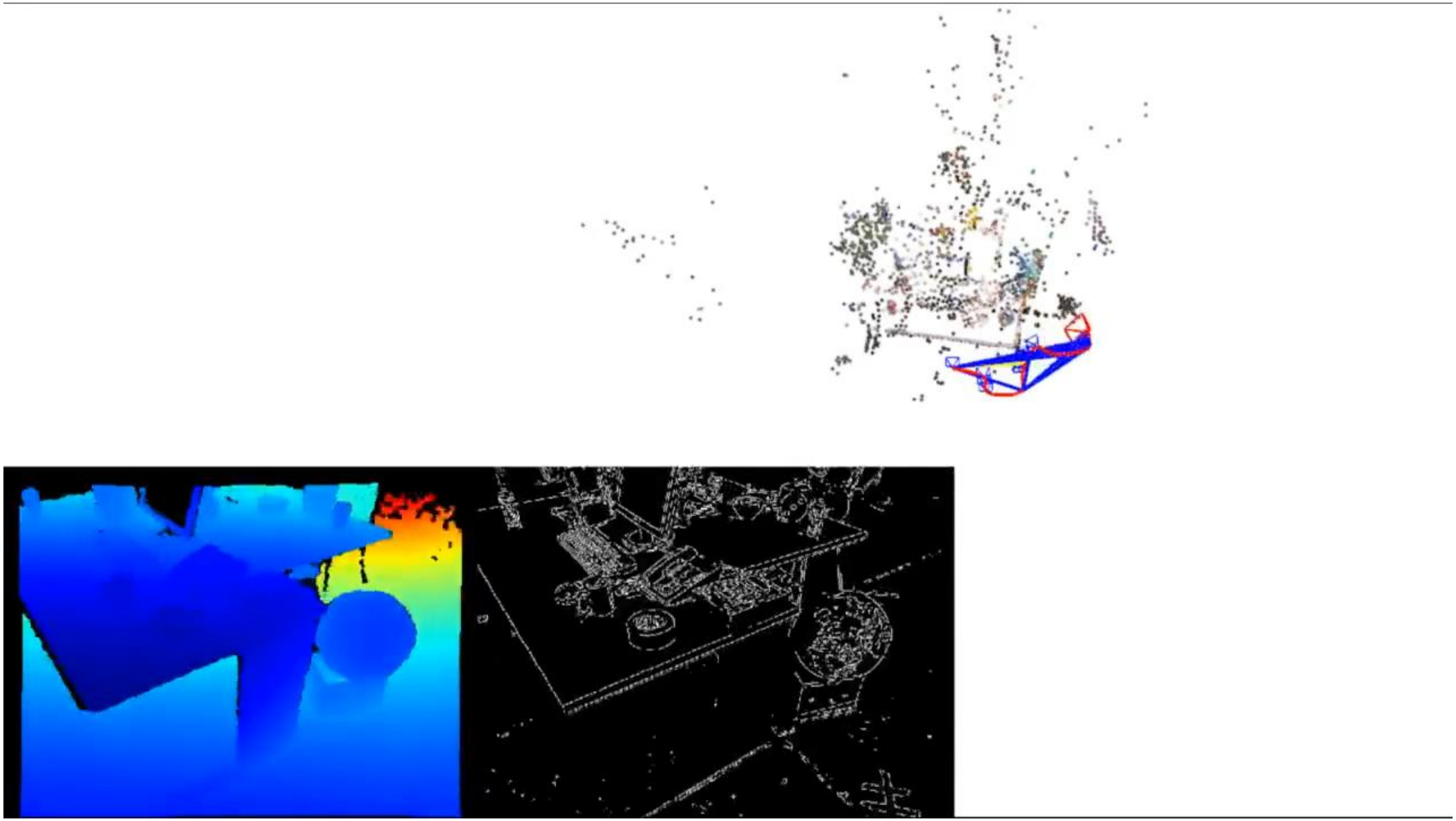


3D scanning - REVO

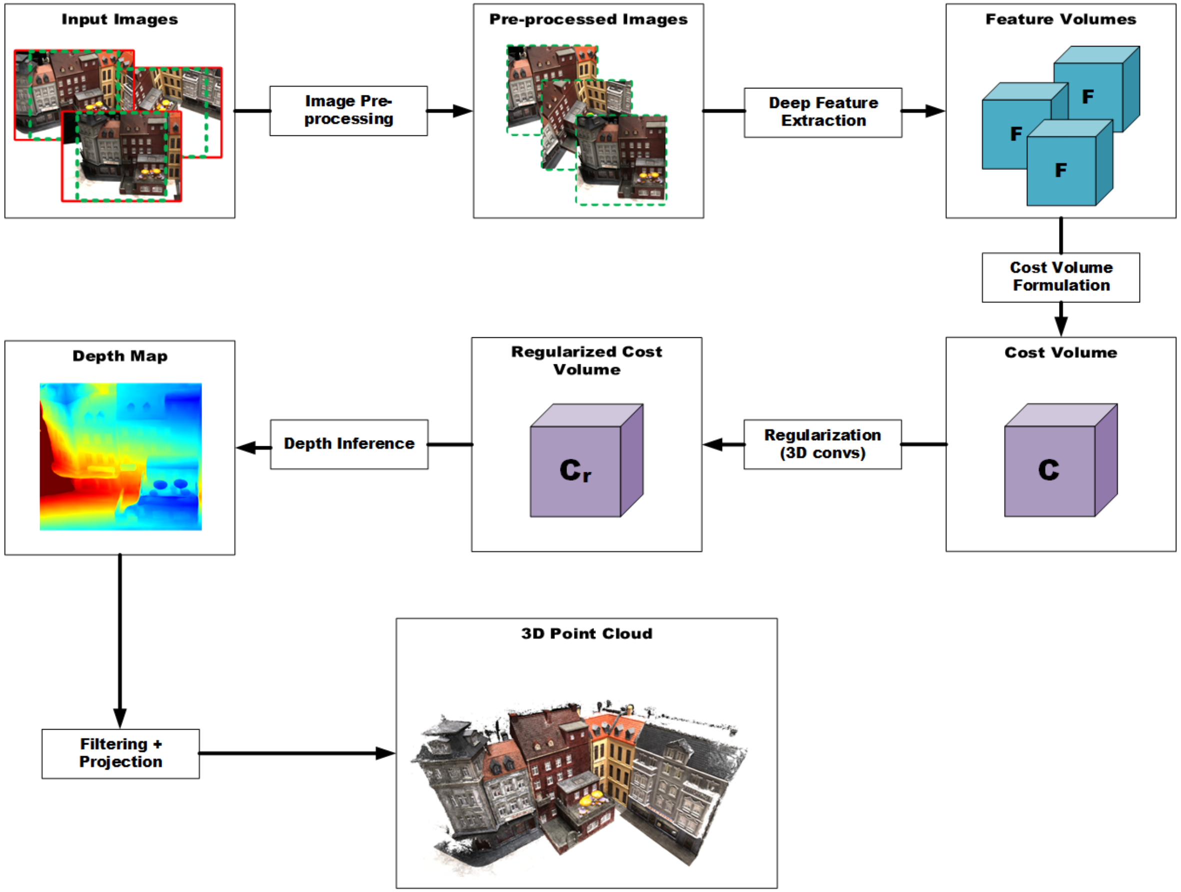
- RGBD recordings with Orbbec Astra Pro



3D scanning - REVO



Multi-View Stereo Pipeline

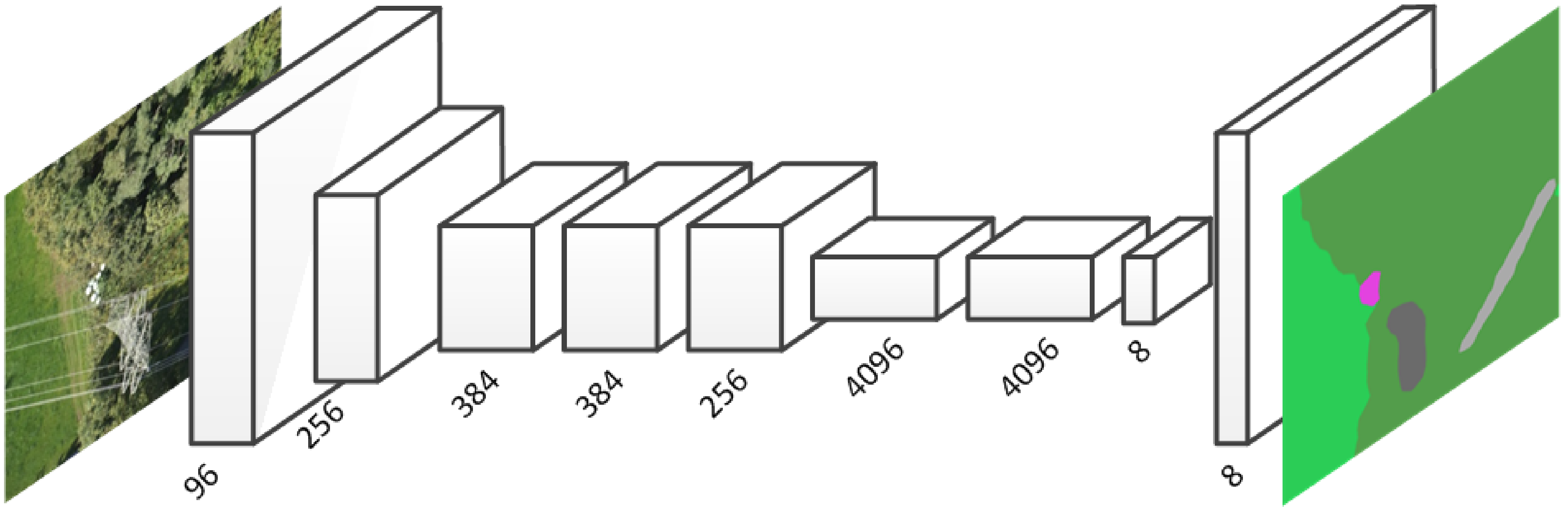


Bridge inspection

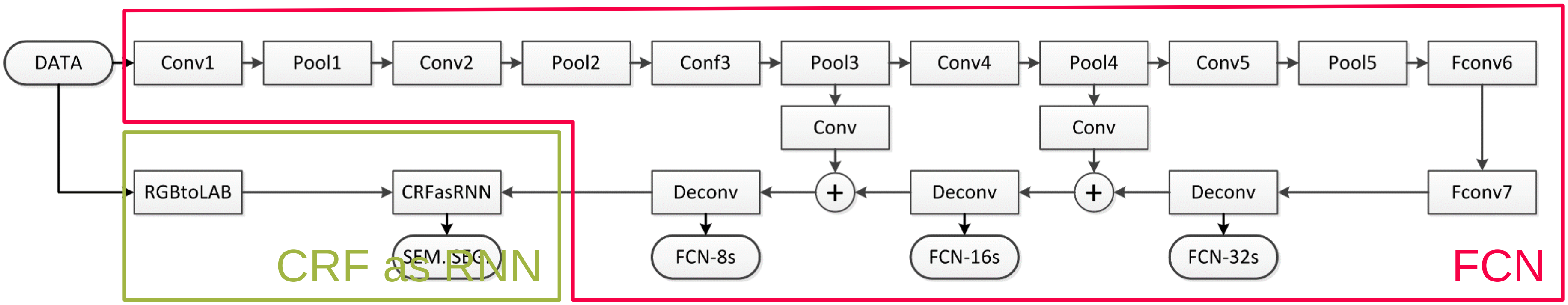
Bridge inspection

Semantic 3D

Semantic 3D

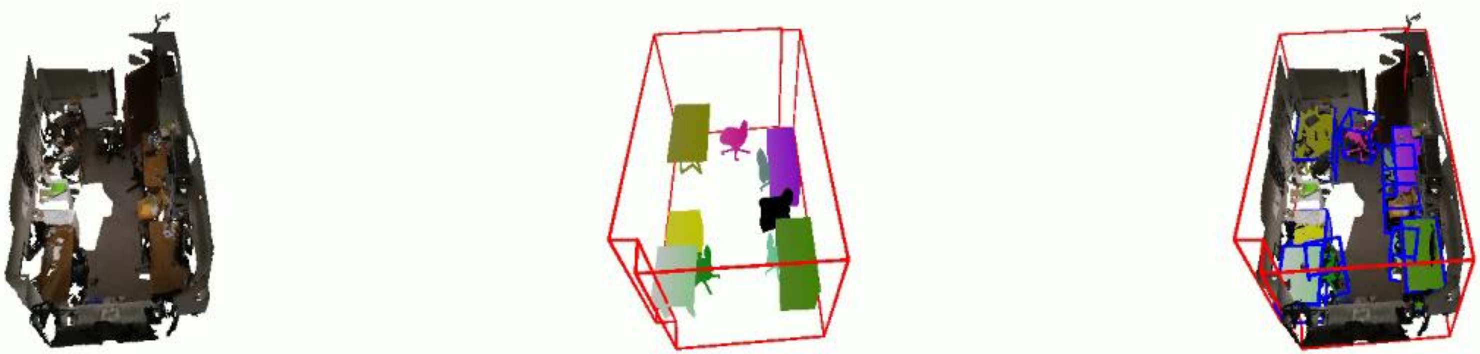


96
256
384
384
256
4096
4096
8
8



DATA
Conv1
Pool1
Conv2
Pool2
Conf3
Pool3
Conv4
Pool4
Conv5
Pool5
Fconv6
Conv
Conv
RGBtoLAB
CRFasRNN
Deconv
Deconv
Deconv
Fconv7
SEM.SEG
FCN-8s
FCN-16s
FCN-32s
CRF as RNN
FCN

3D scene understanding



Embedded AI – Dedicated processors allow integration of deep learning

Embedded AI – Object detection

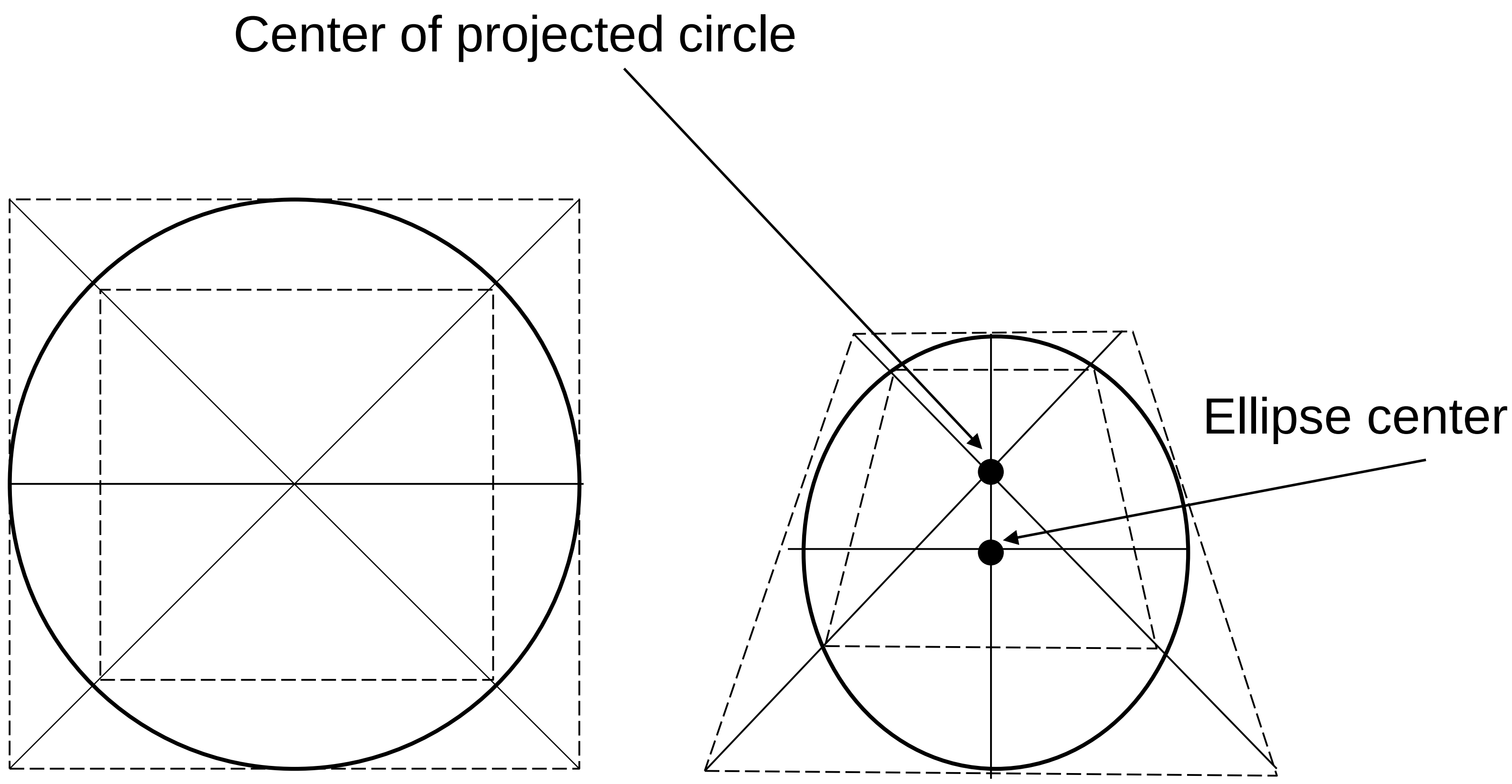
Topics

- Projective geometry
- Parameterization of rigid transformations
- Polynomial systems in computer vision
- Root-solving
- Projective geometric algebra (Müller)
- Path tracing (Coomans)

Projective geometry

[Source: Flickr]

Projective geometry

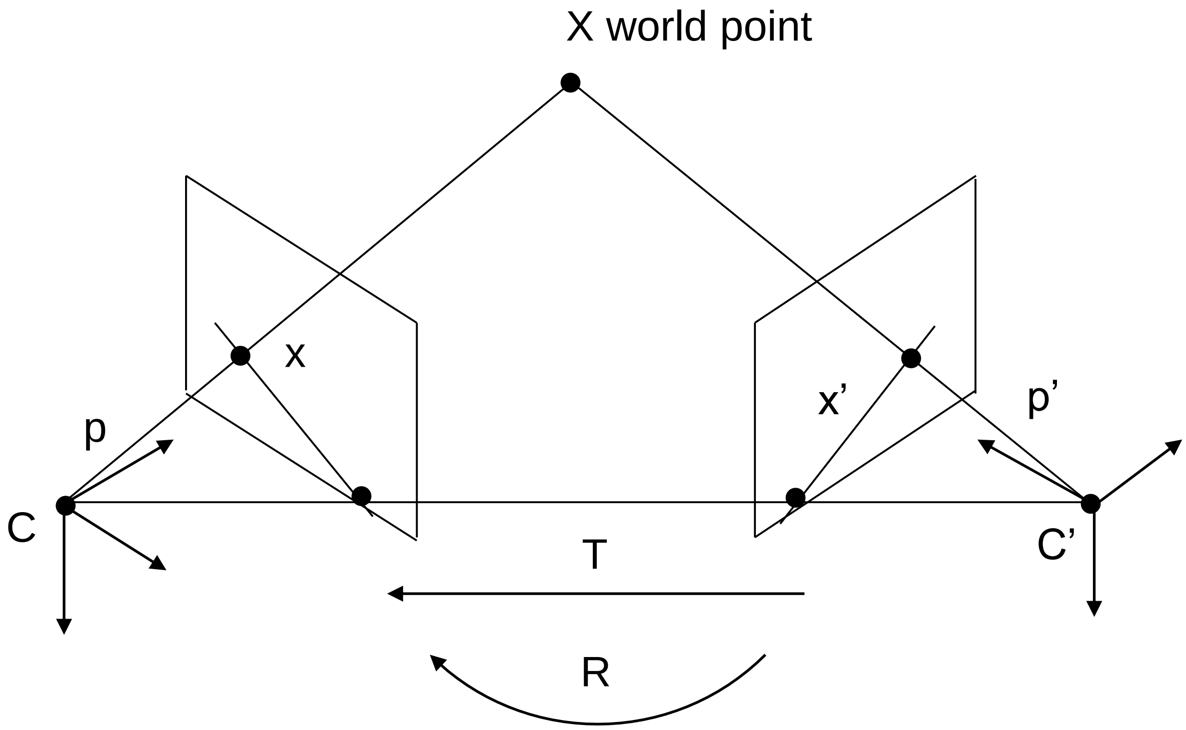


2D circle

Circle after projective transformation

Projective geometry

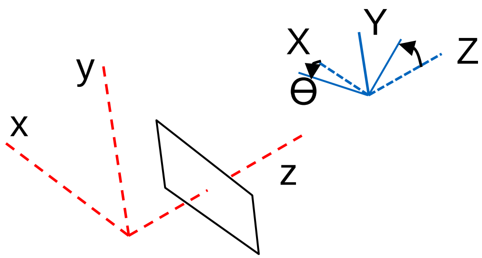
$$x'^T F x = 0 \;\; \ldots \; \textit{Epipolar constraint}$$

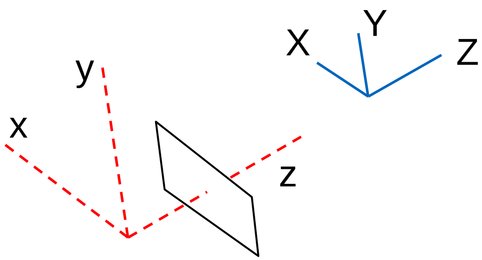


Projective geometry

before rectification

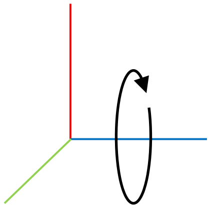
after rectification



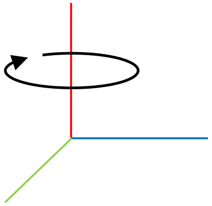


Parameterization of rigid transformations

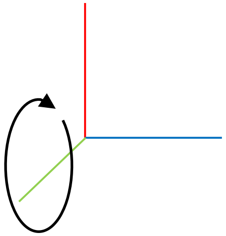
$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$



$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$



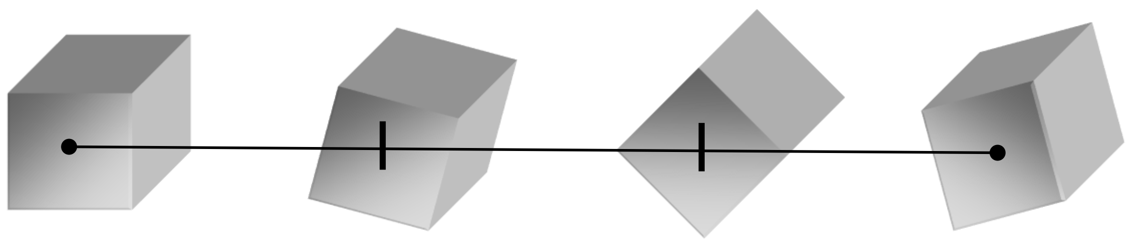
$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Problems with rotation matrices

- Optimization of rotations (bundle adjustment)
  - Newton's method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

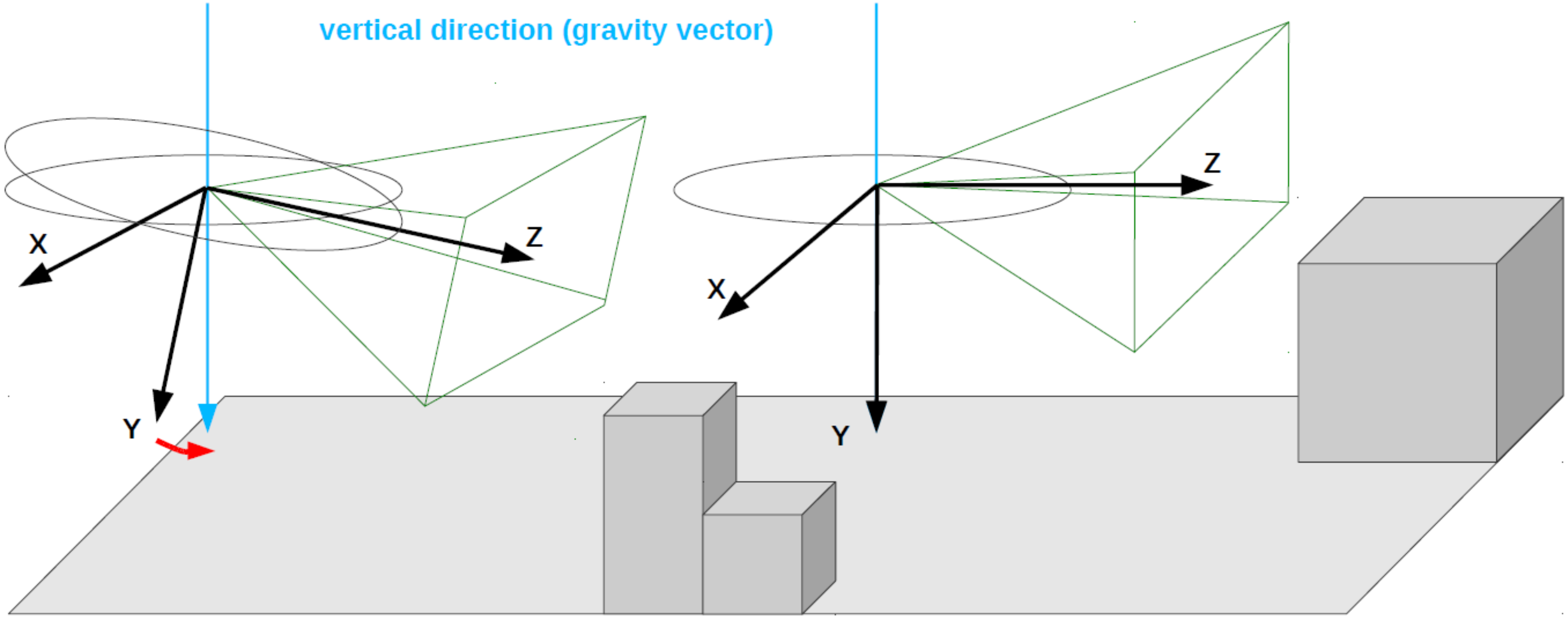
- Linear interpolation



- Filtering and averaging
  - E.g. averaging rotation from IMU or camera pose tracker for AR/VR glasses

Polynomial equation systems in computer vision

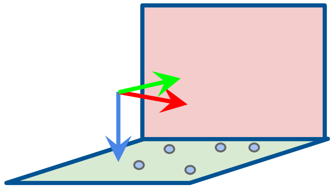
- Assumption: Ground plane normal to gravity vector, walls are vertical
  - IMU measurements can be used to align camera images/features to gravity vector
  - Motion can be computed from 2pt correspondences on the ground



Polynomial equation systems in computer vision
2pt relative pose

Homography Rotation (yaw) Rotation IMU Translation Plane normal

$$\boxed{\mathbf{H}} = \boxed{\mathbf{R}_y}\boxed{\mathbf{R}_x\mathbf{R}_z} + \boxed{\mathbf{t}^T}\boxed{\mathbf{n}}$$



$$\mathbf{H} = \mathbf{R}_y + \mathbf{t}^T\mathbf{n}$$ (for pre-rotated features)

$$\boxed{\mathbf{H}} = \boxed{\mathbf{R_y}} + \boxed{[t_x, t_y, t_z]^T}\boxed{[0, 1, 0]}$$ (H for ground plane)

Homography Rotation (yaw) Translation Plane normal

4 unknowns left, 2 point correspondences give 4 equations

Polynomial systems in computer vision

- P3P, PnP problem:

$$
\begin{array}{l} L_1 \\ L_2 \\ L_3 \end{array}
\left\{ \begin{array}{rcl}
2x^2 + y^2 - 2z + 3z^2 + 5 & = & 0 \\
x^2 + z + z^2 & = & 0 \\
x^2 y^2 + y^2 z^2 - 2 & = & 0
\end{array} \right.
$$

- Solution: Reduction to a single polynomial (several schemes)
- Automatic procedure – Gröbner Basis

Polynomial equation systems

Root solving

- 3pt+IMU, 8^{th} degree polynomial
- 6pt generalized camera, 64^{th} degree polynomial

- Fast method: Sturm bracketing

Projective Geometric Algebra