Mathematical Principles in Visual Computing: Multi-Camera-Systems

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SS 2023

Outline

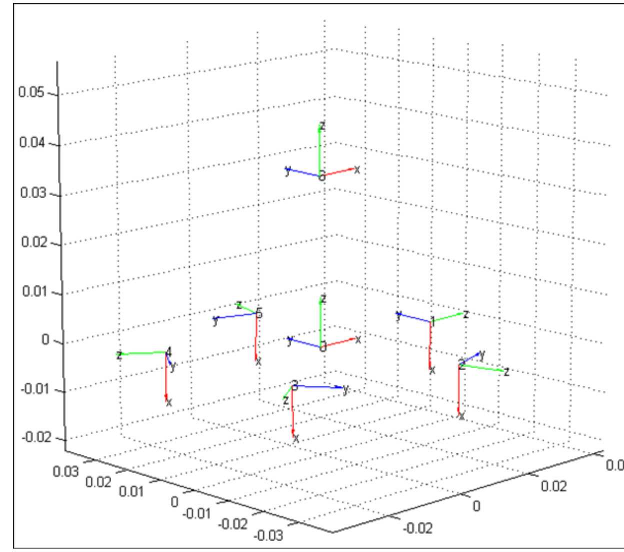
- Multi-Camera-Systems
- The generalized camera
- Plücker coordinates
- Generalized Epipolar Constraint
- Intra- and Inter-camera correspondences
- Generalized PnP

Learning goals

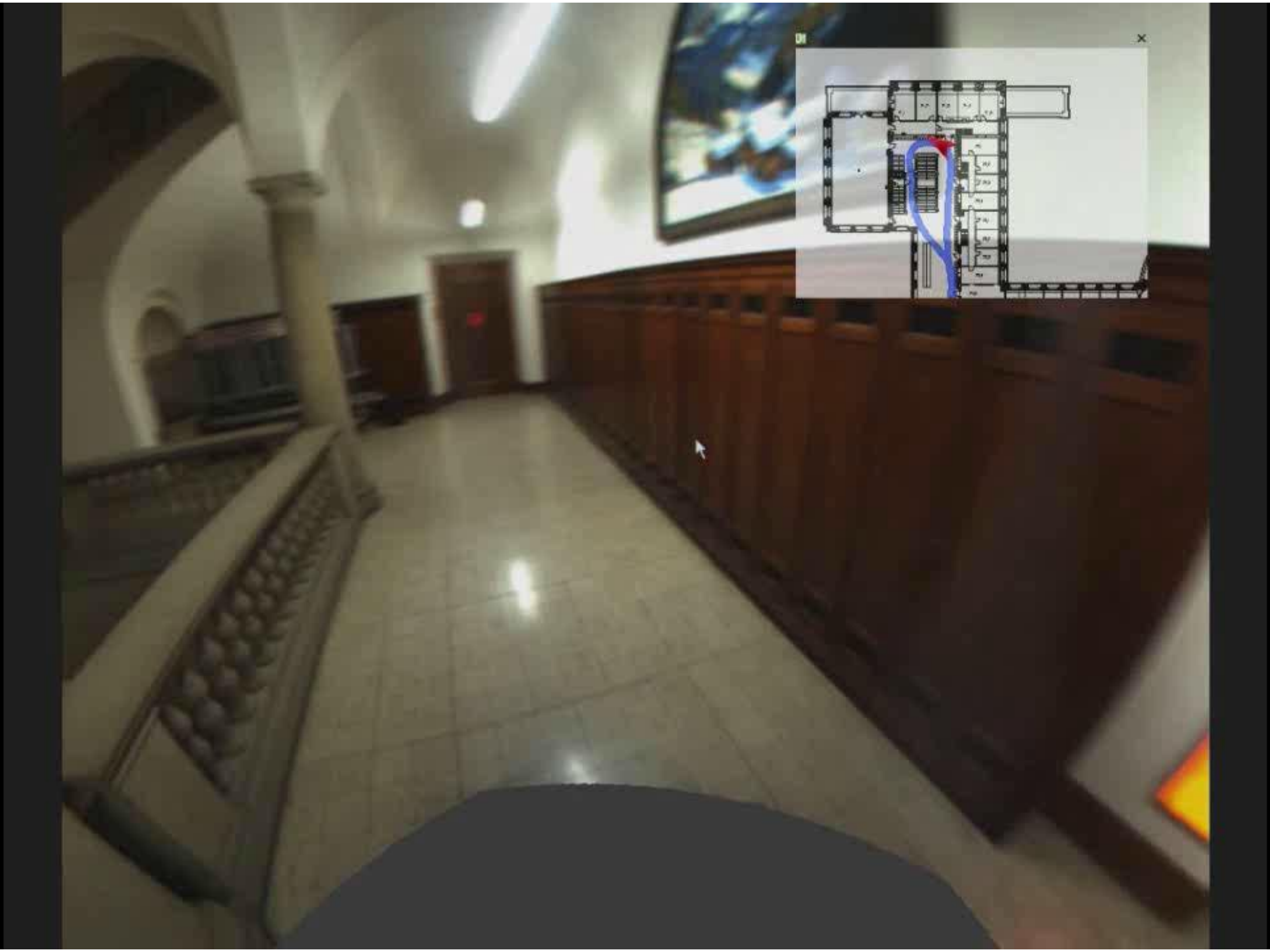
- Understand Plücker-line coordinates
- Understand the use of Plücker-line coordinates to describe multi-camera systems
- Understand the properties of multi-camera systems
- Understand the concept of a generalized camera
- Understand the generalized epipolar concept
- Understand the P3P extension to the generalized camera P3P

PtGrey Ladybug

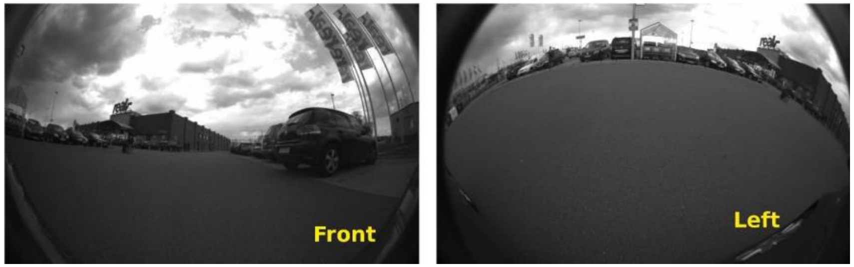
- 6 cameras
- 360 field of view
- Panorama images



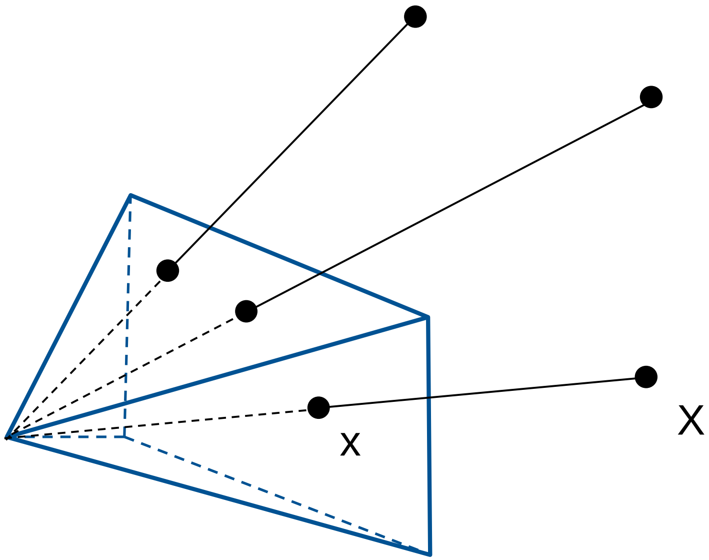
Omnitour



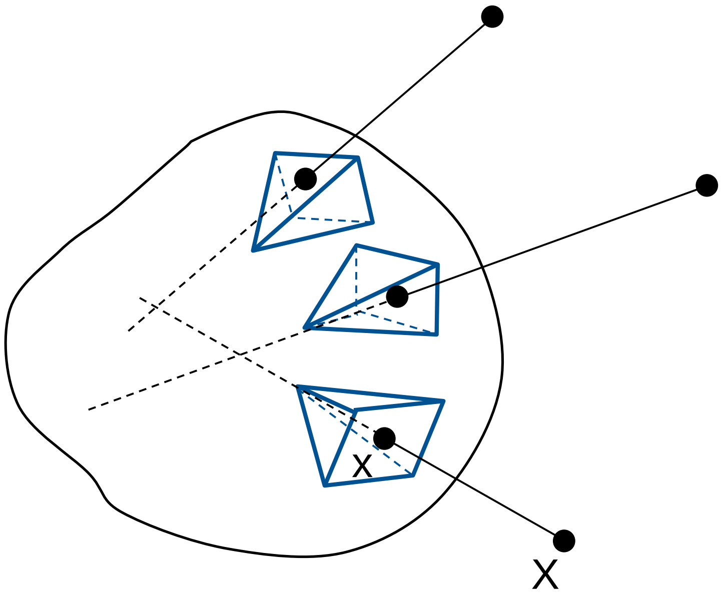
Automotive around view system



The generalized camera

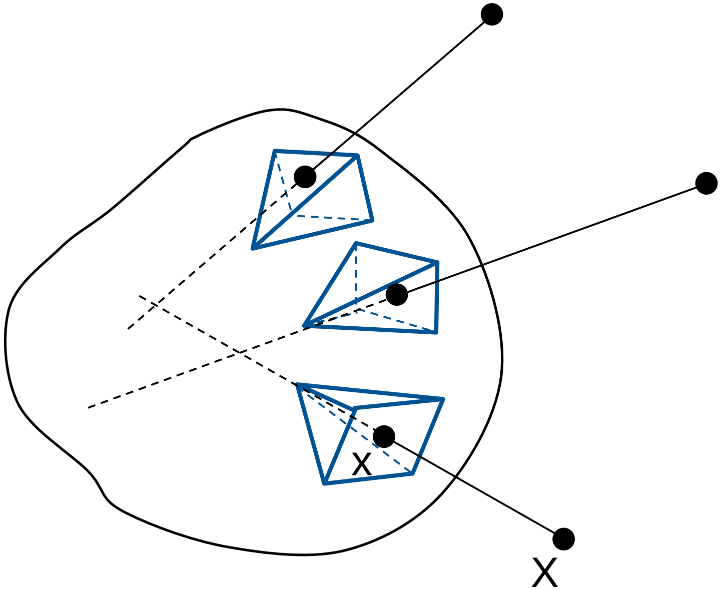


pinhole camera

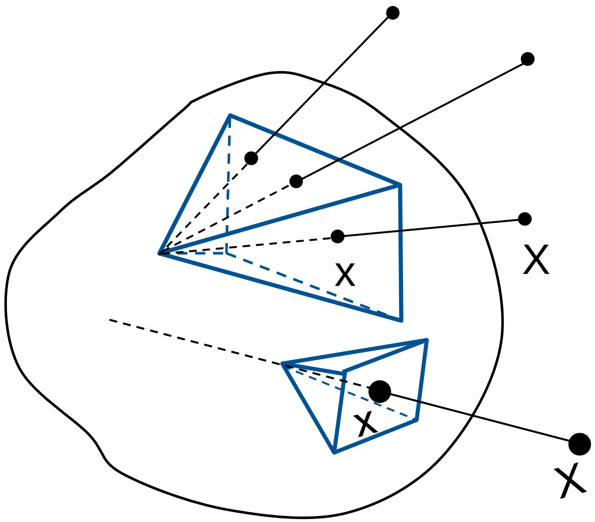


generalized camera

The generalized camera

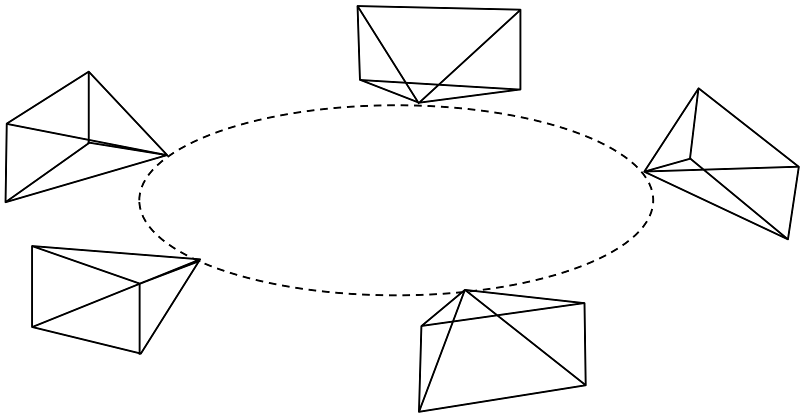


generalized camera

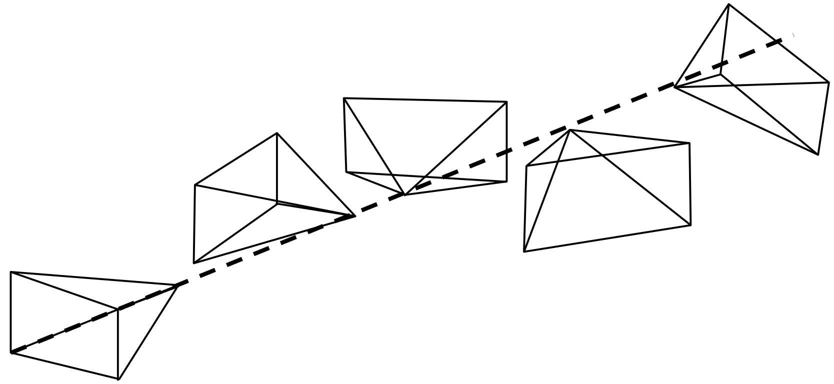


mixed configuration

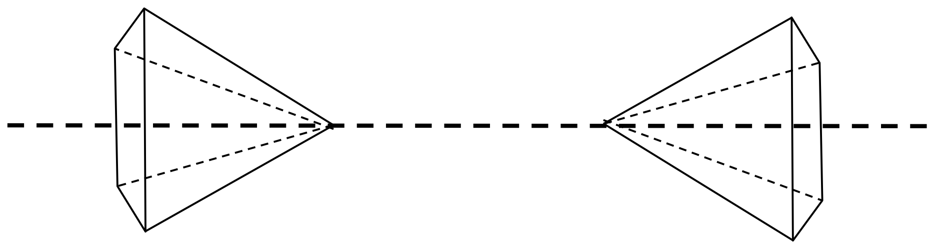
Multi camera systems



general non-axial camera
(locally-central)

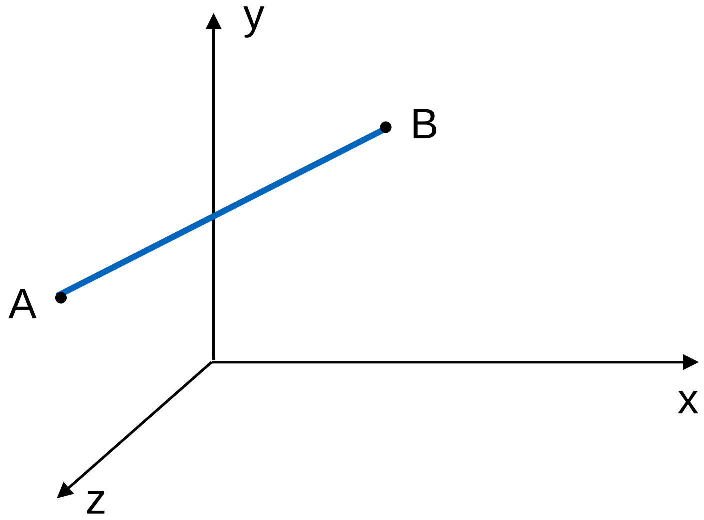


axial camera



axial camera

Plücker coordinates



- Plücker line matrix (4x4) $L = AB^T - BA^T$

- Plücker coordinates (6-vector)

$$L = \begin{bmatrix} A_4B_1 - A_1B_4 \\ A_4B_2 - A_2B_4 \\ A_4B_3 - A_3B_4 \\ A_3B_2 - A_2B_3 \\ A_1B_3 - A_3B_1 \\ A_2B_1 - A_1B_2 \end{bmatrix}$$

Plücker coordinates

- 6-vector L consists of 2 parts

$$a^T = (L_1 \quad L_2 \quad L_3) \qquad b^T = (L_4 \quad L_5 \quad L_6)$$

$$a^T b = 0$$

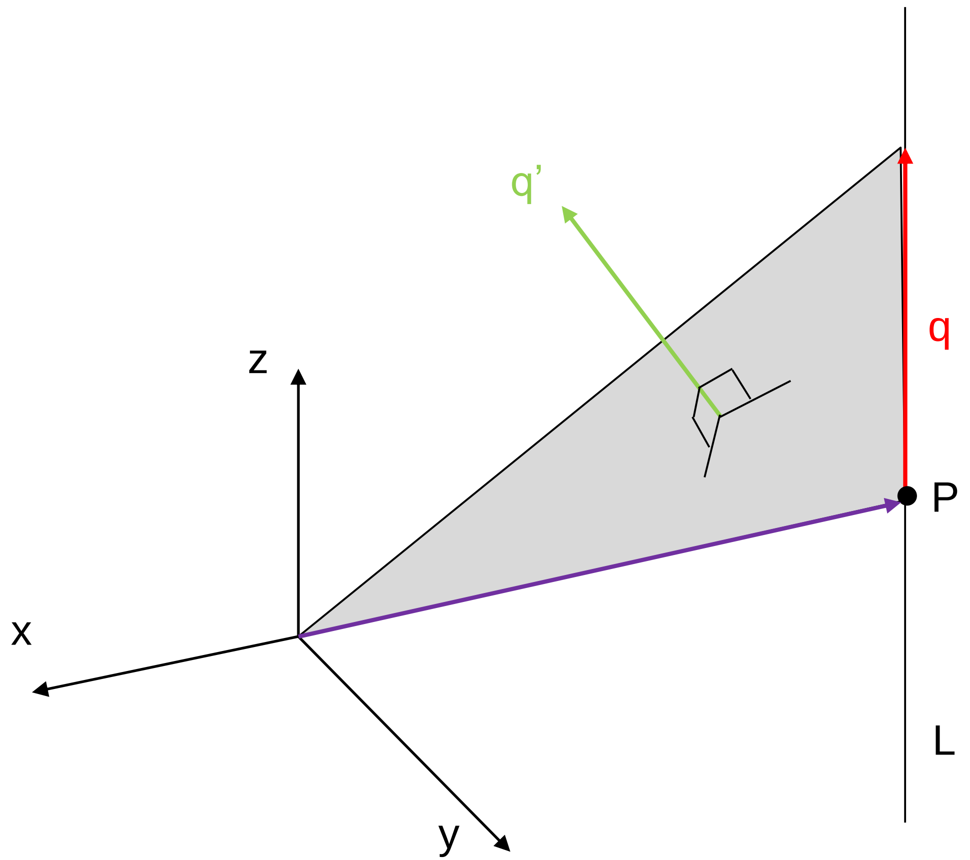
- Rigid transformation
  - Point: $$X' = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} X$$
  - Plücker coordinate $$\begin{bmatrix} a' \\ b' \end{bmatrix} = \begin{bmatrix} R & 0 \\ -[t]_x R & R \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Plücker coordinates

- Line intersection of L_1 and L_2

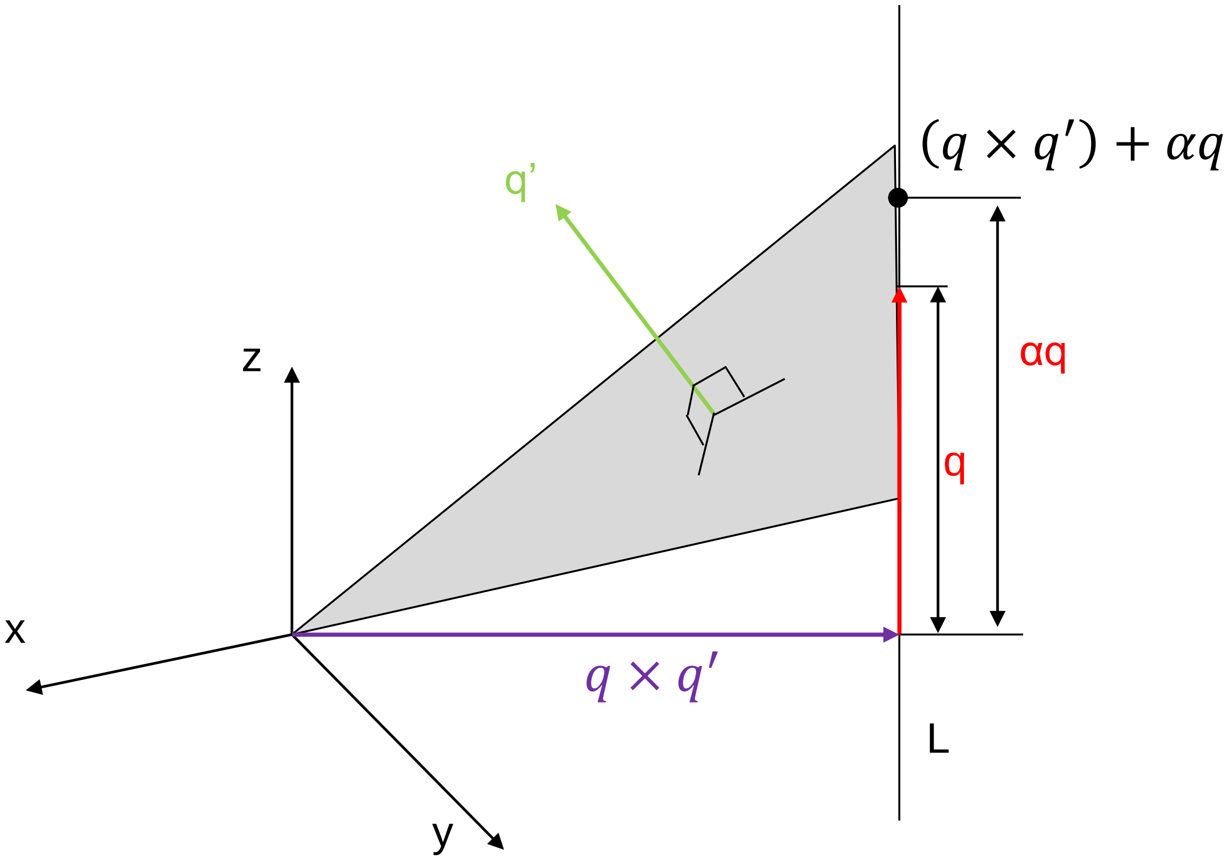
$$L_2^T \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} L_1 = a_2^T b_1 + b_2^T a_1 = 0$$

Plücker coordinates: Geometric interpretation



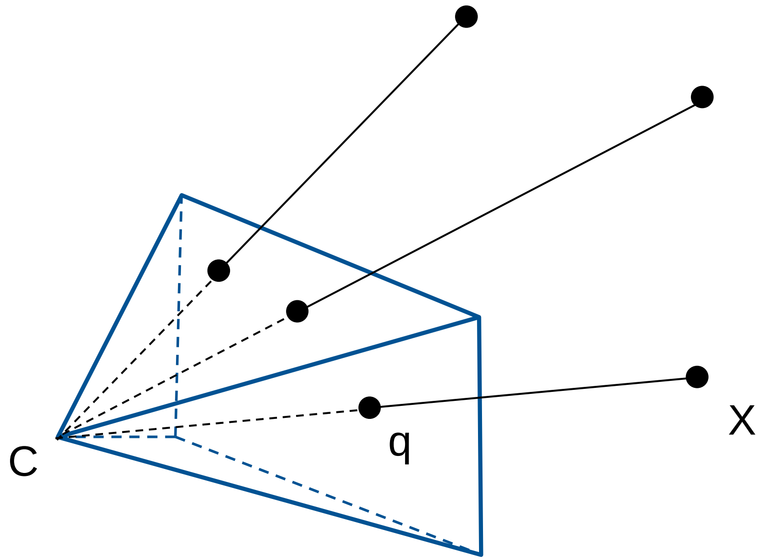
$$L = (q, q')$$
$$q' = P \times q$$
$$q^T q' = 0$$

Plücker coordinates: Geometric interpretation



- All points on the line L are expressed by the two vectors q × q′ and αq, where α is a scalar.

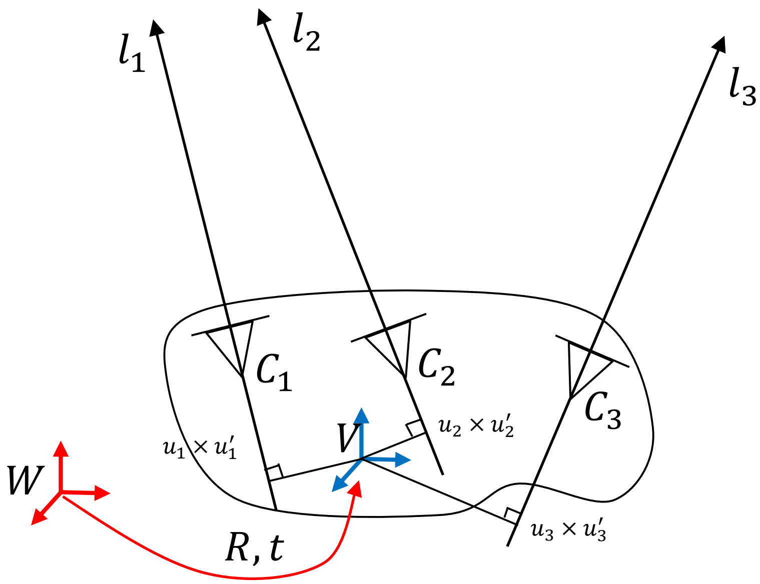
Plücker coordinates: Pinhole camera



$$L = (q, q')$$
$$q' = C \times q$$
$$P = C = (0,0,0)$$
$$L = (q, (0\ 0\ 0)')$$

- Camera center is point C
- Plücker coordinate is just standard homogeneous coordinate in this case

Plücker coordinates: Generalized camera

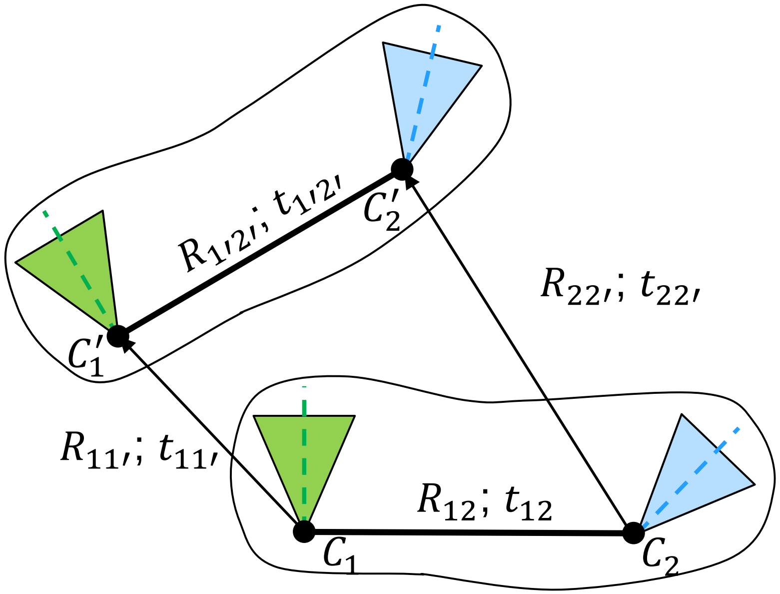


- Camera centers are not at the origin (0,0,0)

$$L_1 = (R_1 x_1, C_1 \times R_1 x_1)$$

- R_1,C_1 … camera center and orientation in a common coordinate system
- x_1 … homogeneous (normalized) image coordinate

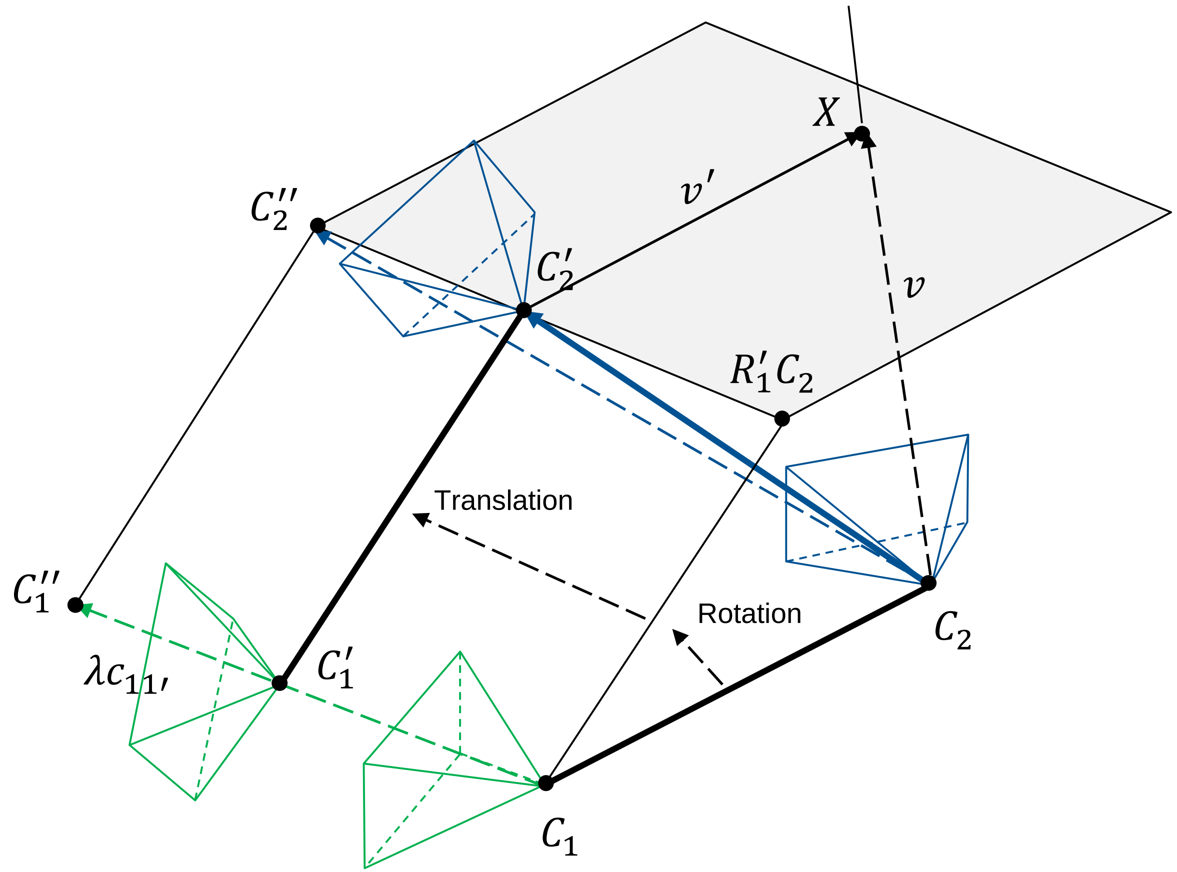
Relative motion



$$R_{11'} = R_{22'}$$
$$t_{11'} \neq t_{22'}$$

- Relative rotations are the same for all cameras
- Relative translations are all different due to the lever arm (direction as well as length).
- Motion is defined as full 6DOF as compared to single camera case with 5DOF (length of translation is defined!)

Relative motion, scale estimation



Generalized epipolar constraint (GEC)

- Line correspondence L<-> L'

$$L = (q_1^T, q'^T_1)^T$$
$$L' = (q_2^T, q'^T_2)^T$$

- Relative motion transforms Plücker coordinates (light ray)

$$L' = \begin{pmatrix} Rq_1 \\ (Rq'_1 + t \times (Rq_1) \end{pmatrix}$$

- Generalized epipolar constraint (GEC) defines intersection of two light rays ([R,T]L,L')

$$q_2^T q'_1 + q'^T_2 q_1 = 0$$
$$q_1 \rightarrow Rq_1$$
$$q'_1 \rightarrow Rq'_1 + t \times (Rq_1)$$
$$q_2^T \left(Rq'_1 + t \times (Rq_1) \right) + q'^T_2 (Rq_1) = 0$$
$$q_2^T Rq'_1 + q_2^T [t]_x Rq_1 + q'^T_2 Rq_1 = 0$$

Generalized epipolar constraint (GEC)

- Matrix form

$$q_2^T R q'_1 + q_2^T [t]_x R q_1 + q'^T_2 R q_1 = 0$$

$$L_2^T G L_1 = \begin{pmatrix} q_2 \\ q'_2 \end{pmatrix}^T \begin{bmatrix} [t]_x R & R \\ R & 0 \end{bmatrix} \begin{pmatrix} q_1 \\ q'_1 \end{pmatrix} = 0$$

$$G = \begin{bmatrix} [t]_x R & R \\ R & 0 \end{bmatrix}$$

- G … generalized essential matrix (6x6)
- G contains essential matrix! ($E = [t]_x R$)

Linear algorithm for G

$$L_2^T G L_1 = \begin{pmatrix} q_2 \\ q_2' \end{pmatrix}^T \begin{bmatrix} [t]_x R & R \\ R & 0 \end{bmatrix} \begin{pmatrix} q_1 \\ q_1' \end{pmatrix} = 0$$

$$L'^T G L = \begin{pmatrix} x' \\ v' \times x' \end{pmatrix}^T \begin{bmatrix} E & R \\ R & 0 \end{bmatrix} \begin{pmatrix} x \\ v \times x \end{pmatrix} = 0$$

- E is 3x3, R is 3x3, i.e. in total 18 unknowns
- Each line correspondence gives 1 equation like

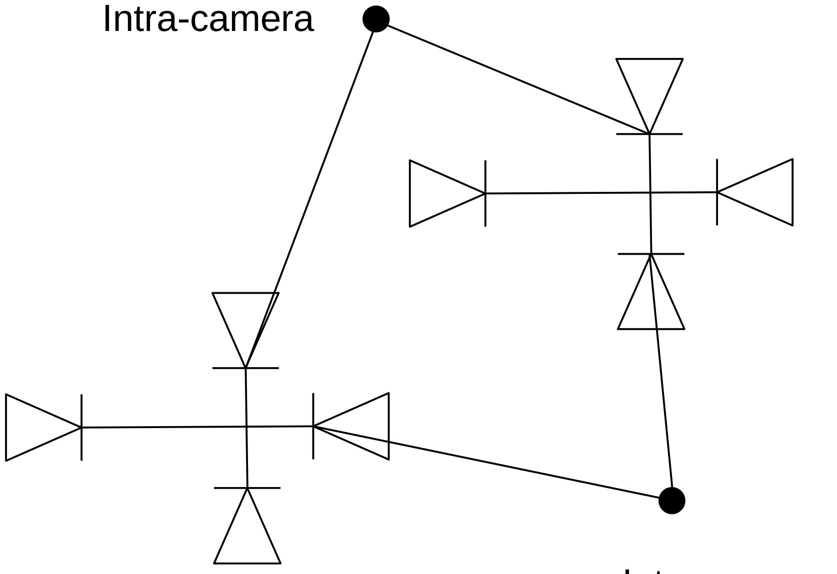
$$x'^T E x + (v' \times x')^T R x + x'^T R (v \times x) = 0$$

- Linear solution needs 17 point correspondences to compute G

Linear algorithm for G

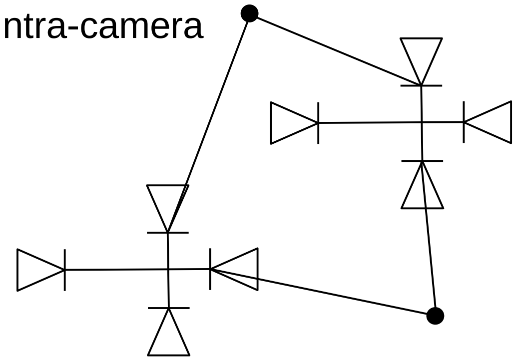
$$
A_i^T y = \begin{bmatrix}
x_1' x_1 \\
x_1' x_2 \\
x_1' x_3 \\
x_2' x_1 \\
x_2' x_2 \\
x_2' x_3 \\
x_3' x_1 \\
x_3' x_2 \\
x_3' x_3 \\
(v_2' x_3' - v_3' x_2') x_1 + x_1' (v_2 x_3 - v_3 x_2) \\
(v_2' x_3' - v_3' x_2') x_2 + x_1' (v_3 x_1 - v_1 x_3) \\
(v_2' x_3' - v_3' x_2') x_3 + x_1' (v_1 x_2 - v_2 x_1) \\
(v_3' x_1' - v_1' x_3') x_1 + x_2' (v_2 x_3 - v_3 x_2) \\
(v_3' x_1' - v_1' x_3') x_2 + x_2' (v_3 x_1 - v_1 x_3) \\
(v_3' x_1' - v_1' x_3') x_3 + x_2' (v_1 x_2 - v_2 x_1) \\
(v_1' x_2' - v_2' x_1') x_1 + x_3' (v_2 x_3 - v_3 x_2) \\
(v_1' x_2' - v_2' x_1') x_2 + x_3' (v_3 x_1 - v_1 x_3) \\
(v_1' x_2' - v_2' x_1') x_3 + x_3' (v_1 x_2 - v_2 x_1)
\end{bmatrix}^T
\begin{bmatrix}
E_{11} \\ E_{12} \\ E_{13} \\ E_{21} \\ E_{22} \\ E_{23} \\ E_{31} \\ E_{32} \\ E_{33} \\ R_{11} \\ R_{12} \\ R_{13} \\ R_{21} \\ R_{22} \\ R_{23} \\ R_{31} \\ R_{32} \\ R_{33}
\end{bmatrix}
$$

Two types of correspondences



- Intra-camera correspondences: Correspondences from the same camera
- Inter-camera correspondences: Correspondences from different cameras

Two types of correspondences



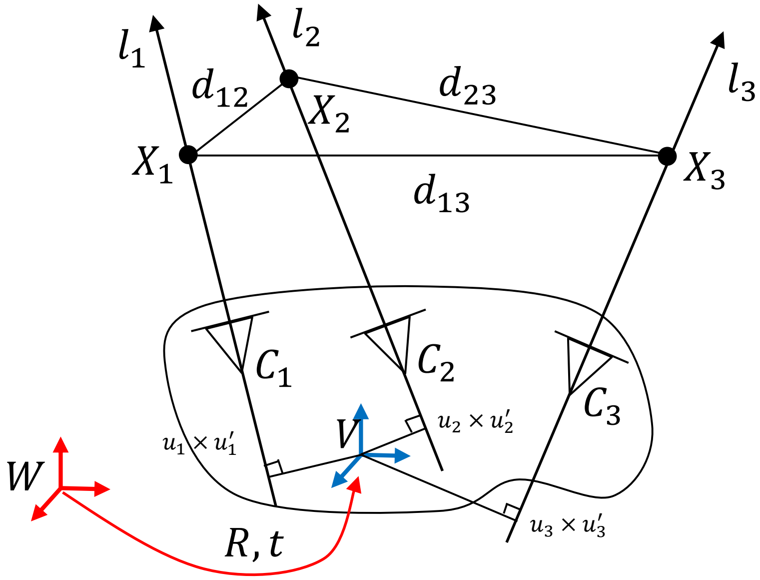
- Problem when rotation is identity and there are only intra-camera correspondences. $R = I \qquad t'_C = t_C$

$$x'^T E x + (t'_C \times x')^T R x + x'^T R (t_C \times x) = 0$$

$$x'^T E x = 0$$

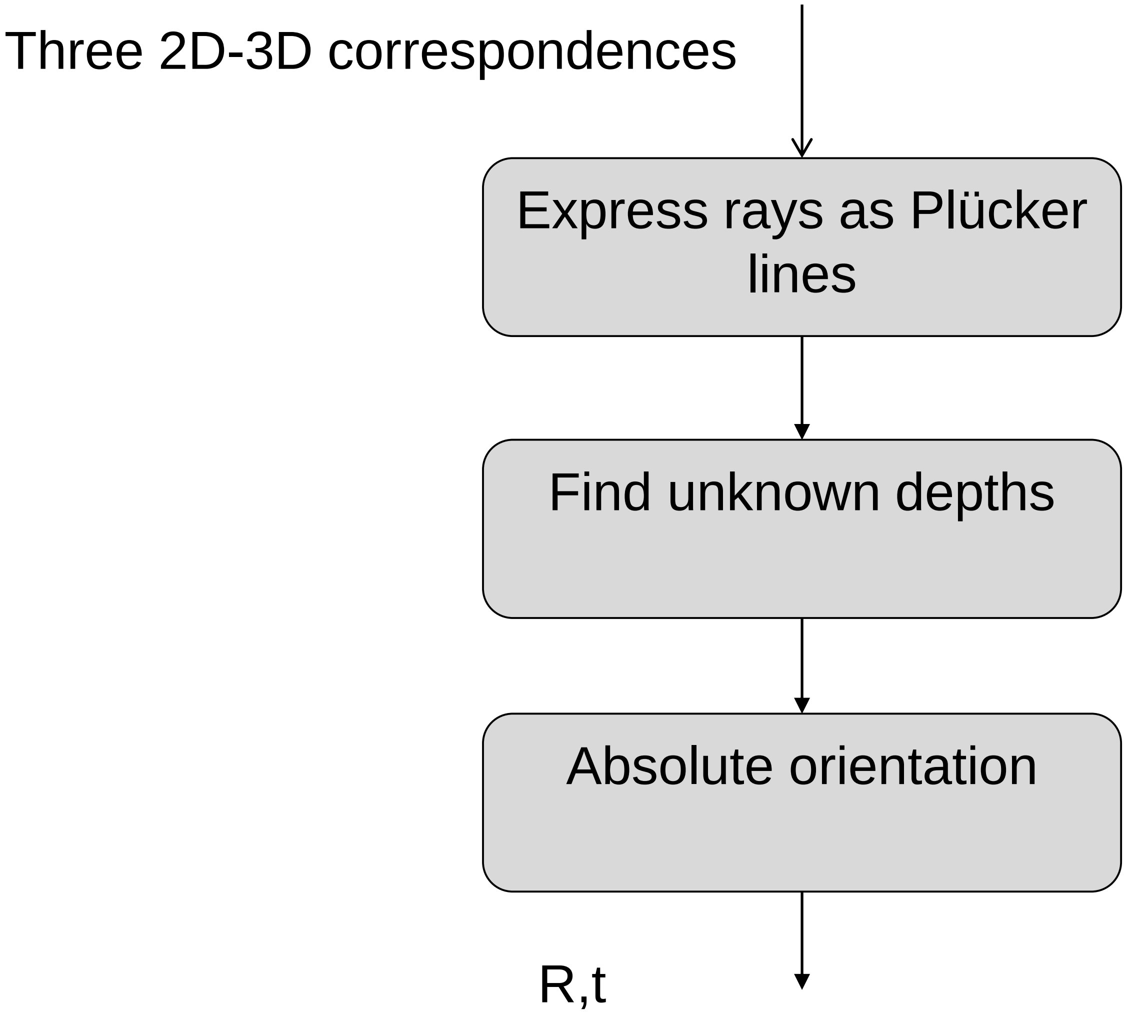
- Scale not observable anymore

Generalized PnP

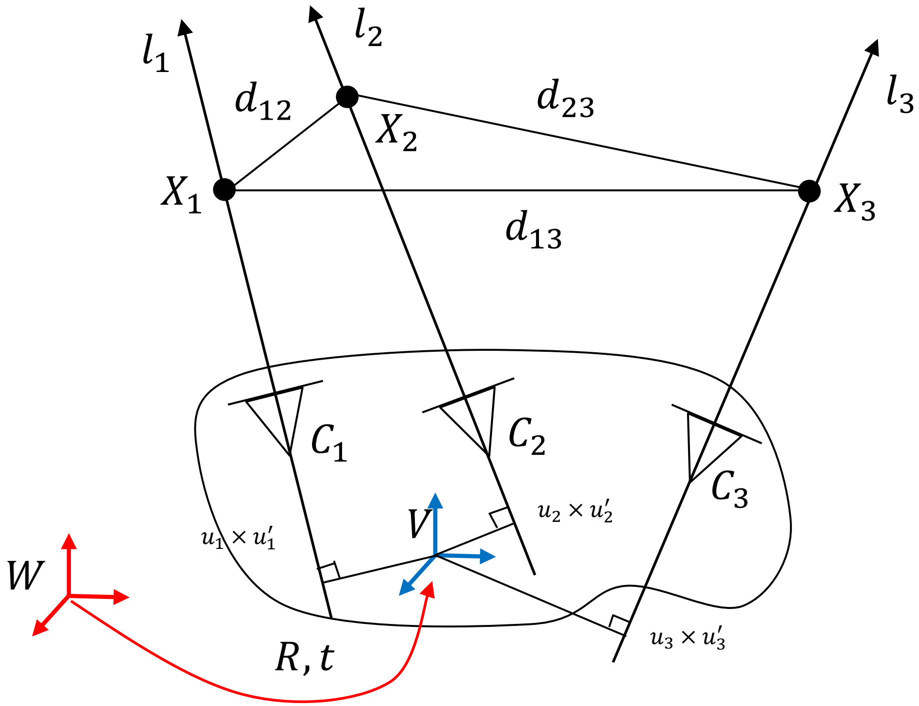


- Very similar to perspective PnP
- Use Plücker coordinates
- But: How many points needed? How many solutions?
- We will show that 3 points are needed and this yields 8 solutions.

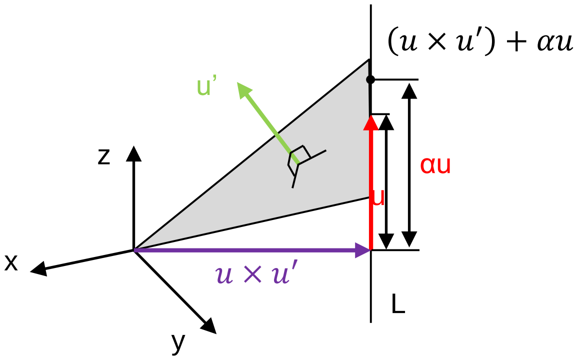
Generalized PnP



Generalized PnP



Plücker coordinates



$$l_i = (u_i, u'_i)$$

$$X_i^V = u_i \times u'_i + \lambda_i u_i$$

$$d_{ij} = \|X_i - X_j\|^2 = \|X_i^V - X_j^V\|^2$$ depths

$$d_{ij} = \|X_i - X_j\|^2 = \|(u_i \times u'_i + \lambda_i u_i) - (u_j \times u'_j + \lambda_j u_j)\|^2$$

Generalized PnP

- 3 distances lead to 3 equations with unknown depths (lambda)

$$k_{11}\lambda_1^2 + (k_{12}\lambda_2 \ + k_{13})\lambda_1 \ + (k_{14}\lambda_2^2 + k_{15}\lambda_2 \ + k_{16}) = 0$$
$$k_{21}\lambda_1^2 + (k_{22}\lambda_3 \ + k_{23})\lambda_1 \ + (k_{24}\lambda_3^2 + k_{25}\lambda_3 \ + k_{26}) = 0$$
$$k_{31}\lambda_2^2 + (k_{32}\lambda_3 \ + k_{33})\lambda_2 \ + (k_{34}\lambda_3^2 + k_{35}\lambda_3 \ + k_{36}) = 0$$

- Eliminating variables using resultants (determinant of Sylvester matrix) leads to an 8 degree polynomial

$$A\lambda_3^8 + B\lambda_3^7 + \mathrm{C}\lambda_3^6 + \mathrm{D}\lambda_3^5 + \mathrm{E}\lambda_3^4 + \mathrm{F}\lambda_3^3 + \mathrm{G}\lambda_3^2 + \mathrm{H}\lambda_3 \ + I = 0$$

- No closed form solution possible (Root solving with companion matrix or Sturm bracket method)

Recap - Learning goals

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