Mathematical Principles in Visual Computing

Prof. Friedrich Fraundorfer

SS 2023

About me

- Prof. Dr. Friedrich Fraundorfer
- Email: fraundorfer@icg.tugraz.at
- Institute of Computer Graphics and Vision
- Inffeldgasse 16/II
- +43 (316) 873 - **5020**
- Send email to schedule an appointment

Additional lecturer

- Dr. Jörg Müller
- Email: joerg.mueller@icg.tugraz.at
- Institute of Computer Graphics and Vision
- Inffeldgasse 16/II

Lecture schedule

- 01.03.2023 Fraundorfer
- 08.03.2023 Fraundorfer
- 15.03.2023 Fraundorfer
- 22.03.2023 Fraundorfer
- 29.03.2023 Fraundorfer
- 19.04.2023 Fraundorfer
- 26.04.2023 Fraundorfer
- 03.05.2023 Fraundorfer
- 10.05.2023 Fraundorfer
- 17.05.2023 Fraundorfer
- 24.05.2023 Fraundorfer
- 31.05.2023 Müller
- 07.06.2023 Müller
- 14.06.2023 Müller
- 21.06.2023 Müller
- 28.06.2023 Exam

Tutor

- Lukas Radl
- Email: radl@student.tugraz.at
- Responsible for questions about classroom assignments
- Q&A slots with tutor
- Q&A in TC forum or e-mail

Course grading

- 3 class room assignments (50% of grade)
  - Math problems
  - Small programming assignments
- Final written exam (50% of grade)

- Written exam at last lecture slot (27 June 2023)
- Submitting the first assignment counts as attempt. A grade will be issued in this case.

- *“§ 22 para. 4: In order to assist students in completing their degrees in a timely manner, all courses with continual assessment must allow students to submit, supplement or repeat in any case at least one partial course requirement to be determined by the course director, by no later than four weeks after the course has ended.”*
- This one course requirement is the examination.

Assignments

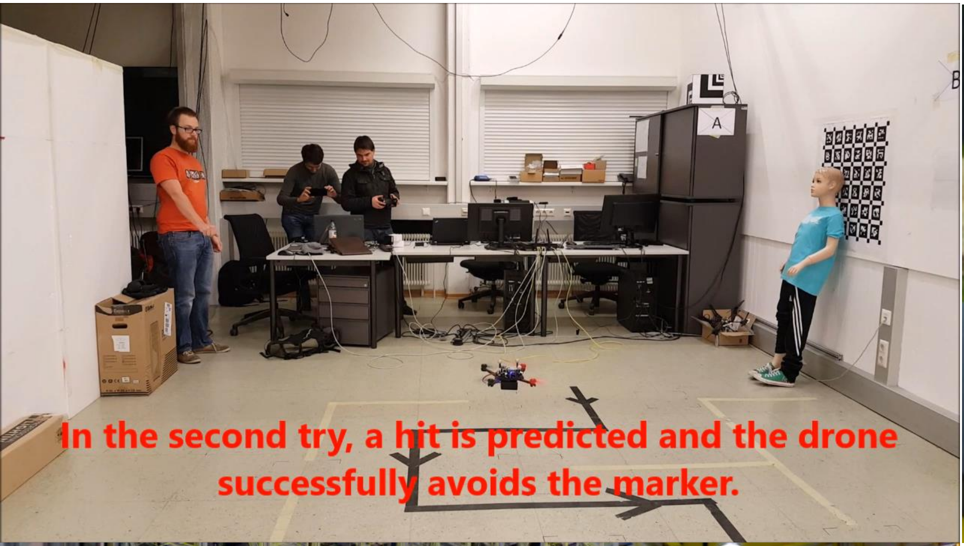
- Individual work, no group work
- Electronic submission using the TeachCenter (Hand-writing and scanning is ok)

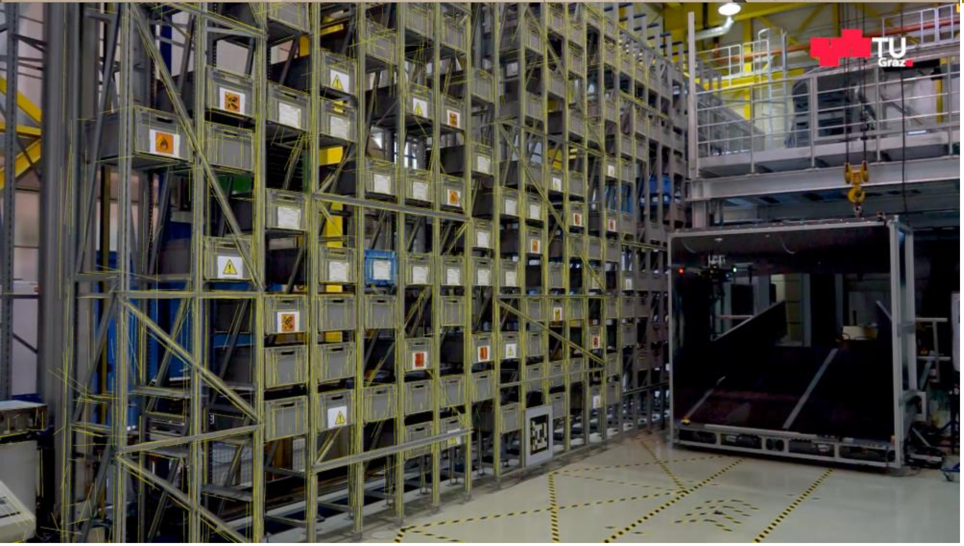
- Schedule:
  - Assignment 1
    - Handout: 15.3.2023
    - Deadline: 2.5.2023
  - Assignment 2
    - Handout: 3.5.2023
    - Deadline: 30.5.2023
  - Assignment 3
    - Handout: 31.5.2023
    - Deadline: 20.6.2023

Lecture material

- Slides are the main material
- Links to relevant publications and book sections will be given
- Lecture recordings from last years are available in the Teach Center

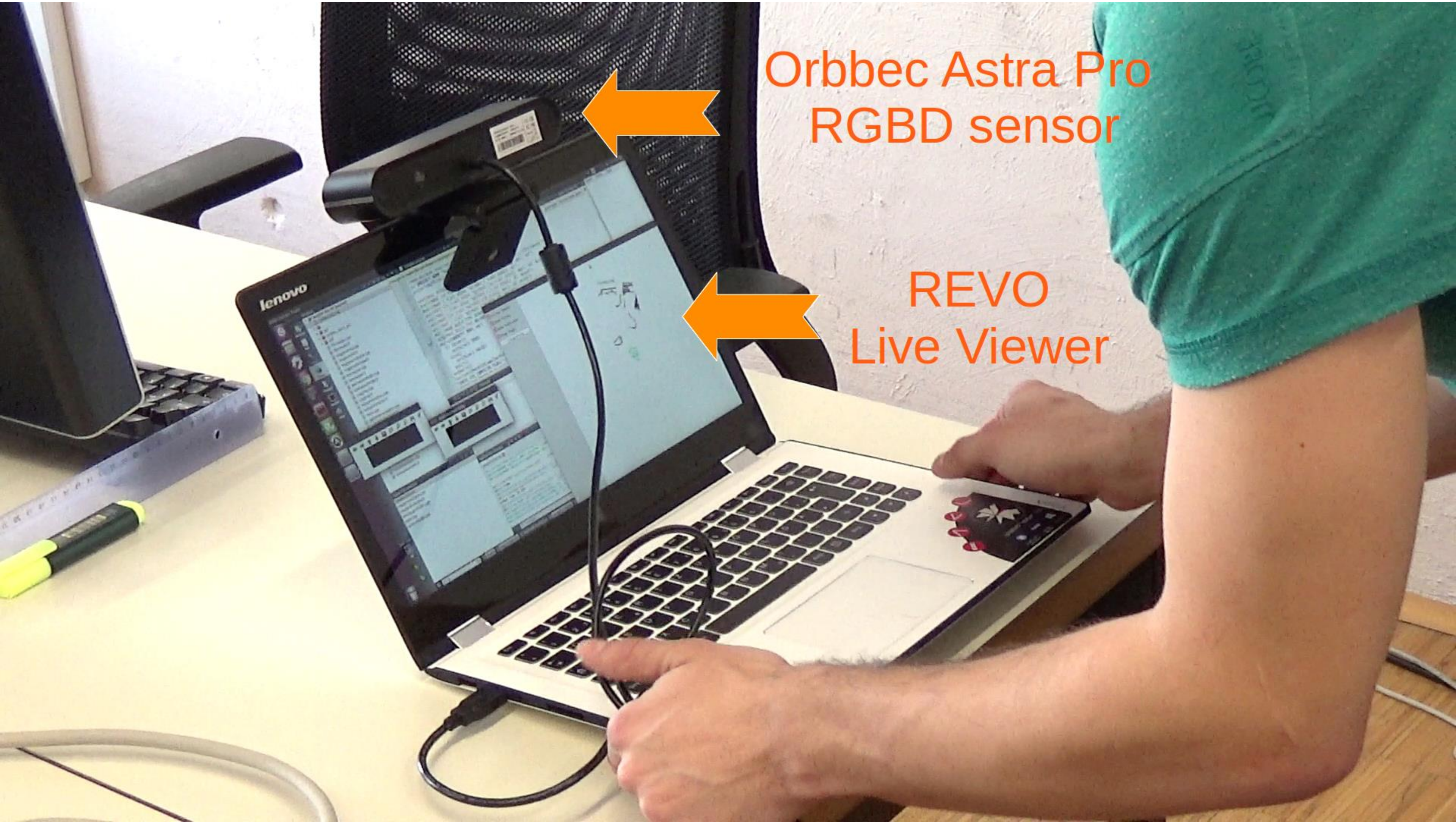
Research areas



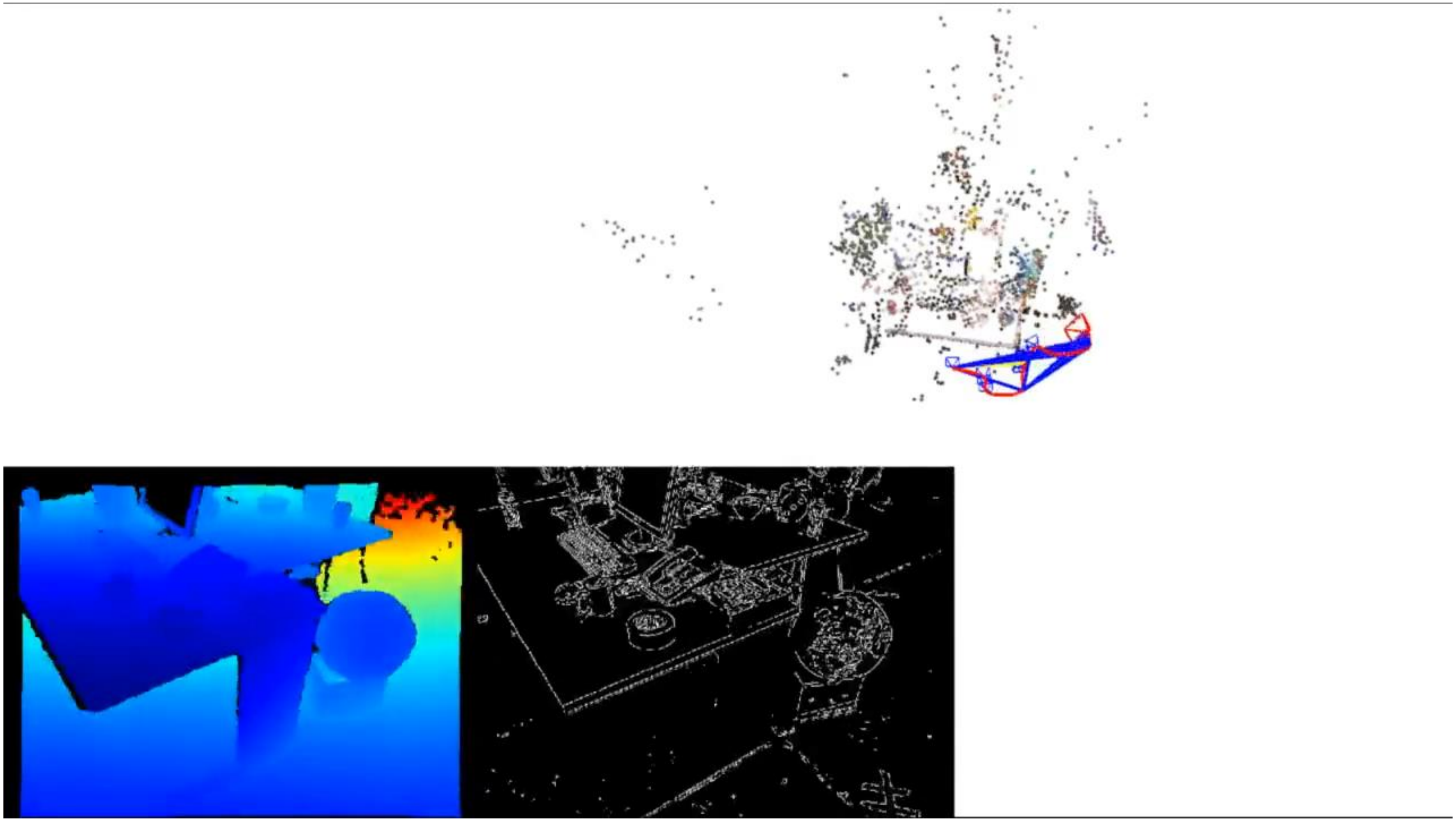


3D scanning - REVO

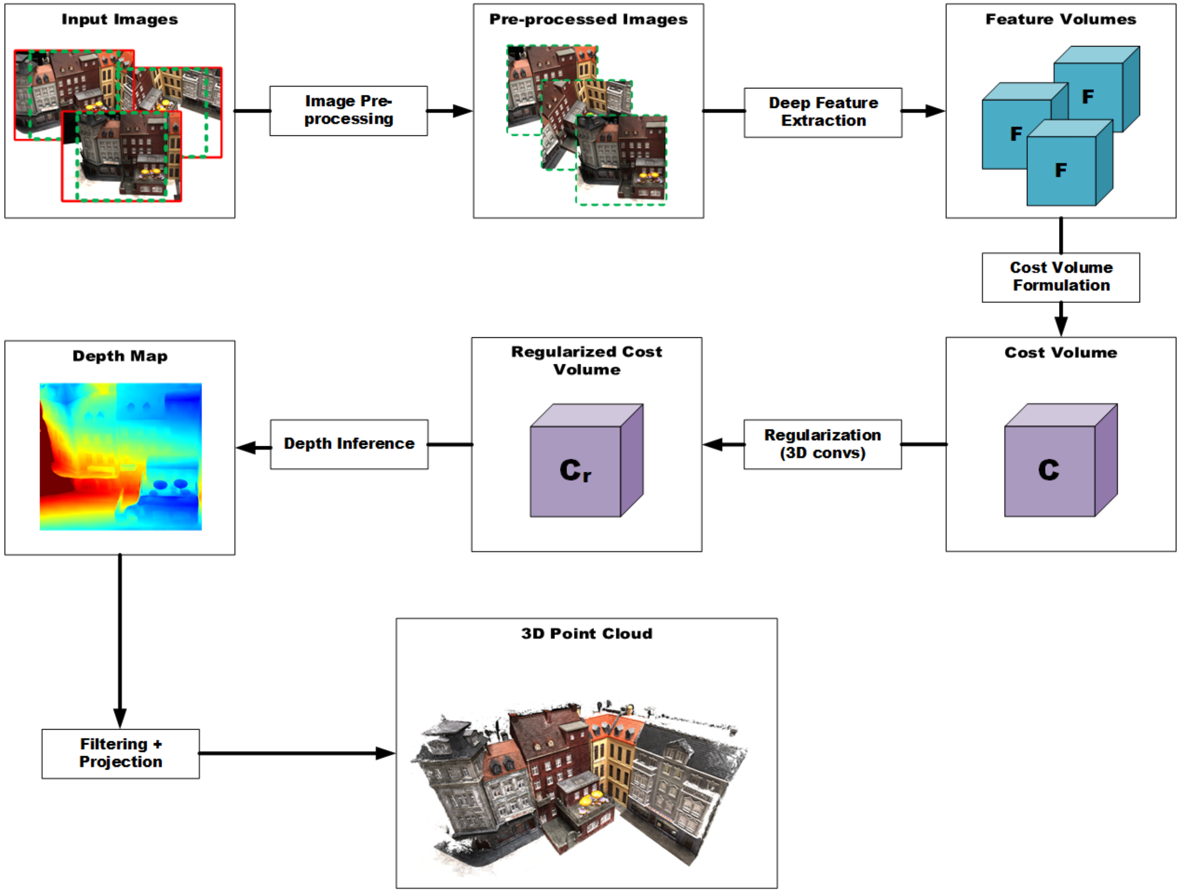
- RGBD recordings with Orbbec Astra Pro



3D scanning - REVO



Multi-View Stereo Pipeline

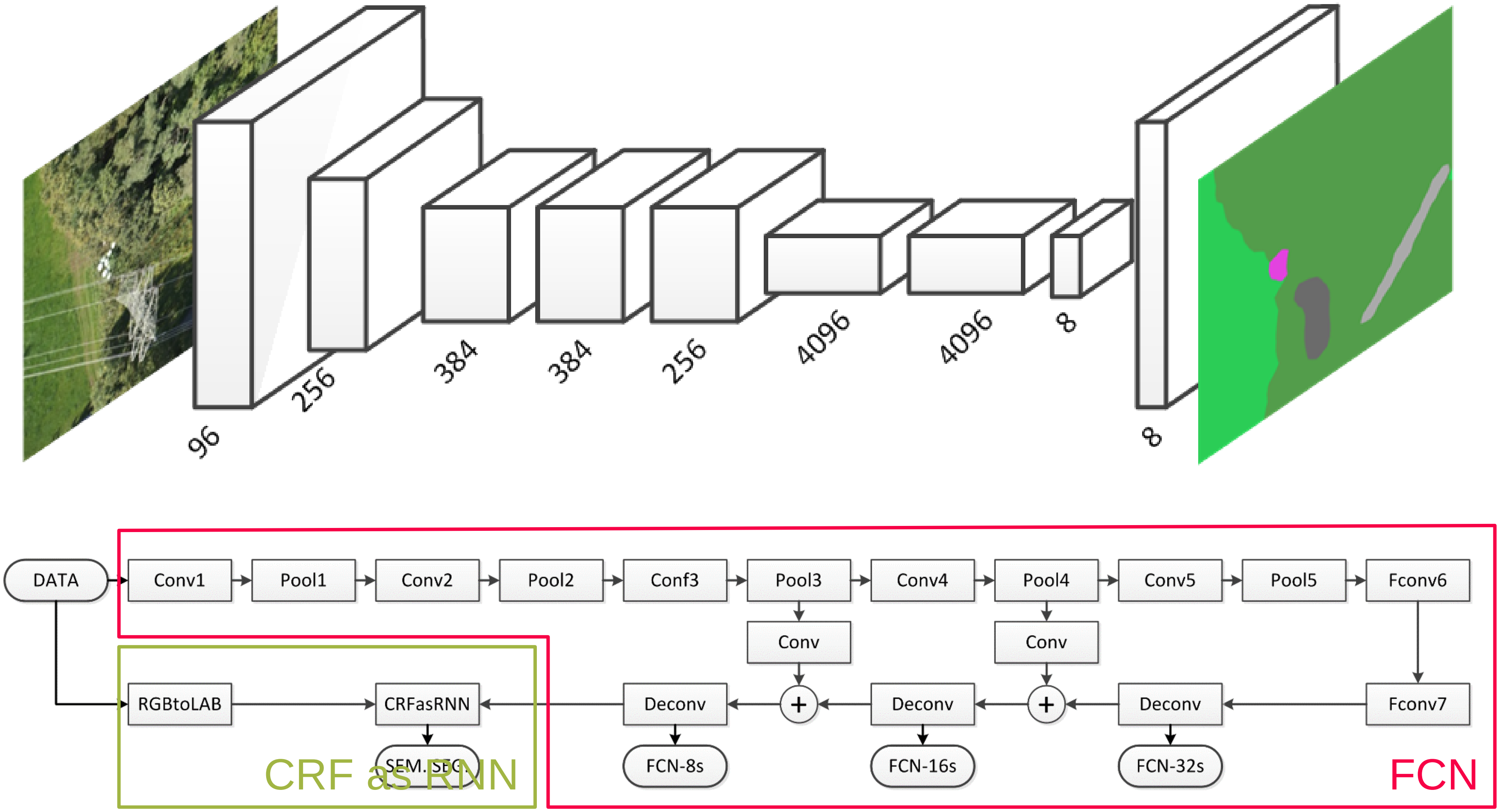


Bridge inspection

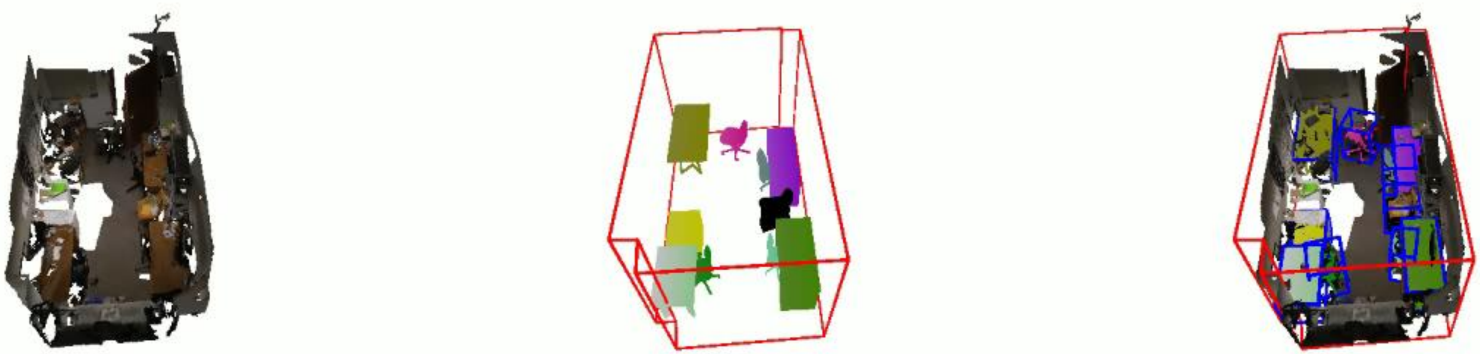
Bridge inspection

Semantic 3D

Semantic 3D


96
256
384
384
256
4096
4096
8
8
DATA
Conv1
Pool1
Conv2
Pool2
Conf3
Pool3
Conv4
Pool4
Conv5
Pool5
Fconv6
Conv
Conv
RGBtoLAB
CRFasRNN
Deconv
Deconv
Deconv
Fconv7
SEM.SEG
FCN-8s
FCN-16s
FCN-32s
CRF as RNN
FCN

3D scene understanding



Embedded AI – Dedicated processors allow integration of deep learning

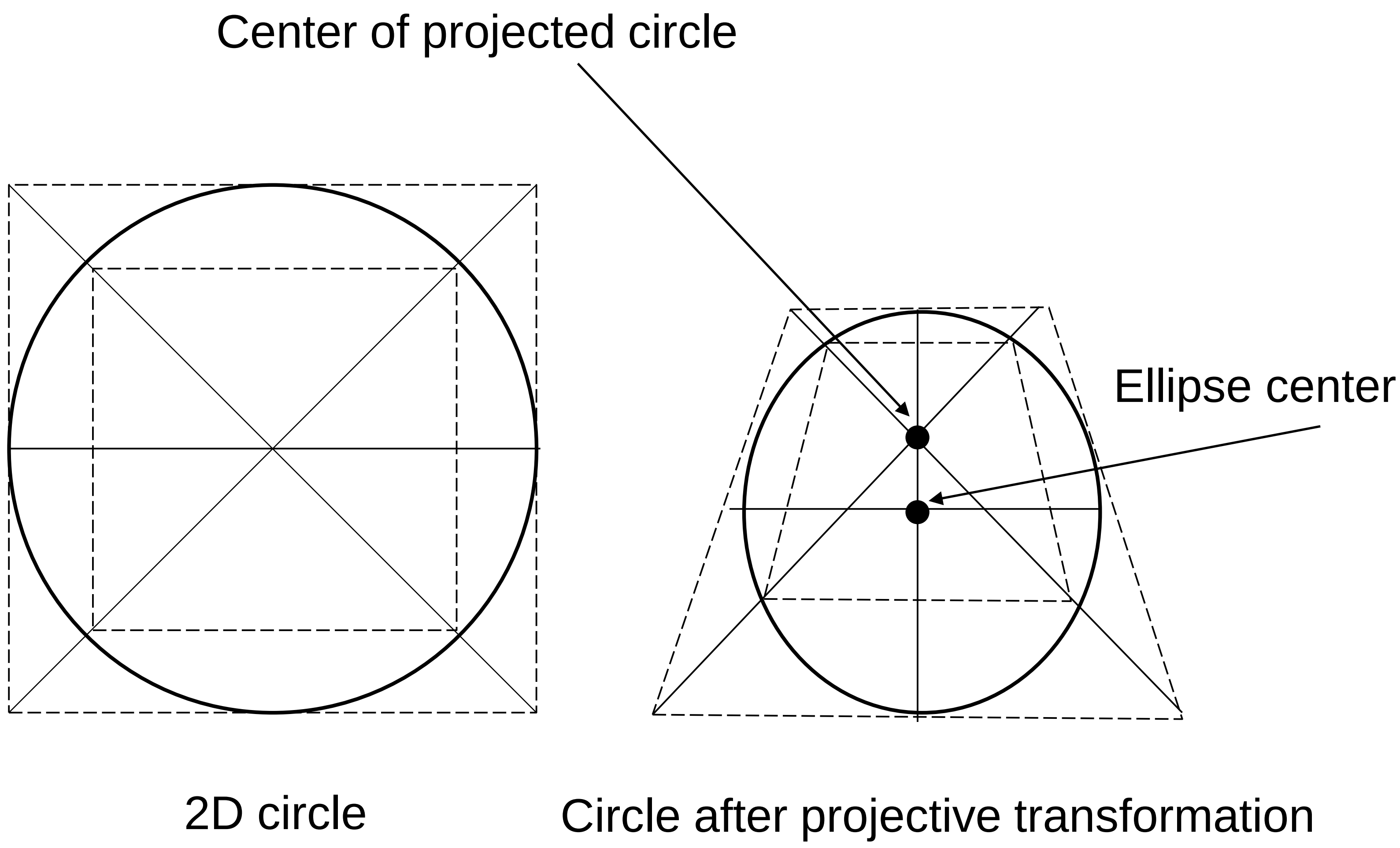
Embedded AI – Object detection

Topics

- Projective geometry
- Geometry of multi-view camera system
- Parameterization of rigid transformations
- Robust estimation (Ransac, Robust cost functions)
- Polynomial systems in computer vision
- Root-solving
- Projective geometric algebra (Müller)

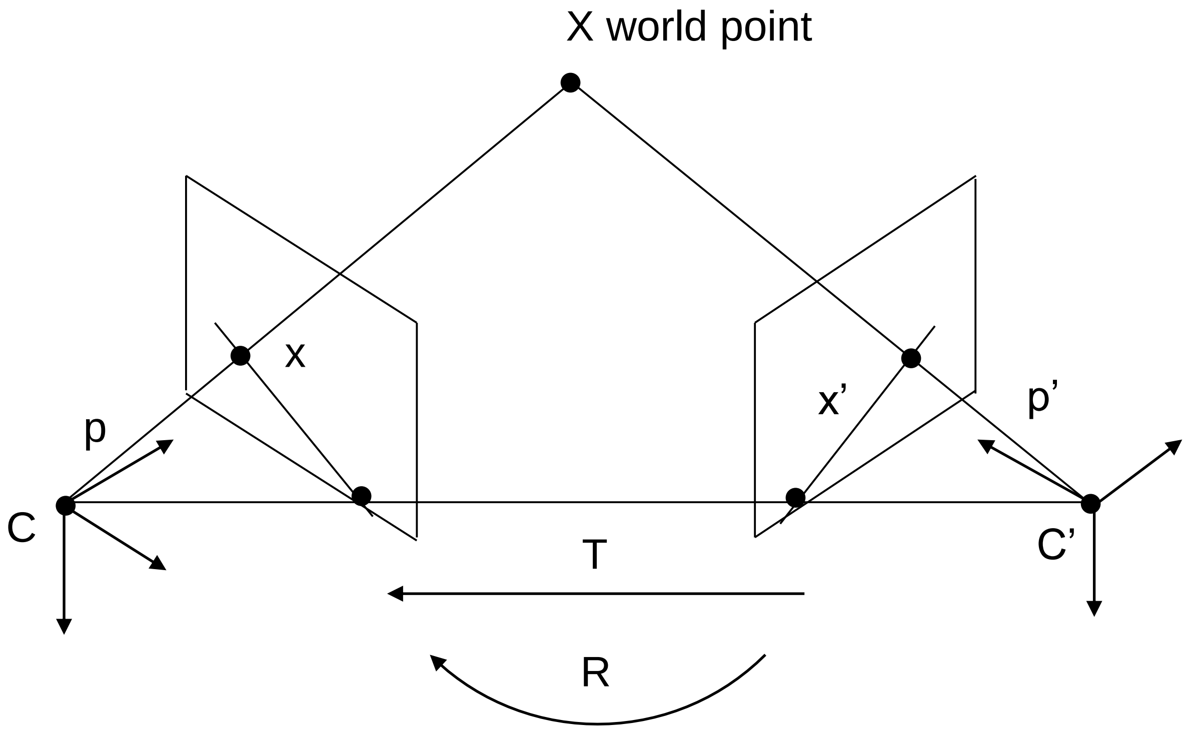
Projective geometry

Projective geometry

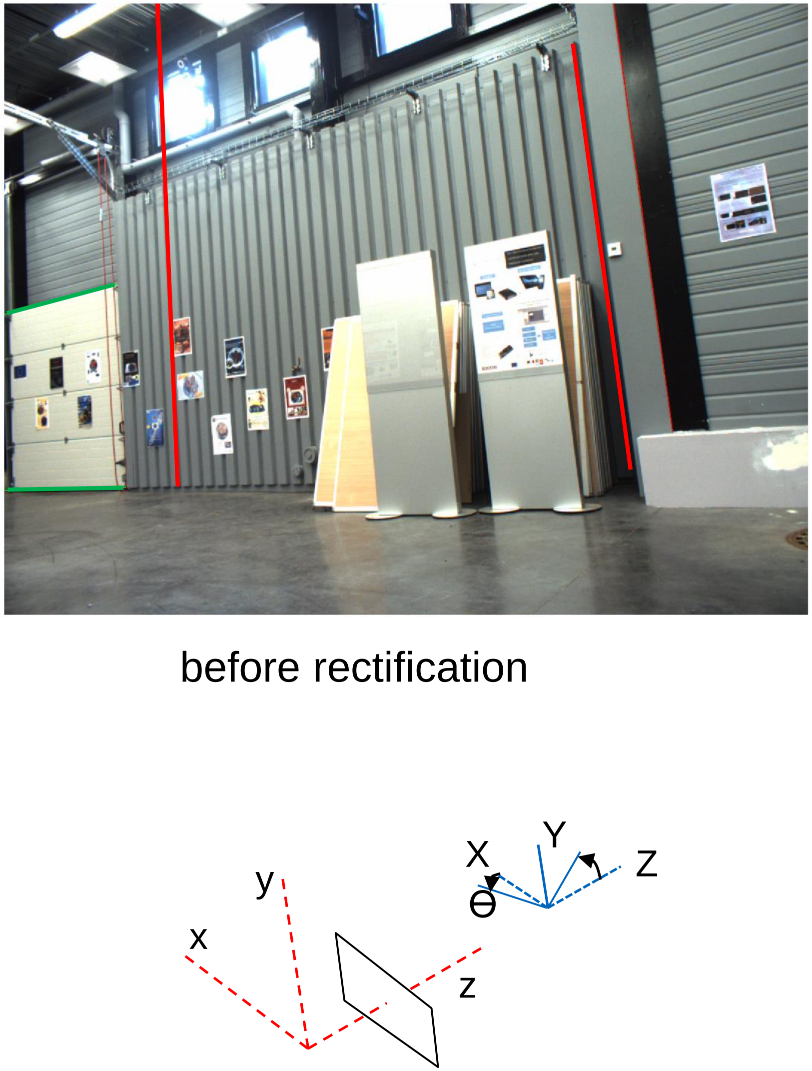


Projective geometry

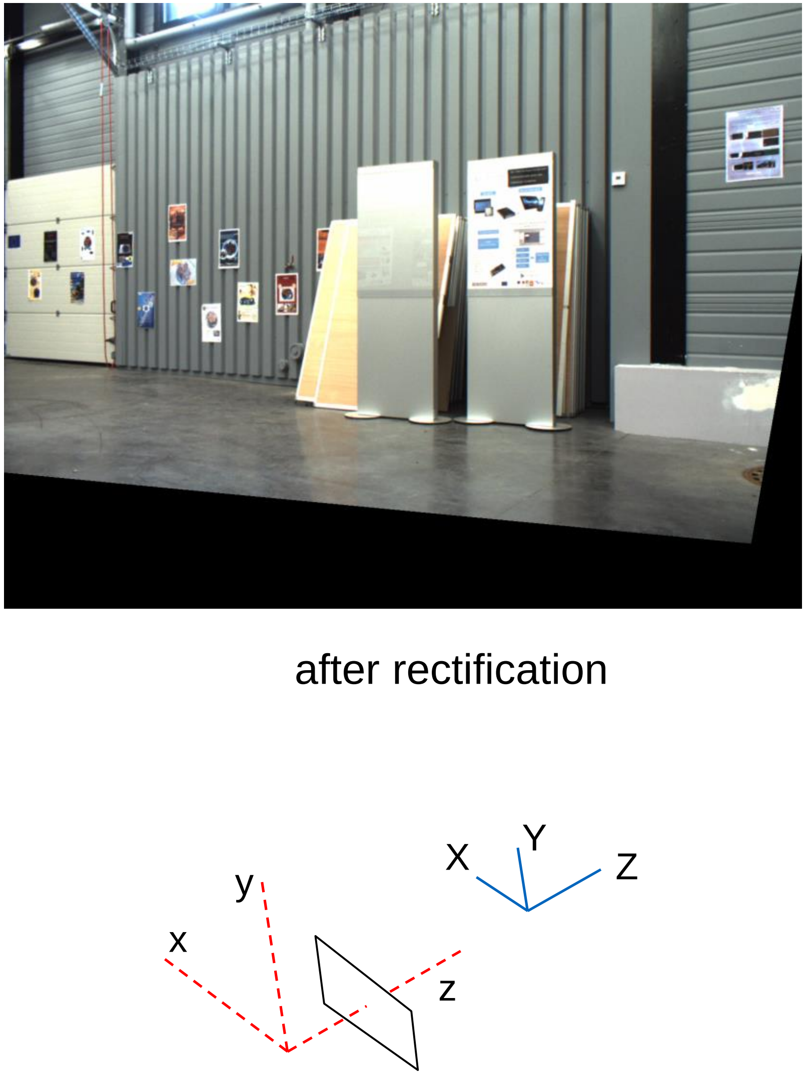
$$x'^T F x = 0 \;\; \ldots \textit{Epipolar constraint}$$



Projective geometry



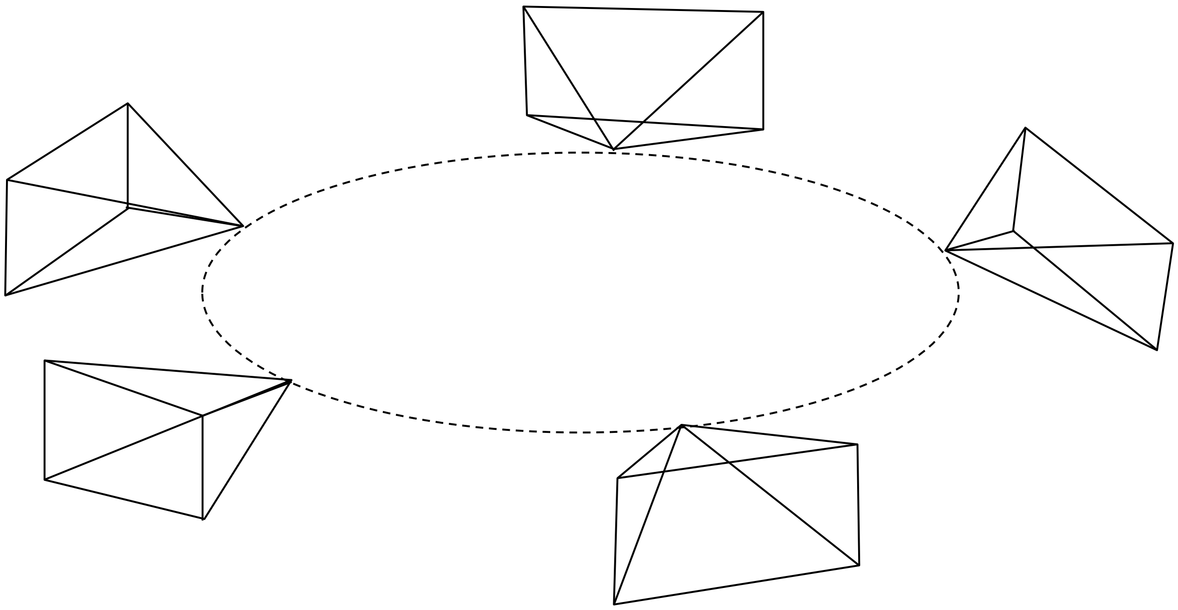
before rectification



after rectification

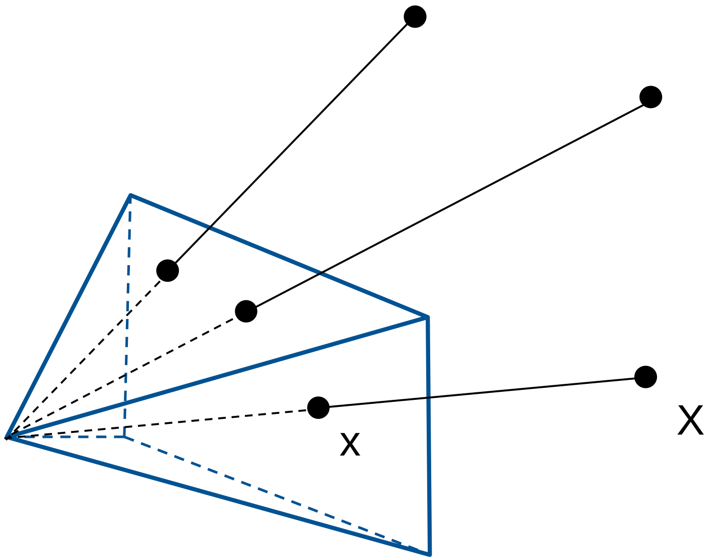
Geometry of multi-view camera system

$$l'^T E_G l = 0 \quad \textit{... generalized Epipolar constraint}$$

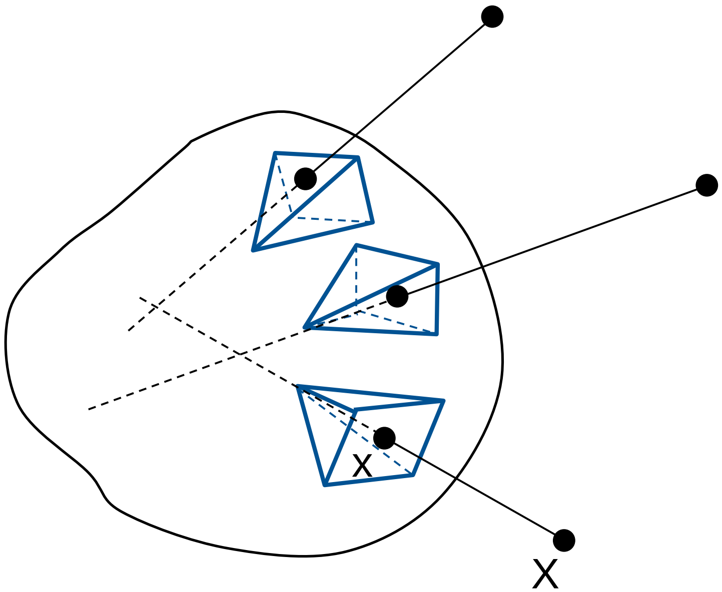


Geometry of multi-view camera system

Geometry of multi-view camera system



pinhole camera

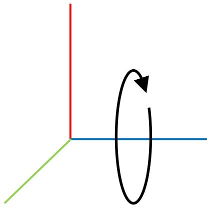


generalized camera

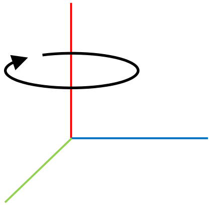
Hololens is a multi-camera system

Parameterization of rigid transformations

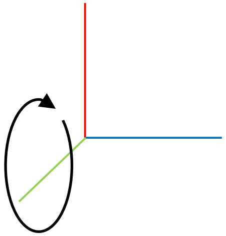
$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$



$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

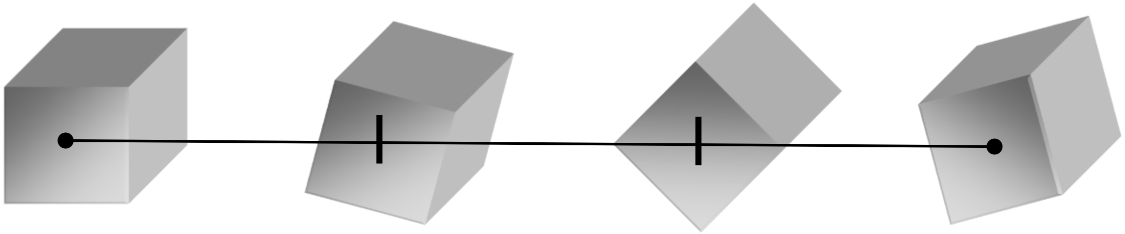


$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Problems with rotation matrices

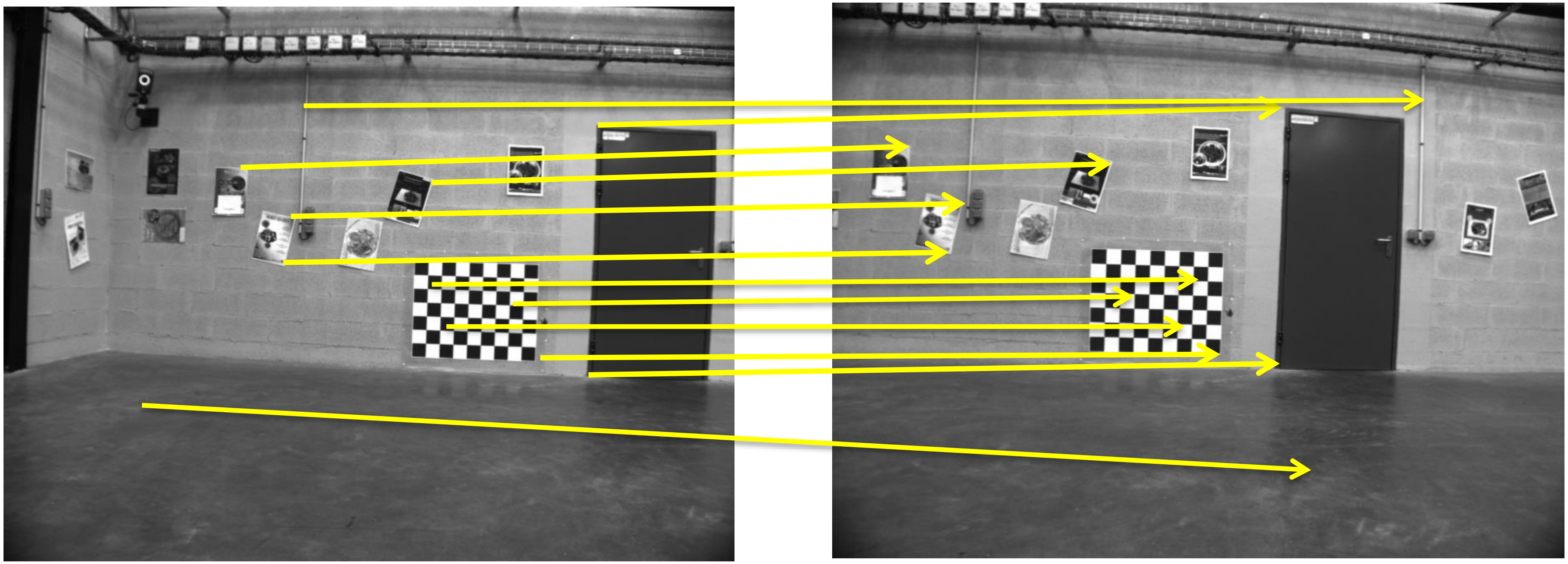
- Optimization of rotations (bundle adjustment)
  - Newton's method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
- Linear interpolation



- Filtering and averaging
  - E.g. averaging rotation from IMU or camera pose tracker for AR/VR glasses

Robust estimation

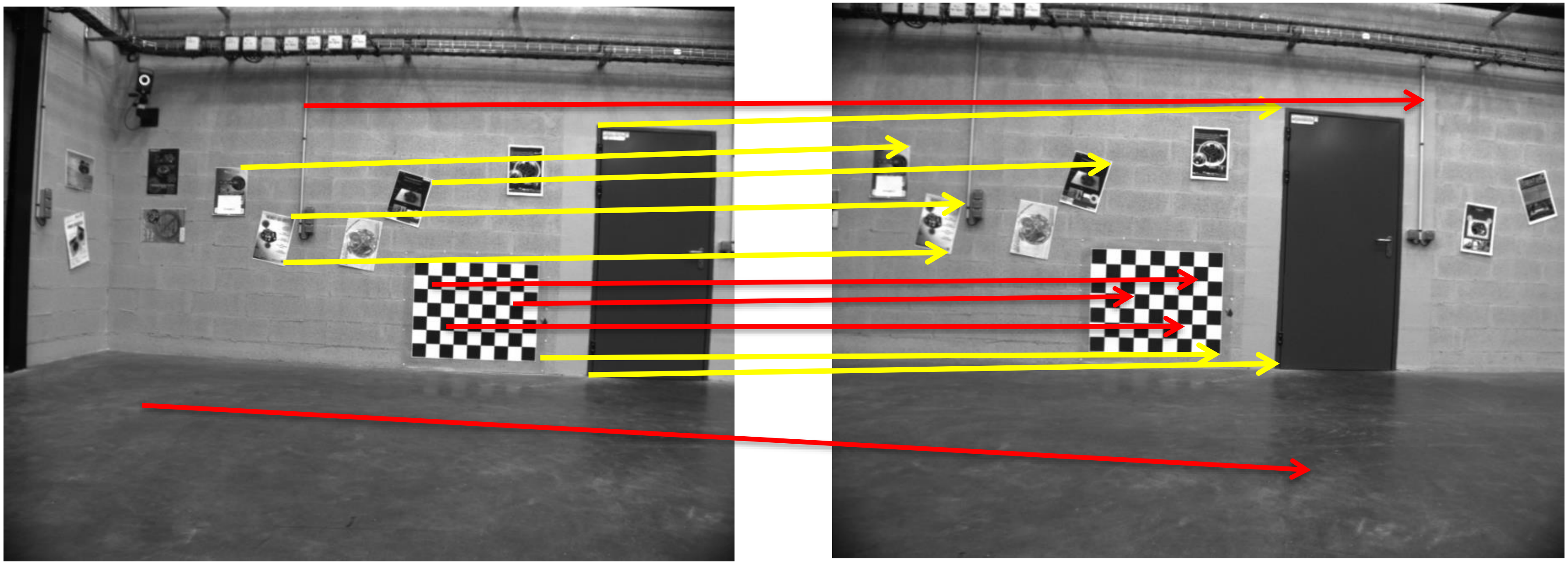
- Motion estimation needs to be robust against mismatches



Yellow: automatically generated image matches
(contain mis-matches)

Robust estimation

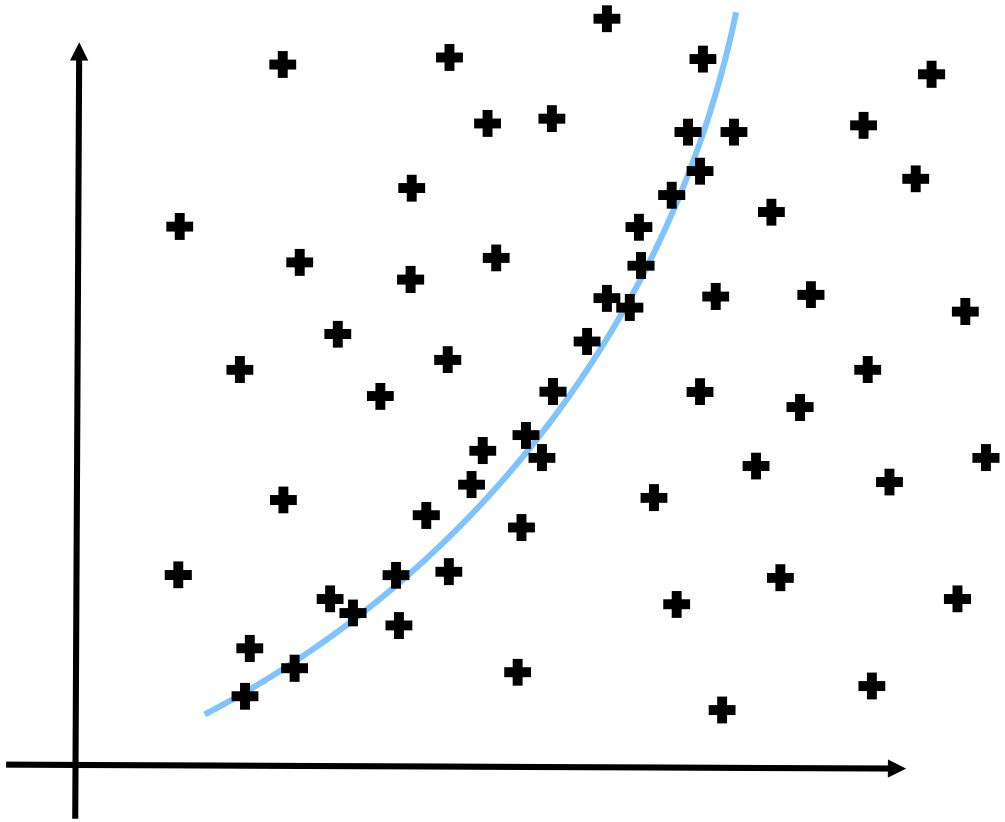
- All feature matches need to follow the same motion



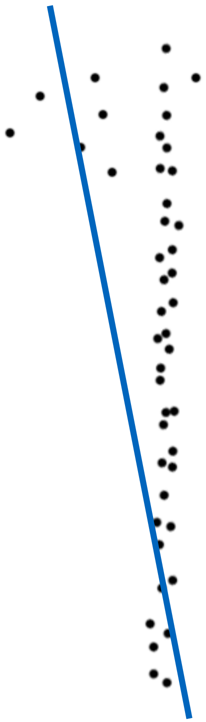
Yellow: correct matches
Red: incorrect matches.

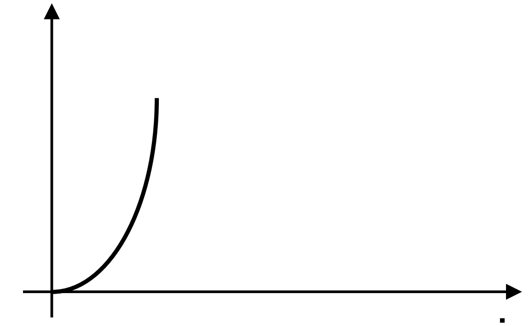
Robust estimation

- Ransac – Random sample consensus



Robust estimation

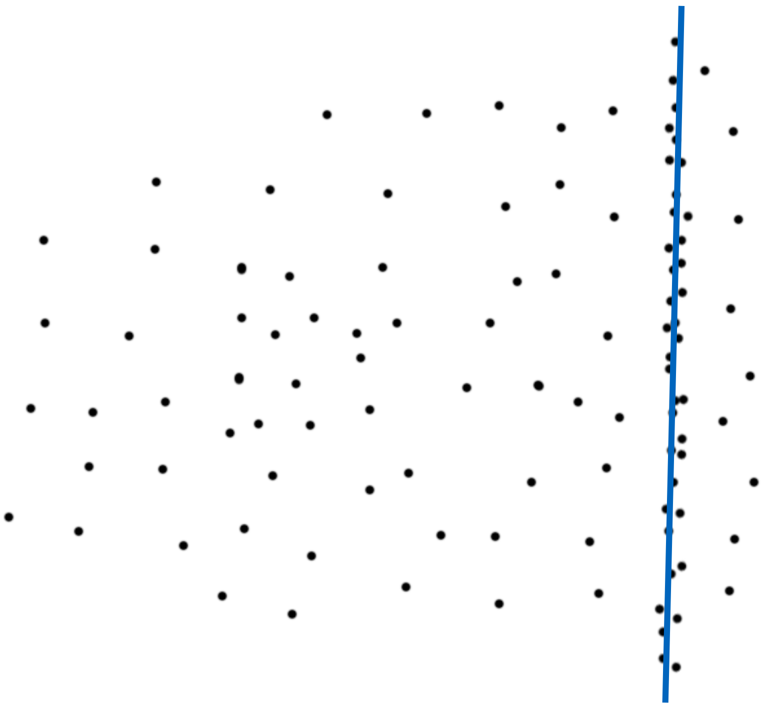

L2 solution

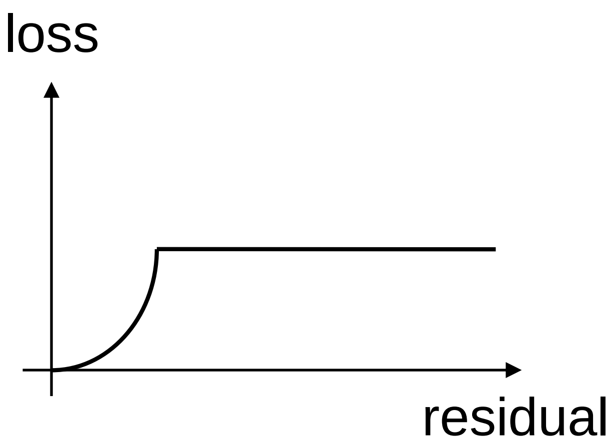


$$\min_x \sum_i r_i^2(x)$$

- Outliers lead to wrong estimate

Robust estimation

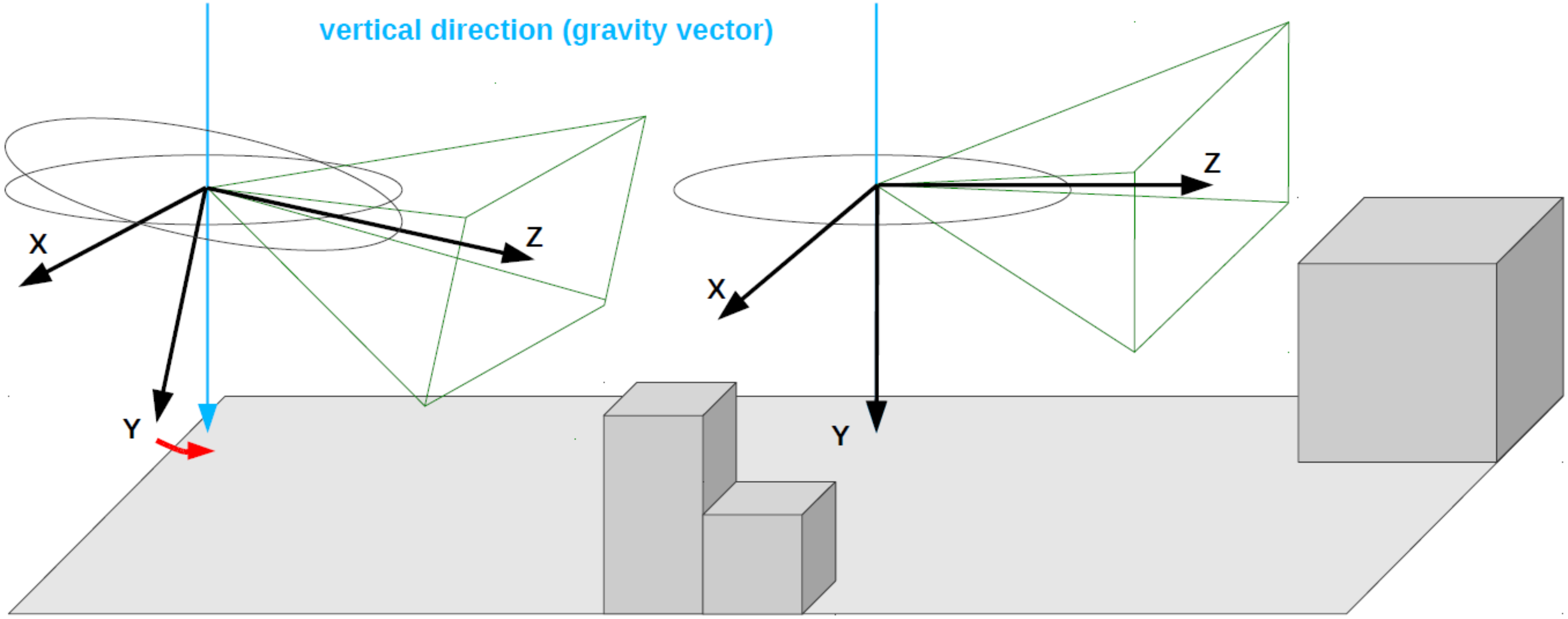




$$\min_{x} \sum_{i} \min(r_i^2(x), t^2)$$

Polynomial equation systems in computer vision

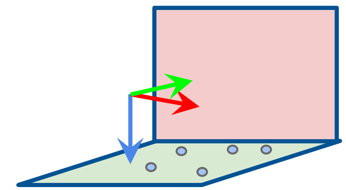
- Assumption: Ground plane normal to gravity vector, walls are vertical
  - IMU measurements can be used to align camera images/features to gravity vector
  - Motion can be computed from 2pt correspondences on the ground



Polynomial equation systems in computer vision
2pt relative pose

Homography — Rotation (yaw) — Rotation IMU — Translation — Plane normal

$$\mathbf{H} = \mathbf{R}_y \mathbf{R}_x \mathbf{R}_z + \mathbf{t}^T \mathbf{n}$$



$$\mathbf{H} = \mathbf{R}_y + \mathbf{t}^T \mathbf{n}$$ (for pre-rotated features)

$$\mathbf{H} = \mathbf{R_y} + [t_x, t_y, t_z]^T [0, 1, 0]$$ (H for ground plane)

Homography — Rotation (yaw) — Translation — Plane normal

4 unknowns left, 2 point correspondences give 4 equations

Polynomial systems in computer vision

- P3P, PnP problem:

$$\begin{array}{l} L_1 \\ L_2 \\ L_3 \end{array} \begin{cases} 2x^2 + y^2 - 2z + 3z^2 + 5 & = & 0 \\ x^2 + z + z^2 & = & 0 \\ x^2y^2 + y^2z^2 - 2 & = & 0 \end{cases}$$

- Solution: Reduction to a single polynomial (several schemes)
- Automatic procedure – Gröbner Basis

Polynomial equation systems

Root solving

- 3pt+IMU, 8th degree polynomial
- 6pt generalized camera, 64th degree polynomial

- Fast method: Sturm bracketing

Projective Geometric Algebra