Mathematical Principles in Visual Computing

Prof. Friedrich Fraundorfer

SS 2022
About me

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- Institute of Computer Graphics and Vision
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- Send email to schedule an appointment
Additional lecturer

- Assoc. Prof. Dr. Markus Steinberger
- Email: steinberger@icg.tugraz.at
- Institute of Computer Graphics and Vision
- Inffeldgasse 16/II
Lecture schedule

- 02.03.2022 Fraundorfer
- 09.03.2022 Fraundorfer
- 16.03.2022 Fraundorfer
- 23.03.2022 Fraundorfer
- 30.03.2022 Fraundorfer
- 06.04.2022 Fraundorfer
- 27.04.2022 Fraundorfer
- 04.05.2022 Fraundorfer
- 11.05.2022 Fraundorfer
- 18.05.2022 Fraundorfer
- 25.05.2022 Fraundorfer
- 01.06.2022 Steinberger
- 08.06.2022 Steinberger
- 15.06.2022 Steinberger
- 22.06.2022 Steinberger
- 29.06.2022 Exam
Tutor

- Christian Sormann
- Email: christian.sormann@icg.tugraz.at
- Responsible for questions about classroom assignments
- Q&A slots with tutor
- Q&A in TC forum or e-mail
Course grading

- 3 class room assignments (50% of grade)
  - Math problems
  - Small programming assignments
- Final written exam (50% of grade)

- Written exam at last lecture slot (29 June 2022)
- Submitting the first assignment counts as attempt. A grade will be issued in this case.

- “§ 22 para. 4: In order to assist students in completing their degrees in a timely manner, all courses with continual assessment must allow students to submit, supplement or repeat in any case at least one partial course requirement to be determined by the course director, by no later than four weeks after the course has ended.”
- This one course requirement is the examination.
Assignments

- Individual work, no group work
- Electronic submission using the TeachCenter (Hand-writing and scanning is ok)

Schedule:

- Assignment 1
  - Handout: 16.3.2022
  - Deadline: 3.5.2022
- Assignment 2
  - Handout: 4.5.2022
  - Deadline: 31.5.2022
- Assignment 3
  - Handout: 1.6.2022
  - Deadline: 28.6.2022
Lecture material

- Slides are the main material
- Links to relevant publications and book sections will be given
- Lecture recordings from last year are available in the Teach Center
Research areas

In the second try, a hit is predicted and the drone successfully avoids the marker.
3D scanning - REVO

- RGBD recordings with Orbbec Astra Pro
3D scanning - REVO
Semantic 3D
Semantic 3D

FCN as RNN
Single image room layout estimation

General 3D Room Layout from a Single View by Render-and-Compare. Sinisa Stekovic, Shreyas Hampali, Mahdi Rad, Sayan Deb Sarkar, Friedrich Fraundorfer, Vincent Lepetit
https://github.com/vevenom/RoomLayout3D_RandC
Single image room layout estimation
Embedded AI – Dedicated processors allow integration of deep learning
Embedded AI – Object detection
Topics

- Projective geometry
- Geometry of multi-view camera system
- Parameterization of rigid transformations
- Robust estimation (Ransac, Robust cost functions)
- Polynomial systems in computer vision
- Root-solving
- Projective geometric algebra (Steinberger)
- Mesh Matrix Basics (Steinberger)
Projective geometry
Projective geometry

Center of projected circle

2D circle

Circle after projective transformation

Ellipse center
Projective geometry

\[ x'^T F x = 0 \quad \ldots \text{Epipolar constraint} \]
Projective geometry

before rectification

after rectification
Geometry of multi-view camera system

\[ l'^T E_G l = 0 \quad \text{... generalized Epipolar constraint} \]
Geometry of multi-view camera system
Geometry of multi-view camera system

pinhole camera

generalized camera
Hololens is a multi-camera system
Parameterization of rigid transformations

\[ R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \]

\[ R_y(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \]

\[ R_z(\gamma) = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \]
Problems with rotation matrices

- Optimization of rotations (bundle adjustment)
  - Newton’s method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

- Linear interpolation

- Filtering and averaging
  - E.g. averaging rotation from IMU or camera pose tracker for AR/VR glasses
Robust estimation

- Motion estimation needs to be robust against mismatches

Yellow: automatically generated image matches (contain mis-matches)
Robust estimation

- All feature matches need to follow the same motion

Yellow: correct matches
Red: incorrect matches.
Robust estimation

- Ransac – Random sample consensus
Robust estimation

- Outliers lead to wrong estimate

\[ \min_x \sum_i r_i^2(x) \]
Robust estimation

\[
\min_x \sum_i \min(r_i^2(x), t^2)
\]
Polynomial equation systems in computer vision

- Assumption: Ground plane normal to gravity vector, walls are vertical
  - IMU measurements can be used to align camera images/features to gravity vector
  - Motion can be computed from 2pt correspondences on the ground
Polynomial equation systems in computer vision

2pt relative pose

\[
H = \begin{bmatrix} R_y \\ R_x \\ R_z \end{bmatrix} + t^T n
\]

\[
H = R_y + t^T n \quad \text{(for pre-rotated features)}
\]

\[
H = R_y + \begin{bmatrix} t_x, t_y, t_z \end{bmatrix}^T [0, 1, 0] \quad \text{(H for ground plane)}
\]

4 unknowns left, 2 point correspondences give 4 equations
Polynomial systems in computer vision

- P3P, PnP problem:

\[
\begin{align*}
L_1 & : 2x^2 + y^2 - 2z + 3z^2 + 5 &= 0 \\
L_2 & : x^2 + z + z^2 &= 0 \\
L_3 & : x^2y^2 + y^2z^2 - 2 &= 0
\end{align*}
\]

- Solution: Reduction to a single polynomial (several schemes)
- Automatic procedure – Gröbner Basis
Polynomial equation systems

Root solving
- 3pt+IMU, 8\textsuperscript{th} degree polynomial
- 6pt generalized camera, 64\textsuperscript{th} degree polynomial

- Fast method: Sturm bracketing