Mathematical Principles in Vision and Graphics: Multi-Camera-Systems

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Outline

- Multi-Camera-Systems
- The generalized camera
- Plücker coordinates
- Generalized Epipolar Constraint
- Intra- and Inter-camera correspondences
- Generalized PnP
Learning goals

- Understand Plücker-line coordinates
- Understand the use of Plücker-line coordinates to describe multi-camera systems
- Understand the properties of multi-camera systems
- Understand the concept of a generalized camera
- Understand the generalized epipolar concept
- Understand the P3P extension to the generalized camera P3P
PtGrey Ladybug

- 6 cameras
- 360 field of view
- Panorama images
Omnitour
Automotive around view system

- Right Camera
- Front Camera
- Left Camera
- Rear Camera
- GPS/INS

Front View
Left View
Rear View
Right View
The generalized camera

pinhole camera
generalized camera
The generalized camera

generalized camera

mixed configuration
Multi-view camera systems

general non-axial camera (locally-central)

axial camera

axial camera
Plücker coordinates

- Plücker line matrix (4x4)

\[
L = AB^T - BA^T
\]

\[
L = \begin{bmatrix}
A_4B_1 - A_1B_4 \\
A_4B_2 - A_2B_4 \\
A_4B_3 - A_3B_4 \\
A_3B_2 - A_2B_3 \\
A_1B_3 - A_3B_1 \\
A_2B_1 - A_1B_2
\end{bmatrix}
\]

- Plücker coordinates (6-vector)
Plücker coordinates

- 6-vector L consists of 2 parts
  \[ a^T = (L_1 \ L_2 \ L_3) \quad b^T = (L_4 \ L_5 \ L_6) \]
  \[ a^T b = 0 \]

- Rigid transformation
  - Point:
    \[ X' = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} X \]
  - Plücker coordinate
    \[ \begin{bmatrix} a' \\ b' \end{bmatrix} = \begin{bmatrix} R & 0 \\ -[t]_x R & R \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \]
Plücker coordinates

- Line intersection of $L_1$ and $L_2$

\[
L_2^T \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} L_1 = a_2^T b_1 + b_2^T a_1 = 0
\]
Plücker coordinates: Geometric interpretation

\[ L = (q, q') \]
\[ q' = P \times q \]
\[ q^T q' = 0 \]
All points on the line L are expressed by the two vectors $q \times q'$ and $\alpha q$, where $\alpha$ is a scalar.
Plücker coordinates: Pinhole camera

- Camera center is point C
- Plücker coordinate is just standard homogeneous coordinate in this case

\[ L = (q, q') \]
\[ q' = C \times q \]
\[ P = C = (0,0,0) \]
\[ L = (q, (0 0 0)') \]
Plücker coordinates: Generalized camera

- Camera centers are not at the origin (0,0,0)

\[ L_1 = (R_1 x_1, C_1 \times R_1 x_1) \]

- \( R_1, C_1 \) ... camera center and orientation in a common coordinate system
- \( x_1 \) ... homogeneous (normalized) image coordinate
Relative motion

- Relative rotations are the same for all cameras
- Relative translations are all different due to the lever arm (direction as well as length).
- Motion is defined as full 6DOF as compared to single camera case with 5DOF (length of translation is defined!)

\[
R_{11}', = R_{22}', \\
t_{11}', \neq t_{22}',
\]

\[
R_{11}; t_{11}; \\
R_{22}; t_{22}; \\
R_{12}; t_{12}
\]
Relative motion, scale estimation
Generalized epipolar constraint (GEC)

- Line correspondence \( L <-> L' \)
  \[
  L = (q^T_1, q'^T_1)^T \\
  L' = (q^T_2, q'^T_2)^T
  \]

- Relative motion transforms Plücker coordinates (light ray)
  \[
  L' = \begin{pmatrix} Rq_1 \\ (Rq'_1 + t \times (Rq_1)) \end{pmatrix}
  \]

- Generalized epipolar constraint (GEC) defines intersection of two light rays ([R,T]L,L')
  \[
  q^T_2 q'_1 + q'^T_2 q_1 = 0 \\
  q_1 \rightarrow Rq_1 \\
  q'_1 \rightarrow Rq'_1 + t \times (Rq_1) \\
  q^T_2 \left( Rq'_1 + t \times (Rq_1) \right) + q'^T_2 (Rq_1) = 0 \\
  q^T_2 Rq'_1 + q^T_2 [t]_x Rq_1 + q'^T_2 Rq_1 = 0
  \]
Generalized epipolar constraint (GEC)

- Matrix form

\[ q_2^T R q_1' + q_2^T [t]_x R q_1 + q_2'^T R q_1' = 0 \]

\[ L_2^T GL_1 = \begin{pmatrix} q_2 \\ q_2' \end{pmatrix}^T \begin{bmatrix} [t]_x & R \\ R & 0 \end{bmatrix} \begin{pmatrix} q_1 \\ q_1' \end{pmatrix} = 0 \]

\[ G = \begin{bmatrix} [t]_x & R \\ R & 0 \end{bmatrix} \]

- G … generalized essential matrix (6x6)
- G contains essential matrix! (E = [t]_x R)
Linear algorithm for G

$$L_2^T G L_1 = \begin{pmatrix} q_2 \\ q'_2 \end{pmatrix}^T \begin{bmatrix} [t]_x & R & R \\ R & 0 \end{bmatrix} \begin{pmatrix} q_1 \\ q'_1 \end{pmatrix} = 0$$

$$L'^T G L = \begin{pmatrix} x' \\ v' \times x' \end{pmatrix}^T \begin{bmatrix} E & R \\ R & 0 \end{bmatrix} \begin{pmatrix} x \\ v \times x \end{pmatrix} = 0$$

- E is 3x3, R is 3x3, i.e. in total 18 unknowns
- Each line correspondence gives 1 equation like

  $$x'^T Ex + (v' \times x')^T Rx + x'^T R(v \times x) = 0$$

- Linear solution needs 17 point correspondences to compute G
Linear algorithm for G

\[ A_i^T y = \begin{bmatrix} x_1' x_1 \\ x_1' x_2 \\ x_1' x_3 \\ x_2' x_1 \\ x_2' x_2 \\ x_2' x_3 \\ x_3' x_1 \\ x_3' x_2 \\ x_3' x_3 \end{bmatrix}^T \begin{bmatrix} E_{11} \\ E_{12} \\ E_{13} \\ E_{21} \\ E_{22} \\ E_{23} \\ E_{31} \\ E_{32} \\ E_{33} \\ R_{11} \\ R_{12} \\ R_{13} \\ R_{21} \\ R_{22} \\ R_{23} \\ R_{31} \\ R_{32} \\ R_{33} \end{bmatrix} \]
Two types of correspondences

- Intra-camera correspondences: Correspondences from the same camera
- Inter-camera correspondences: Correspondences from different cameras
Two types of correspondences

- Problem when rotation is identity and there are only intra-camera correspondences. \( R = I \quad t'_c = t_c \)

\[
x'^TEx + (t'_c \times x')^TRx + x'^TR(t_c \times x) = 0
\]

\[
x'^TEx = 0
\]

- Scale not observable anymore
Generalized PnP

- Very similar to perspective PnP
- Use Plücker coordinates
- But: How many points needed? How many solutions?
- We will show that 3 points are needed and this yields 8 solutions.
Generalized PnP

Three 2D-3D correspondences

Express rays as Plücker lines

Find unknown depths

Absolute orientation

\( R, t \)
Generalized PnP

\[ d_{ij} = \|X_i - X_j\|^2 = \|X_i^V - X_j^V\|^2 \quad \text{depths} \]

\[ d_{ij} = \|X_i - X_j\|^2 = \|(u_i \times u'_i + \lambda_i u_i) - (u_j \times u'_j + \lambda_j u_j)\|^2 \]

Plücker coordinates

\[ l_i = (u_i, u'_i) \]

\[ X_i^V = u_i \times u'_i + \lambda_i u_i \]
Generalized PnP

- 3 distances lead to 3 equations with unknown depths (lambda)

\[
\begin{align*}
k_{11} \lambda_1^2 + (k_{12} \lambda_2 + k_{13}) \lambda_1 + (k_{14} \lambda_2^2 + k_{15} \lambda_2 + k_{16}) &= 0 \\
k_{21} \lambda_1^2 + (k_{22} \lambda_3 + k_{23}) \lambda_1 + (k_{24} \lambda_3^2 + k_{25} \lambda_3 + k_{26}) &= 0 \\
k_{31} \lambda_2^2 + (k_{32} \lambda_3 + k_{33}) \lambda_2 + (k_{34} \lambda_3^2 + k_{35} \lambda_3 + k_{36}) &= 0
\end{align*}
\]

- Eliminating variables using resultants (determinant of Sylvester matrix) leads to an 8 degree polynomial

\[
A \lambda_3^8 + B \lambda_3^7 + C \lambda_3^6 + D \lambda_3^5 + E \lambda_3^4 + F \lambda_3^3 + G \lambda_3^2 + H \lambda_3 + I = 0
\]

- No closed form solution possible (Root solving with companion matrix or Sturm bracket method)
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