
Camera Drones

Lecture – Sensor Fusion

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Outline

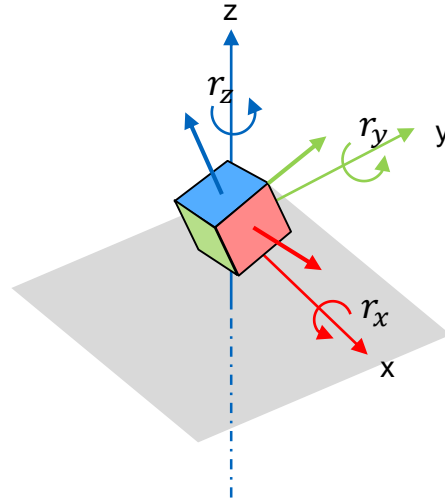
- Mathematical model for an IMU
- Sensor fusion methods
- Sensor fusion with a Kalman filter

Mathematical model for an IMU

- IMU measures attitude with respect to Earth's gravitational field (6DOF-IMU)
- IMU is a combination of an accelerometer (3-axis) and a gyroscope (3-axis)
- Attitude W will be represented through Euler angles in a world coordinate system (roll, pitch, yaw).

$$\mathbf{r} = [r_x, r_y, r_z]^T \text{ [rad]}$$

$$W = \text{euler}(r_x, r_y, r_z)$$



Mathematical model for an IMU

- Gyroscope:

$$\boldsymbol{\omega} = \hat{\boldsymbol{\omega}} + \mathbf{b}_g + \mathbf{n}_g$$

$$\boldsymbol{\omega} = [\omega_x, \omega_y, \omega_z]^T \left[\frac{\text{rad}}{\text{s}} \right]$$

$\boldsymbol{\omega}$... measurement of rotational velocity (angular rate)

$\hat{\boldsymbol{\omega}}$... true rotational velocity

\mathbf{b}_g ... bias of angular measurement

\mathbf{n}_g ... noise of the measurement of the rotational velocity

- Accelerometer:

$$\mathbf{a} = W^T(\hat{\mathbf{a}} - \mathbf{g}) + \mathbf{b}_a + \mathbf{n}_a$$

$$\mathbf{a} = [a_x, a_y, a_z]^T \left[\frac{\text{m}}{\text{s}^2} \right]$$

\mathbf{a} ... measurement of acceleration

$\hat{\mathbf{a}}$... true acceleration

\mathbf{b}_a ... bias of acceleration measurement

\mathbf{n}_a ... noise of acceleration measurement

\mathbf{g} ... acceleration due to gravity

W ... orientation of sensor in world frame

Measuring attitude by integrating rotational velocities

- Attitude update

$$W_{t+1} = \text{euler}(\Delta t \boldsymbol{\omega}_t) W_t$$

- Initial value W_0 must be defined
- This integration leads to drift. Due to bias and noise the attitude deviation will increase continuously.
- Solution: Fusion with measurements of the accelerometer

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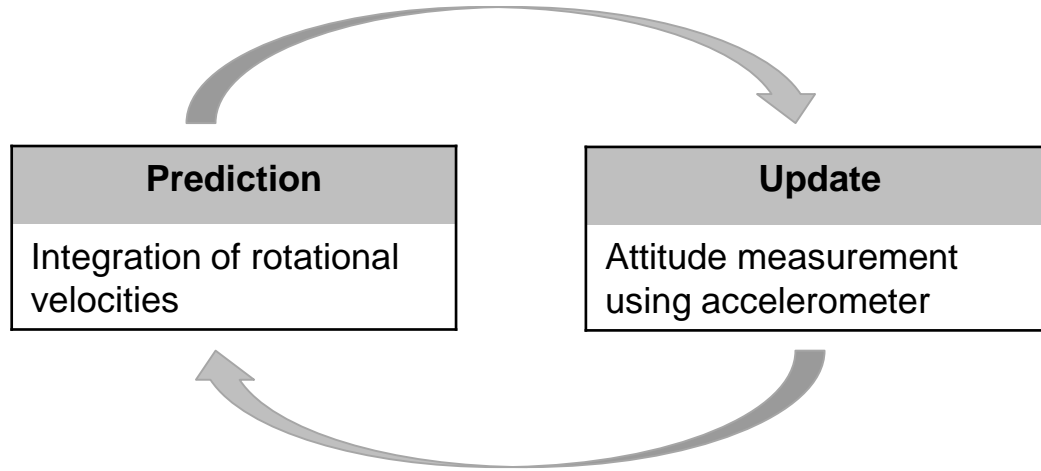
Sensor fusion principle:

- 1. Attitude estimation through integration of rotational velocities**
- 2. Correction of attitude estimate through attitude measurements of accelerometer**

Sensor fusion methods

- Complementary filter
- Madgwick filter
- Mahoney filter
- Kalman filter

Sensor fusion with a Kalman filter



- State vector $x_t = \begin{bmatrix} r_{x,t} \\ r_{y,t} \\ r_{z,t} \end{bmatrix}$

Kalman filter: Prediction

- Prediction of estimated mean and co-variance of state vector by integration of gyroscope measurements
- Constant attitude model

$$\hat{\mu}_{t+1} = A_{t+1}\hat{\mu}_t + B_{t+1}u_{t+1}$$

$$\hat{\Sigma}_{t+1} = A_{t+1}\hat{\Sigma}_t A_{t+1}^T + Q_{t+1}$$

$$A_{t+1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q.... covariance matrix of dynamic noise

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Q.... covariance matrix of dynamic noise

- Gyroscope measurements

$$\mathbf{u}_{t+1} = \boldsymbol{\omega}_t$$

- Integration of gyroscope measurements

$$\mathbf{B}_{t+1} = \begin{bmatrix} \Delta t & 0 & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \end{bmatrix}$$

Kalman filter: Update

- Update estimated mean and co-variance of state vector by comparison with attitude measurement of accelerometer

$$K_{t+1} = \hat{\Sigma}_{t+1} C^T (C \hat{\Sigma}_{t+1} C^T + R)^{-1}$$

$$\mu_{t+1} = \hat{\mu}_{t+1} + K_{t+1} (z_{t+1} - C \hat{\mu}_{t+1})$$

$$\Sigma_{t+1} = \hat{\Sigma}_{t+1} - K_{t+1} C \hat{\Sigma}_{t+1}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

R.... co-variance matrix of measurement noise

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- Attitude measurement of accelerometer

$$z_{t+1} = \begin{bmatrix} r_{x,t} \\ r_{y,t} \\ 0 \end{bmatrix}$$

$$r_{x,t} = \text{atan} \left(\frac{a_y}{\sqrt{a_x^2 + a_z^2}} \right)$$

$$r_{y,t} = \text{atan} \left(\frac{a_x}{\sqrt{a_y^2 + a_z^2}} \right)$$

Learning goals

- What is measured by an IMU and how is it built?
- What is the mathematical model of an IMU?
- How does the integration of rotational velocities work?
- How is sensor fusion using a Kalman filter done?