Biomedical Sensor Systems

Laboratory

Laboratory Tutorial: Oscillation Measurement

Place: BMT02002, Stremayrgasse 16, 2.OG

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Short Description:

In this lab students will perform oscillation measurements implemented by a piezo transducer and appropriate amplifier circuits. The signals will be analysed and interpreted using a storage oscilloscope.

Learning Objectives:

The students are able to …

- … understand the principles of sensors and amplifiers
- … design appropriate amplifier curcuits for given problems
- … handle oscilloscopes and understand the functional principles
- … solve linear ordinary second-order differential equations with constant coefficients

Content

1. Theory

1.1. Analogue measurement

Why do we need amplifiers?

- measured potentials and currents are often low
- sensor elements are often load-sensitive
- tapping/transforming of measured variables

Requirements:

- low response to measured variable
- high resolution
  small current or voltage signals and changes should be measurable
- defined response characteristics
  the output signal should uniquely depend on the input signal
- good dynamic behavior
  the output signal should follow the input signal as fast as possible
- output stable and insensitive to response
  ouput signals should not be changed by following measurement instruments

1.1.1. Operational Amplifier „OpAmp“

Amplifiers can consist of one or several OpAmps. Figure 1 shows the simplified internal wiring of an OpAmp.

Differential input stage (yellow): differential amplifier with two inputs and constant current source; converts small potential differences to a proportional output current.

Amplifier stage (orange): transforms the small input current into a high output voltage; sets high off-load voltage amplification; frequency-dependent feedback of the capacitor ensures stability and determines the cutoff frequency.

Output stage (blue): often a push-pull, no voltage amplification, acts as current driver for the output, small output resistor which enables a high output current.

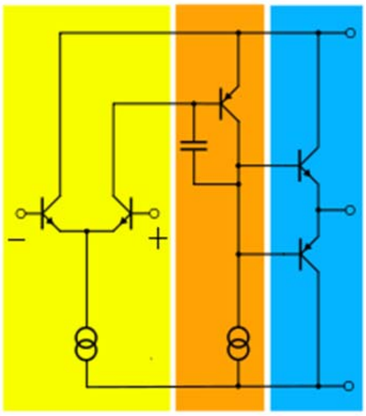
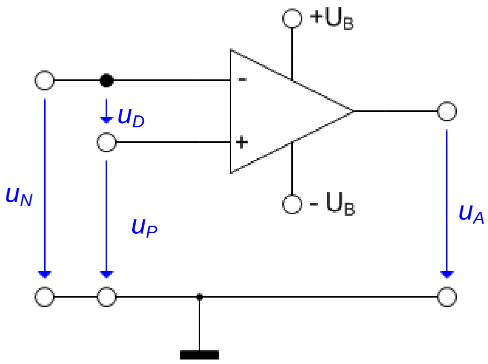


Figure 1: Simplified internal wiring of an OpAmp

Ideal OpAmps are assumed to have a voltage-controlled power supply with open load voltage gain $V_0 \rightarrow \infty$ (see Figure 2). **Real** OpAmps have an open loop voltage gain of about 10^4 to 10^7.



$$u_A = V_0.(u_P - u_N) = V_0.u_D$$

ideal: $V_0 \rightarrow \infty$

real: $V_0 \approx 10^4$ to 10^7

Figure 2: Equivalent curcuit diagram OpAmp

The transfer characteristic of an OpAmp is illustrated in Figure 3. u_A is ***limited by the supply voltage*** U_B → $u_{A,max}$ is always smaller than U_B!

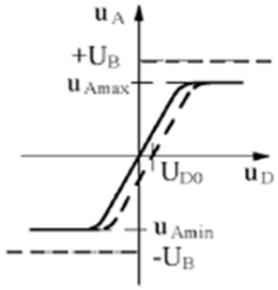


Figure 3: Transfer characteristic OpAmp, U_{D0}: input offset voltage

1.2. Transducers, sensors and actuators

Transducers convert one form of energy to another form of energy. If a transducer converts a measurable quantity (mechanical, thermal, …) into an electrical signal (voltage, current) it is called a sensor.

A sensor (transducer) is the primary element in a measurement chain. Sensors can be divided into active and passive sensors. Active sensors convert non-electrical energy into electrical energy (voltage) without auxiliary voltage. Passive sensors change their electrical properties under the influence of non-electrical variables. Table 1 lists some active and passive sensors and their variables.

Active sensors	**non-electrical parameter**
thermal element	temperature, radiation
photo element	light, temperature
electrochemical element	ph-Value, redox potential
piezo sensor	force, pressure, tension, acceleration, temperature
induction sensor	rate of rotation, acceleration

Passive sensors	**influenced electrical parameter**	**non-electrical parameter**
potentiometer	ohmic resistance	length, angle
strain gauge	ohmic resistance	force, pressure, length, angle, strain, torsion
photo resistor, photo transistor, photo diode	ohmic resistance	light variables
resistance thermometer	ohmic resistance	length, angle
transformer	magnetic coupling	length, angle
Hall probe	voltage	length, angle
induction sensor	inductance	length, angle
capacitive sensor	capacitance	length, angle
magnetic sensor	magnetic field	length, angle

Table 1: Overview active and passive sensors

1.1.2. Piezo sensors

Piezoelectric effect

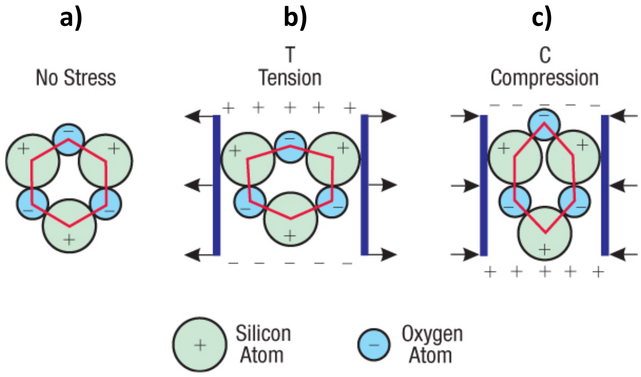


Figure 4: Piezoelectric effect in Quartz. The two oxygen atoms (SiO_2) are combined into a single unit in the drawing. b) longitudinal effect. c) transversal effect

Figure 4 shows the piezoelectric effect of a quartz molecule. Through a force acting on a non-centrosymmetric crystal (like quartz), the differently charged ions (Si and O_2) shift in the crystal lattice and an electric charge is generated on the surface of the crystal. I.e., mechanical deformation like tension or compression causes a charge shifting and thereby a voltage. Reversely, an applied voltage leads to a deformation of the material. Thus, piezoelectric crystals and ceramics can be both, actuators as well as sensors.

Piezo Sensors

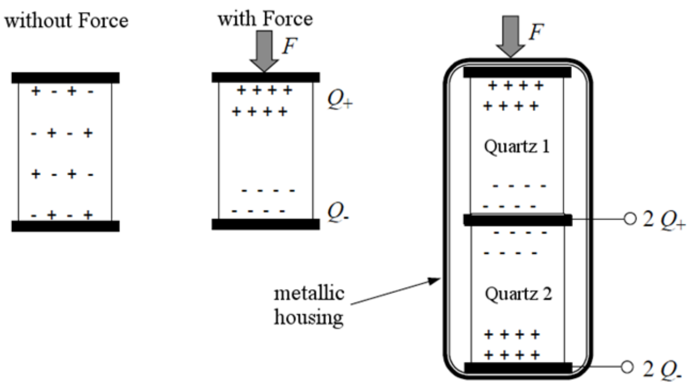


Figure 5: Piezo Sensor for force measurement

Piezo sensors can be used to determine a variety of physical variables (e. g. pressure, acceleration, temperature, strain or force). The function principles are mainly based on the longitudinal effect, transversal effect and shear effect. In Figure 5 two quartz crystals are put together in a mirror-inverted way so that the longitudinal effect becomes effective. The charge *Q* is (at least in a certain range) proportional to the force *F*. The proportionality factor is called coupling factor or piezo module (piezoelectric constant) k_p and has the dimension As /N:

$$Q = k_p . F \, . \tag{1}$$

The charge shift is characterised by displacement of the shift flux density *D* [AS/m^2] which describes the density of the electric field lines with respect to a surface *A*:

$$D = \frac{Q}{A}. \tag{2}$$

The charges are not directly measurably and have to be converted to a proportional voltage U_q using a capacitor *C*. Thereby charge *Q* can be written as

$$Q = C.U_q \Rightarrow U_q = \frac{Q}{C} = \frac{k_p.F}{C} \tag{3}$$

Therefore amplifiers with a great high-ohmic input, so called electrometer amplifiers (non-inverting amplifier) and charge amplifiers are applicable.

Piezo sensor for acceleration measurement

A piezoelectric acceleration sensor consists of two basic components:

- Piezoelectric crystal
- Seismic mass

One side of the piezo disc is attached to the seismic mass, the other side is fixed to the base (see Figure 6). If this combination is vibrated, a force acts on the piezo disk via the mass. The resulting force is the product of acceleration and mass:

$$F = m.a\ .$$

Over a wide frequency range, the sensor base and the mass follow the same motion, causing the sensor to correctly measure the acceleration. A piezoelectric accelerometer can be considered as a mechanical low pass with resonance peak.

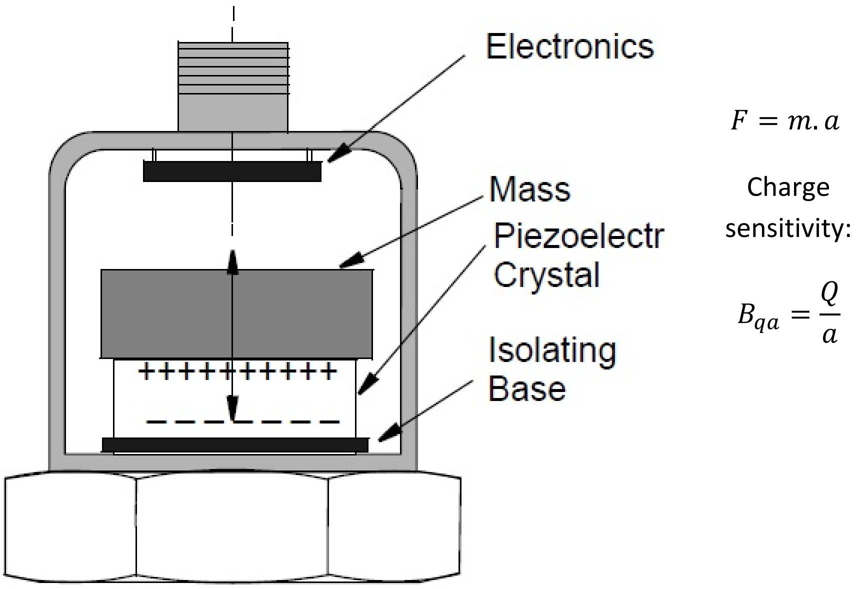


Figure 6: Piezo sensor for acceleration measurement

The basic function, i. e., the conversion of mechanical acceleration into an electrical signal, is the same for all piezoelectric accelerometers. The various design variants are used to adapt to different measuring tasks and to protect against external disturbances. Three mechanisms for the mechanical-electrical conversion are used, from which the type designation of our transducers is derived:

- KS-Type (shear system)
- KD-Type (compression system)
- KB-Type (bending system)

Transfer characteristics of an acceleration sensor

A charge output piezoelectric accelerometer can be considered as either a charge source or a very high internal resistance voltage source. As a result, charge or voltage sensitivity (also called transfer factors) is given to describe the behavior of the sensor versus acceleration. The charge sensitivity B_{qa} is given in Picocoulomb per m/s² or Picocoulomb per g (1 g = 9.81 m/s²). The charge transfer factor is slightly frequency-dependent.

The Voltage sensitivity can be obtained from the known charge sensitivity and the capacitance of the measurement chain.

Electrometer amplifier (non-inverting amplifier)

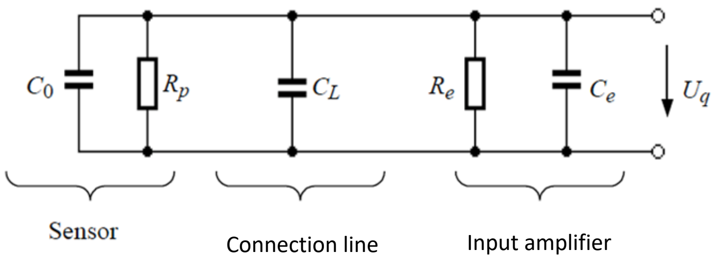


Figure 7: Input/output capacitances and resistors in the electrometer amplifier circuit

The input resistance R_e of the amplifier as well as the loss resistance R_p have to be very high-ohmic. Otherwise the charge would be compensated quickly. For this circuit, all capacitive components need to be taken into consideration (C_0 of the piezo crystal, C_L of the connection line and the input capacitance of the amplifier C_e). The time constant of the measurement setup as shown in Figure 8 and Figure 8 can be calculated as follows

$$\tau = R_{ges}C_{ges} = \left(\frac{R_e.R_p}{R_e + R_p}\right).(C_0 + C_L + C_e) \qquad R_e \parallel R_p \to \infty \,. \tag{4}$$

Inserting Eq. (3), the output voltage U_a becomes

$$U_a = U_q\left(1+\frac{R_2}{R_1}\right) = \frac{Q}{C_{ges}}.\left(1+\frac{R_2}{R_1}\right) = F.\frac{k_p}{C_{ges}}.\left(1+\frac{R_2}{R_1}\right). \quad (5)$$

Attention:

- $U_q = f(F)$
- unavoidable, external switching capacities lower sensitivity

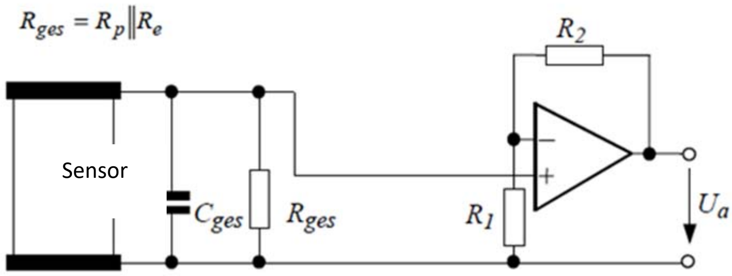


Figure 8: Piezo sensor with electrometer amplifier. The circuit is called electrometer amplifier because of its very high input resistance

Charge amplifier

The charge amplifier converts the charge quantity, proportional to the force, into the voltage u_a. The current $i_q(t)$ is directly derived from the charge shifting and the current $i_k(t)$ from the output voltage of the OpAmp, and the capacitor C. The equations for $i_q(t)$ and $i_k(t)$ can be written as follows

$$i_q(t) = \frac{dQ}{dt} \quad (6)$$

$$i_k(t) = C.\frac{du_a(t)}{dt} \quad (7)$$

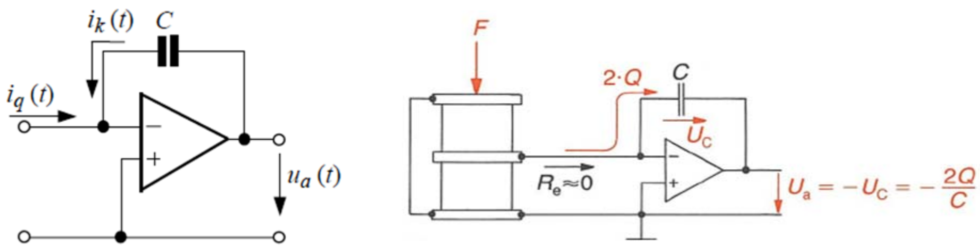


Figure 9: a) Charge amplifier b) Piezo sensor with charge amplifier

As shown in Figure 9 a), $i_q(t) + i_k(t) \approx 0$. After integration, the equation for u_a of the charge amplifier is given by

$$u_a(t) = -\frac{1}{C}\int_0^T iq(t)dt = -\frac{Q}{C} = -\frac{k_p.F}{C}. \quad (8)$$

C is the only decisive capacitance for u_a (instead of the parasitic capacitances of the electrometer amplifier circuit C_0, C_L, C_e). A disadvantage is the restriction to the measurement of alternating parameters, because of charge changes over time.

1.3. Oscilloscope

1.3.1. Functional principle

Oscilloscopes are used to display the change of an electrical signal over time. Basically, all procedures which can be reproduced as a voltage time curve can also be displayed by an oscilloscope. The observed waveform can be analyzed for shape parameters such as amplitude, frequency, rise time, time interval, phase shift, distortion and others.

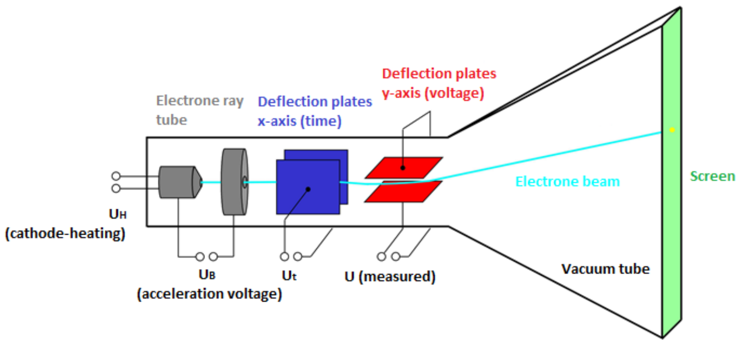


Figure 10: Basic construction analogue oscilloscope

Before the advent of digital electronics, analogue oscilloscopes (Figure 10) used electrode ray tubes to display the signals. Thereby, an electron emitter generates free electrons which are focused and accelerated by high voltage U_B. The measured signal is amplified and is used to diffract the electron beam by deflection plates. Finally the deflected electrons come upon the screen and induce the fluorescing screen material to light up.

In principle, a digital oscilloscope works like an analogue, but the measured signals are converted by an ADC (analog digital converter) before processing and the signals can be stored for further investigations. The digital-analog conversion enables the use of additional implemented analysis software (FFT, average, …), the autoset-autorange function and the pre-trigger allow the recording of slow signal curves. Several ADCs are used to illustrate high frequency signals accurately. In order to be able to represent signals accurately and to avoid aliasing, the conditions of the **sampling theorem** have to be observed.

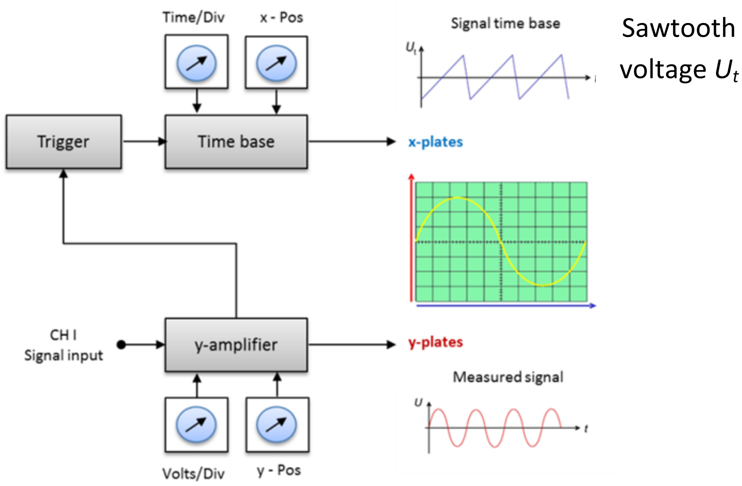


Figure 11: Functional principle oscilloscope

To obtain a temporal representation of the input signal, the light point has to move with constant speed, horizontally along the screen. This happens thanks to a sawtooth voltage U_t generated by the time base (see Figure 11). During the falling edge of U_t the light point is faded out. The rise time of this voltage sets the time dimension of the *X*-axis. K_t (e.g. 0.2 µs/raster element) gives the time per raster element or cm of the *X*-axis.

The **Trigger** starts the sawtooth voltage, which generates the stationary image. To obtain the same sequence in all periods, the measured signal *U* and U_t have to run paired. Therfore a trigger level is used, whereby the sawtooth voltage gets started as soon as *U* exceeds this level.

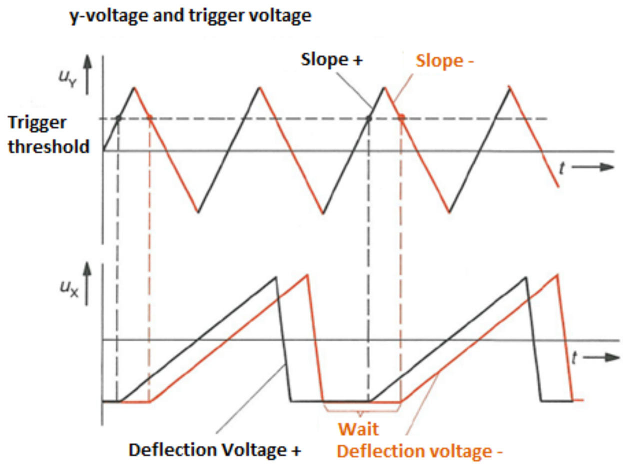
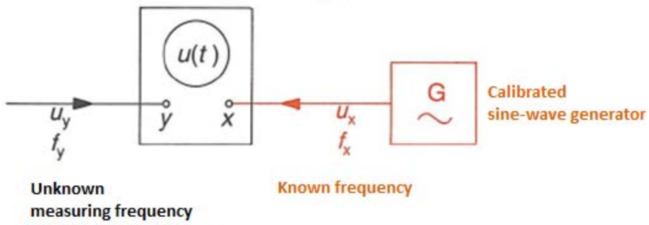


Figure 12: Trigger function

x-y mode: most modern oscilloscopes have several inputs for voltages which can be used to plot one varying voltage versus another. In x-y mode the oscilloscope has the same deflection sensitivity for both the x- and y-direction. This is especially useful for graphing I-V curves (current versus voltage characteristics) for components such as diodes. Lissajous figures, for example, are used to track phase differences between multiple input signals and can be used for frequency measurement as illustrated in Figure 13.

a)



b)

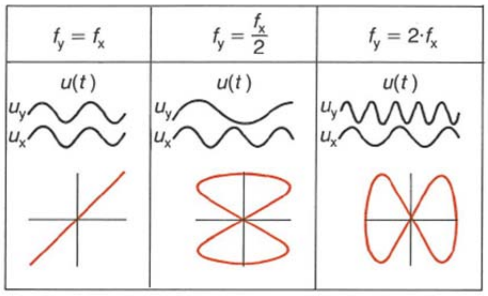


Figure 13: a) Measuring setup for frequency measurement. b) Lissajous figures.

1.3.2. Setup and basic functions

The basic oscilloscope is typically divided into four sections: the display, vertical controls, horizontal controls and trigger controls. In addition to the screen, most display sections are equipped with three basic controls: a focus knob, an intensity knob and a beam finder button.

The vertical section controls the amplitude of the displayed signal. This section carries a Volts-per-Division (Volts/Div) selector knob, an AC/DC/Ground selector switch and the vertical (primary) input for the instrument. Additionally, this section is typically equipped with the vertical beam position knob.

The horizontal section controls the time base or "sweep" of the instrument. The primary control is the Seconds-per-Division (Sec/Div) selector switch. Also included is a horizontal input for plotting dual X-Y axis signals. The horizontal beam position knob is generally located in this section.

The trigger section controls the start event of the sweep and can be set to automatically restart after each sweep or it can be configured to respond to an internal or external event. The principal controls of this section will be the source and coupling selector switches. An external trigger input (EXT Input) and level adjustment will also be included.

2. Preparatory activities for the lab

As preparatory activities for the lab you should:

I. mathematically establish the descriptive equation of a (weak) damped oscillation (see section 2.1)
II. get familiar with the functional principles of an oscilloscope (see Section 2.2)
III. study the data sheet of the accelerometer KS93 (see Section 2.3 and Appendix)

Please include the results in the protocol!

2.1. Mathematical model of a (weak) damped oscillation

Together with the amplifier circuit the piezo sensor provides an electrical signal, corresponding to a damped oscillication. During the lab you will be concerned with oscillations *u*(*t*) with decreasing amplitudes *A* over time. A (linear) signal progression can be described by the following mathematical equation:

$$u(t) = A.e^{-\frac{t}{\tau}}.\left[\cos(\omega t) + \frac{1}{\omega.\tau}.\sin(\omega.t)\right], \tag{9}$$

where *A* is the amplitude at time *t* = 0, *ω* the angular frequency of the oscillation and *τ* the decay coefficient. The differential equation of a free damped oscillation is given by

$$u'' + \frac{k_1}{m}u' + \frac{k_2}{m}u = 0 \tag{10}$$

where *u*(*t*) is the deflection at time *t*, k_1 is the damping coefficient, k_2 the spring constant and *m* the mass of the swinging body. The right side of the equation is set to 0, because of a free oscillation without an exterior time excitation. A quite handy substitution with

$$\frac{2}{\tau} = \frac{k_1}{m} \text{ and } \omega^2 = \frac{k_2}{m}$$

and the consideration of the initial conditions *u*(*t*=0) = u_0 = *A* (deflection at time *t*=0) as well as *u*'(*t*=0 = u'_0 = 0 (the body will not be given an initial speed) lead – after appropriate computation – to equation 9.

Execute that computation and demonstrate that equation 9 is the result of equation 10 regarding the

- initial conditions
- the fact that it's a weak damped oscillation
- correlating given substitutions

Please note: The calculation should be performed **solely** in the time domain without using Laplace transforms.

> ***useful hints:***
>
> *Make use of the fact that it's a weak damped oscillation where ω can be set as*
>
> $$\omega >> \frac{1}{r^2}$$
>
> *consistently you can set expressions like*
>
> $$\sqrt{\frac{1}{\tau^2} - \omega^2} = j.\sqrt{\omega^2 - \frac{1}{r^2}} \cong j.\sqrt{\omega^2} = j.\omega$$

On the basis of your measured results you will determine paramters *A*, *ω* and *τ*. In advance, plan ahead how the determination respectively the calculation can be carried out. Keep in mind that you have to expect corresponding fluctuation ranges for *ω* and *τ*. Hence you should average over several periods.

For this purpose, assume an oscillation as shown in Figure 14:

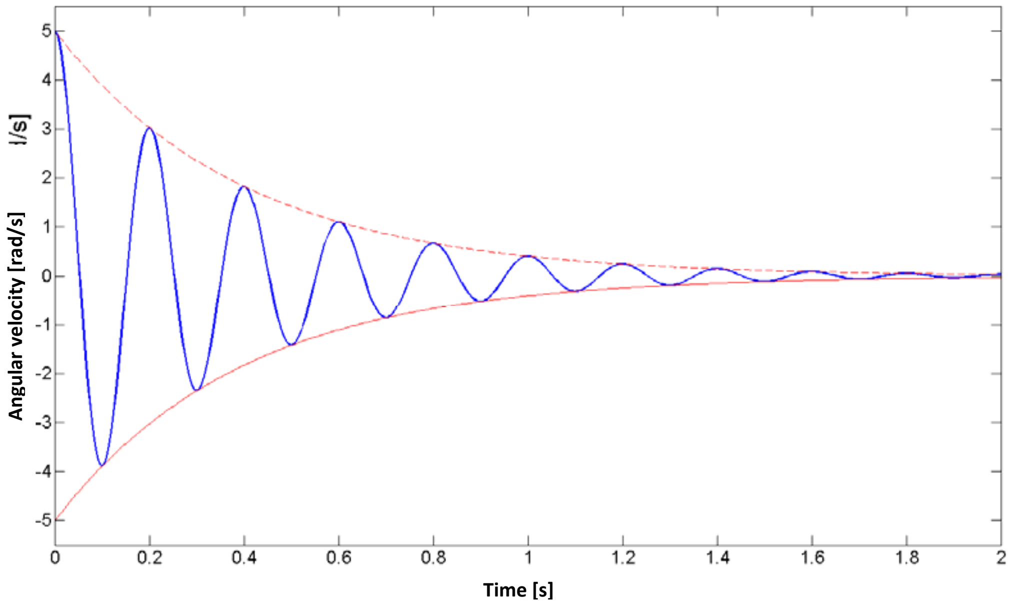


Figure 14: Example of an oscillation course u(t) out of equation 9, with decaying amplitude over time ($A.e^{-\frac{t}{\tau}}$). In this case paramter A = 5 rad/s, τ = 2,5 s and ω = 5 1/s. The red lines show the contour of the oscillation.

2.2. Oscilloscope

The signals will be measured with a digital storage oscilloscope. Could the signals also be measured by an analogue oscilloscope? Explain your answear.

2.3. Accelerometer KS93

From the datasheet extract the following information

- Piezo design
- Charge sensitivity
- Sensor mass
- Measurement range

3. Implementation in the lab

3.1. Dimensioning and completion of appropriate amplifier circuits

Figure 15 and Figure 16 illustrate circuitries of a charge and electrometer amplifier, respectively. Dimension the circuitry under the following conditions:

- the sensor will be the miniature-accelerometer sensor KS93
- measured frequencies are in the range of 20 to 80 Hz
- an LM324 will be used as operational amplifier
- capacitors are available in the range of 1 pF to 22 nF
- resistors up to 1 MΩ are available

useful hints:

- you can find all data sheets in the appendix
- consider, the input of the op-amp should always be on defined levels
- also consider the power supply of the LM324

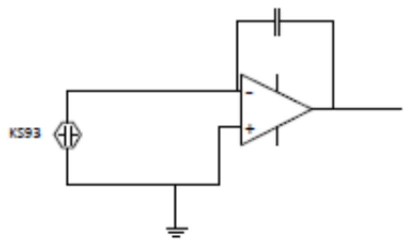


Figure 15: Incomplete charge amplifier

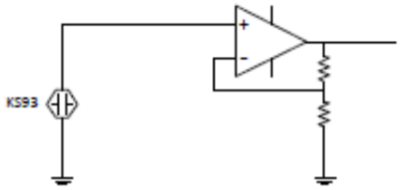


Figure 16: Incomplete electrometer amplifier

3.2. Charge amplifier

Tension the ruler at 10 cm and 20 cm. Deflect it slightly first (1) and greater secondly (2) and record the oscillation until the ruler is resting again.

Please note: in case of large oscillations the saturation region can be reached depending on the dimension of your charge amplifier!

Conduct and answer following tasks and questions for both deflections:

a) Implement your designed charge amplifier circuit on a patch panel.
b) Metrologically determine the frequency of the fundamental oscillation with the help of an oscilloscope.
c) Does the frequency remain constant?
d) Determine the damping coefficient under assuming a damped harmonic oscillation.
e) Additionally, switch a capacitor parallel to the input. The capacitor should simulate a changed interconnected capacitance. Are there changes in the output signal? Justify your observation.

3.3. Electrometer amplifier

Repeat tasks a) to e) of 3.2 with an electrometer amplifier.

4. Appendix