

# Messen optischer Größen, Messen aus Bildern

## Übersicht

Optische Strahlung, Sensorik

Geometrie, Photogrammetrie

- Kamerakalibrierung
- Stereo

Menschliche Wahrnehmung

- Neurophysiologie
- Kognitive Psychologie

Digitale Bildanalyse

- Digitales Rasterbild, Kenngrößen
- Bildverarbeitungsoperationen, Segmentation
- Salient point detection + description

# Bildverarbeitungsoperationen

Eingabebild  $E(x,y)$ , Ausgabebild  $A(x,y)$

**Funktion**  $f: E(x,y) \rightarrow A(x,y)$

Bildkoordinaten  $(x,y)$  ... „Ortsbereich“

**Transformation**  $T: E(x,y) \rightarrow F(u,v)$

$T$  transformiert in einen anderen „Bereich“

z.B.: „*Frequenzbereich*“  $(u,v)$  bei der Fourier-Transformation

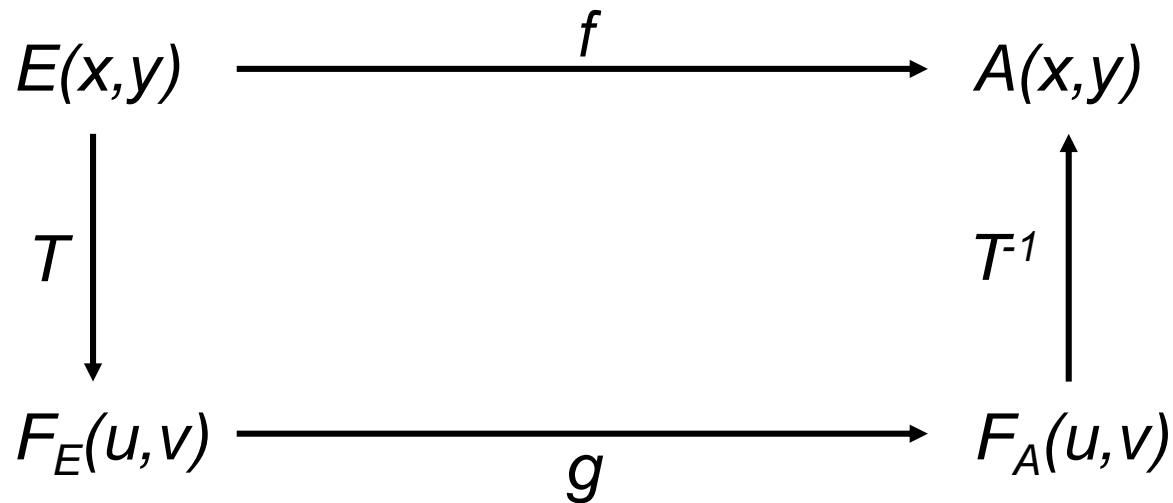
Wozu Transformationen?

z.B.: Komplexität, geschlossene Formulierung e. Problems, etc.

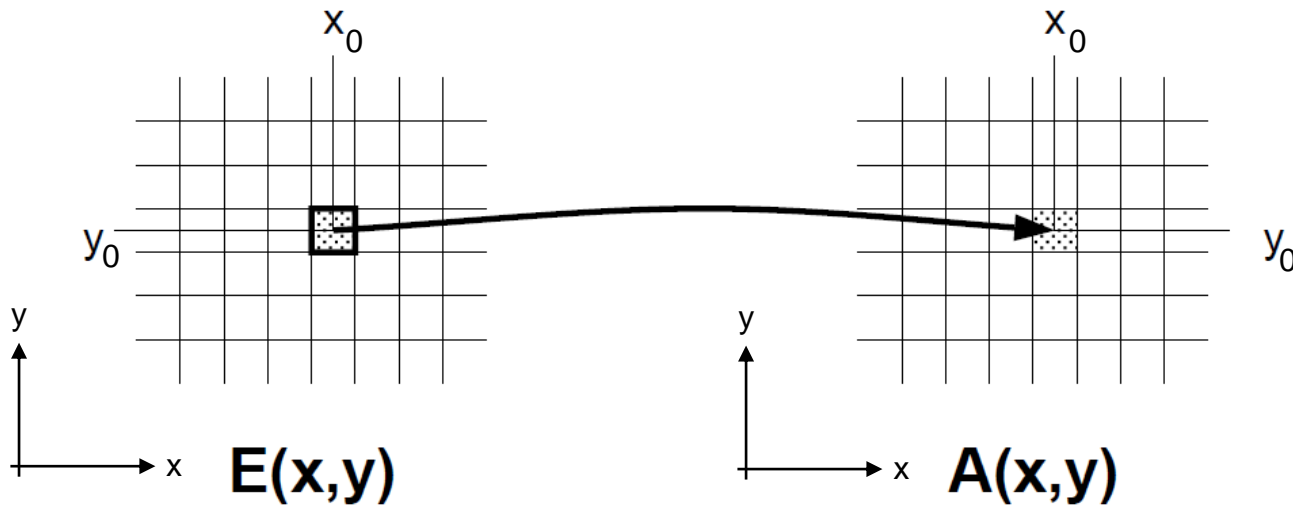
# Ein Resultat, viele mögliche Lösungswege



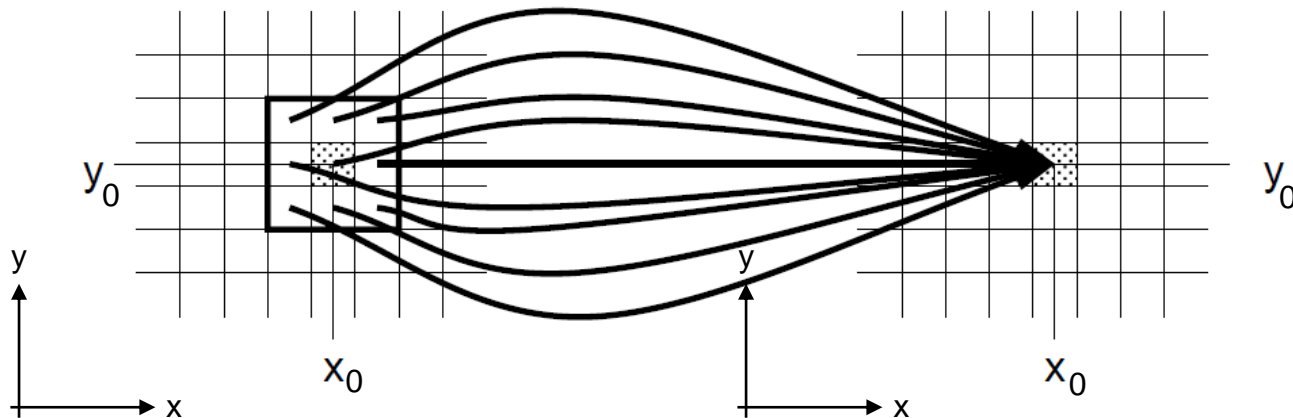
```
>> E=imread('peppers.jpg');
>> G5=fspecial('gaussian',15,5);
>> A=imfilter(E,G5);
```



# Punkt- und „lokale“ Operationen



**Punkt-  
operation**

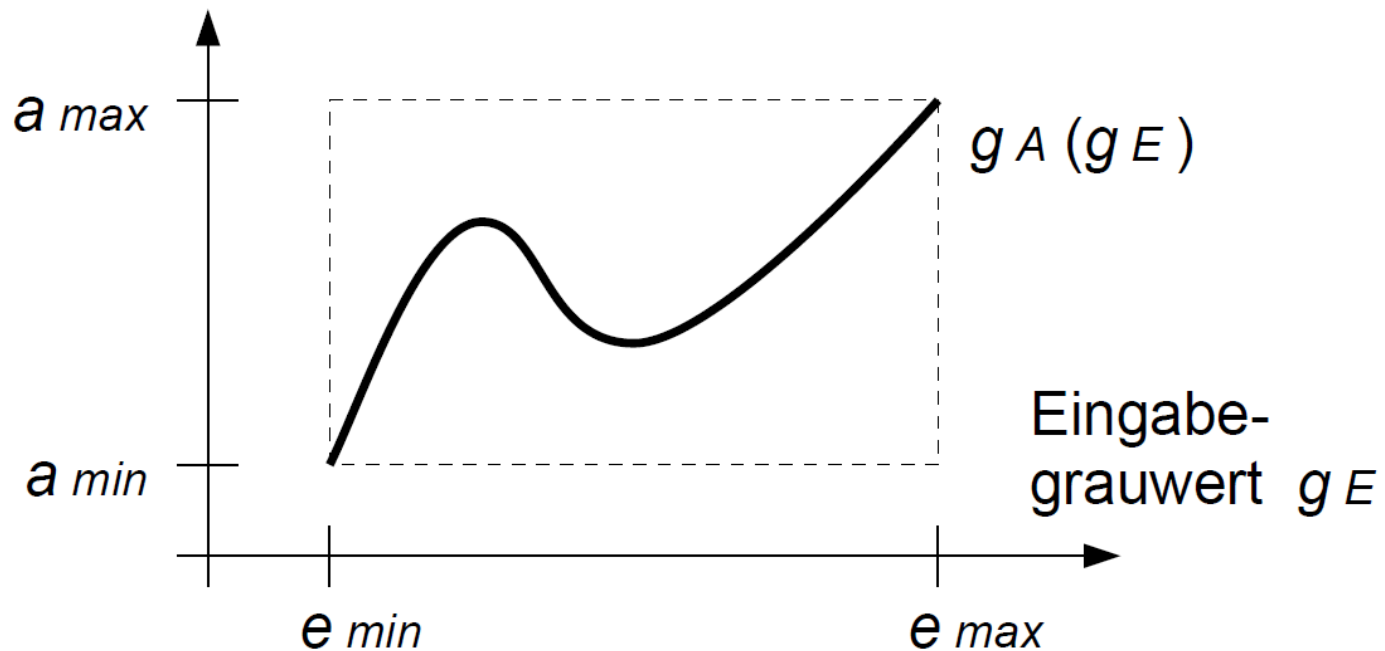


**Lokale  
Operation**

# Punktoperationen $A(x,y)=f(E(x,y))$

Allgemein: beliebige Grauwertübertragungsfunktion

Ausgabegrauwert  $g_A$



# Punktoperationen – Beispiele (1)

Schwellwert  $S$ : 
$$A(x, y) = \begin{cases} 0 & \dots E(x, y) < S \\ 1 & \dots E(x, y) \geq S \end{cases}$$


 $E(x, y)$ 

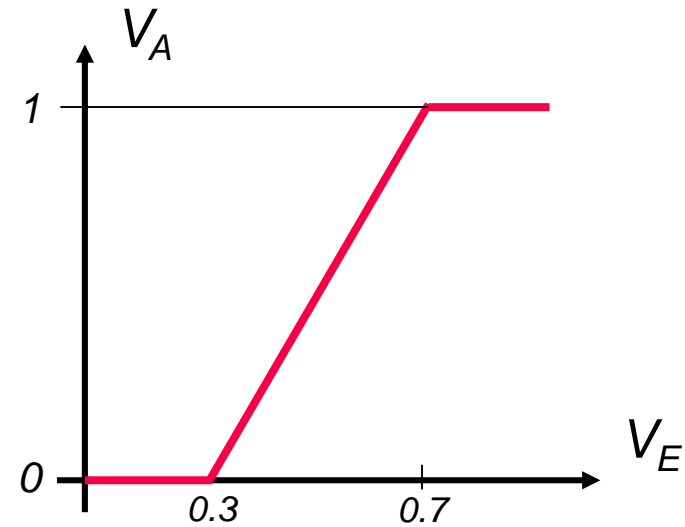
 $A(x, y)$

# Punktoperationen – Beispiele (2)

Kontraständerung:

$$A(x, y) = kE(x, y) + d$$

(cropping auf [min,max])



$E(x,y)$



$A(x,y)$

# Punktoperationen – Beispiele (3)

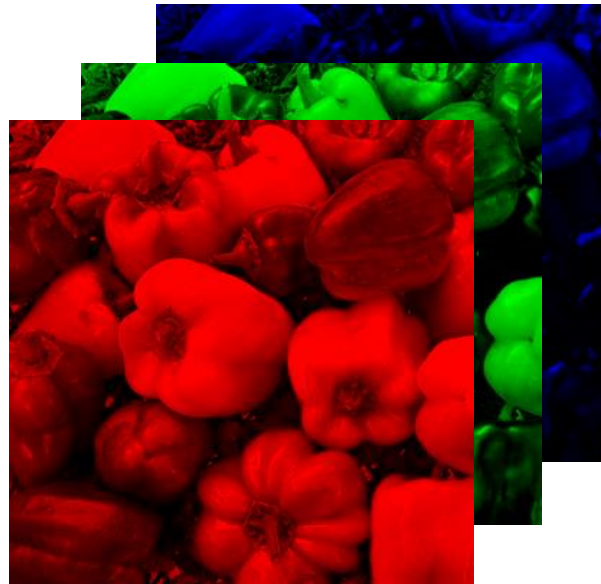
Punktoperation auf mehreren Bildern: Farbe  $\rightarrow$  monochrom

$$A(x, y) = \frac{1}{3} E_r(x, y) + \frac{1}{3} E_g(x, y) + \frac{1}{3} E_b(x, y)$$

```
>> IN=imread('peppers.jpg');
>> BW=IN(:,:,1)/3+IN(:,:,2)/3+IN(:,:,3)/3;
>> imwrite(BW,'peppers_BW.jpg');
```



$E(x, y)$



$E_r, E_g, E_b$



$A(x, y)$



# Punktoperationen – Beispiele (4)

Punktoperation auf mehreren Bildern: Bewegung in Video

$$A(x, y) = |E_1(x, y) - E_2(x, y)|$$



$E_1(x, y)$



$E_2(x, y)$



$A(x, y)$

Anmerkung: idealisierte, vereinfachte Darstellung aus [Pinz, Bildverstehen, 1994]

# Bewegung in Video (1)

```

>> frame1=imread('jogger_0.jpg');
>> frame2=imread('jogger_1.jpg');
>> f1_hsv=rgb2hsv(frame1);
>> f2_hsv=rgb2hsv(frame2);
>> f1_v=f1_hsv(:,:,3);
>> f2_v=f2_hsv(:,:,3);
>> f_diff=abs(f1_v-f2_v);
>> imshow(f_diff);
    
```

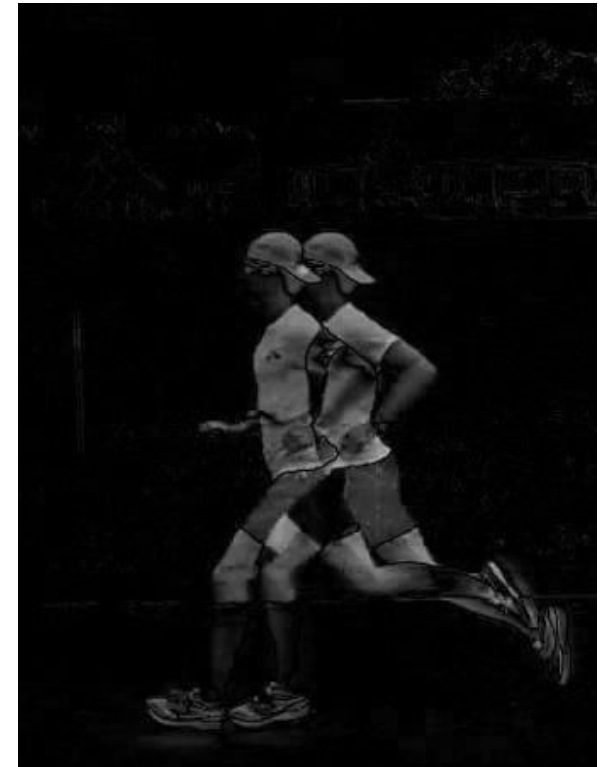
jogger\_0



jogger\_1



f\_diff

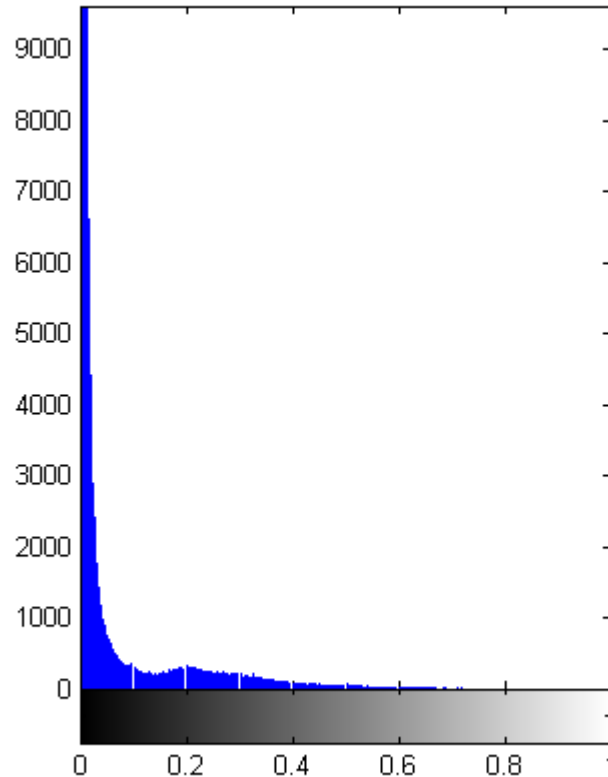


# Bewegung in Video (2)

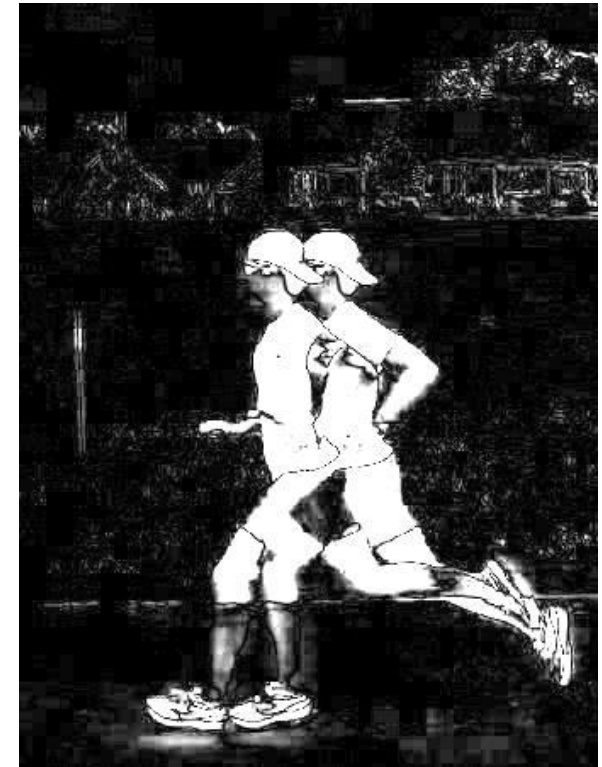
f\_diff



imhist(f\_diff)

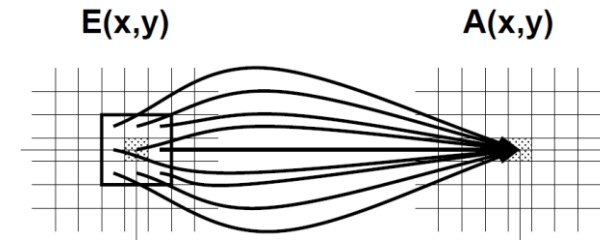


8\*f\_diff



Auch Bewegung im Hintergrund!  
→ Kamera-Pan

# Lokale Fensteroperation



Allgemein: lokales Fenster  $W_E$  von  $E$ :

$$W_E(x, y, \delta) = \begin{pmatrix} E(x - \delta, y - \delta) & \dots & E(x + \delta, y - \delta) \\ \vdots & E(x, y) & \vdots \\ E(x - \delta, y + \delta) & \dots & E(x + \delta, y + \delta) \end{pmatrix}, \quad A(x, y) = f(W_E(x, y, \delta))$$

Insbesondere: *Faltung*  $A = W * E$

$$A(x, y) = \sum_{s=-\delta}^{\delta} \sum_{t=-\delta}^{\delta} W(s, t) \cdot E(x + s, y + t)$$

Achtung: Behandlung des Bildrandes definieren!

# Beispiele zur Faltung – Hochpass

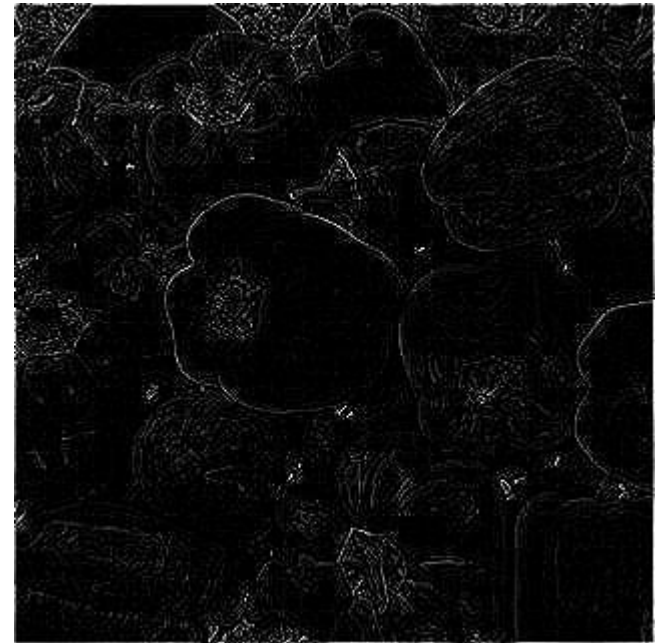
$$W = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

```
>> E=im2double(imread('peppers_val.jpg'));
>> W=[0,-1,0;-1,4,-1;0,-1,0];
>> A=imfilter(E,W,'conv');
>> imshow(A);
```

$E(x,y)$



$A(x,y)$



# Beispiele zur Faltung – Tiefpass

$$W = \frac{1}{9} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

```
>> E=imread('peppers_val.jpg');
>> W=[1,1,1;1,1,1;1,1,1];
>> A=imfilter(E,W,'conv');
>> imshow(A/9);
```

$E(x,y)$



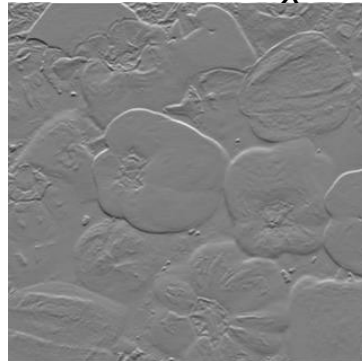
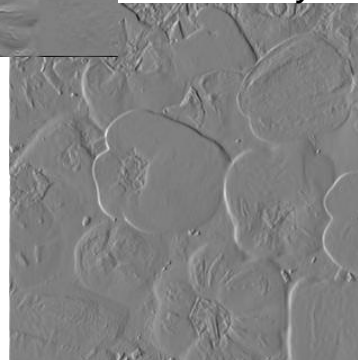
$A(x,y)$

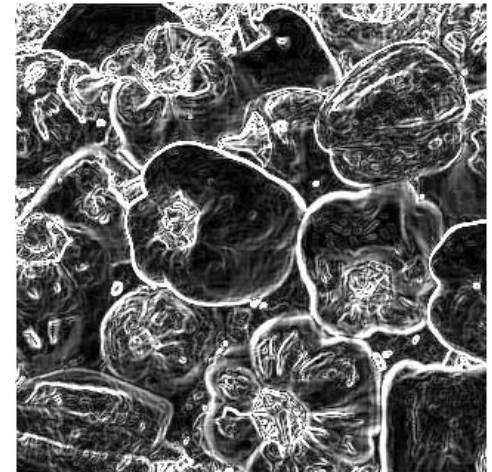


# Beispiele zur Faltung – Kantendetektion mit dem Sobeloperator

$$S_x = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{pmatrix}, S_y = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{pmatrix}, S_A = \sqrt{(S_x * E)^2 + (S_y * E)^2}$$

 $E(x,y)$ 

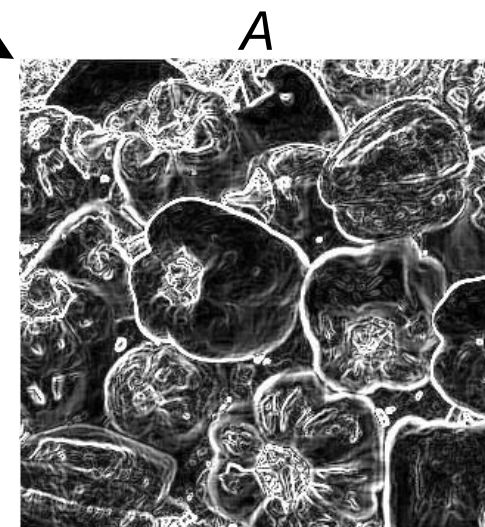
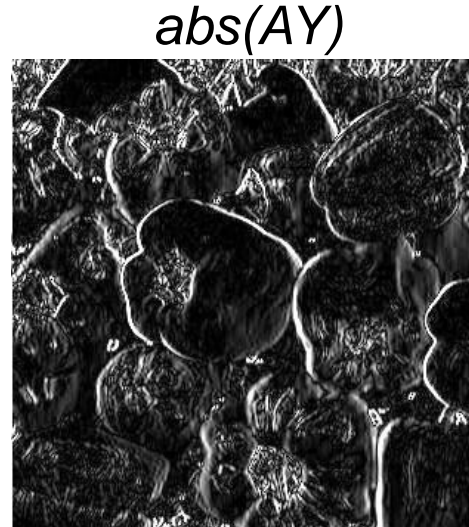
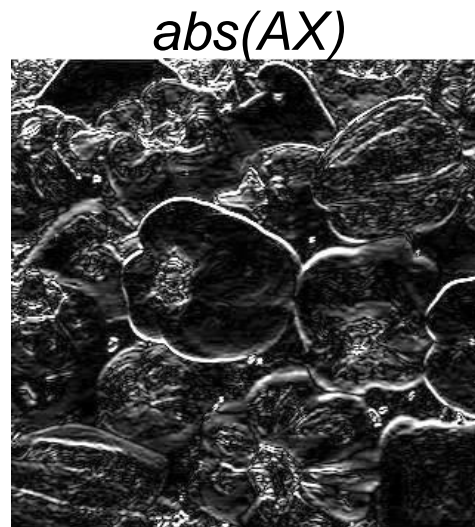
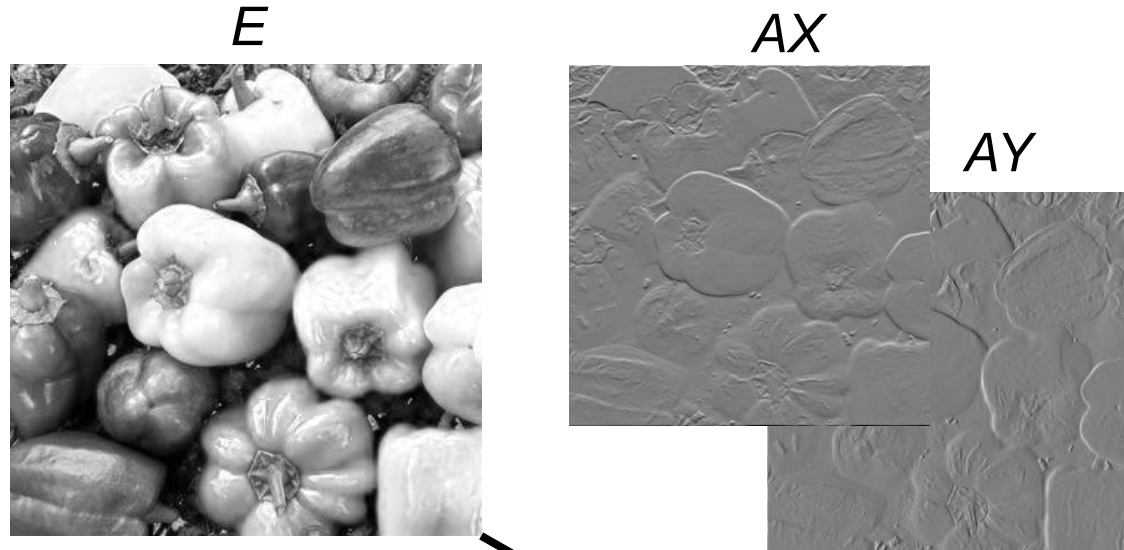
 $E * S_x$ 

 $E * S_y$ 

 $[-4,4]$ 

 $A(x,y)$ 


# Matlab Code zum Sobeloperator

```

>> E=im2double(imread('peppers_val.jpg'));
>> SX=[1,2,1;0,0,0;-1,-2,-1];
>> SY=[1,0,-1;2,0,-2;;1,0,-1];
>> AX=imfilter(E,SX,'conv');
>> AY=imfilter(E,SY,'conv');
>> imwrite(AX,'sobel_X.jpg');
>> imwrite(AY,'sobel_Y.jpg');
>> A=abs(AX)+abs(AY);
>> imshow(A);
>> imwrite(abs(AX),'abs_sobel_X.jpg');
>> imwrite(abs(AY),'abs_sobel_Y.jpg');
>> imwrite(A,'sobel_amplitude.jpg');
    
```





# Globale Operationen $A(u, v) = f(E)$

## Beispiel Fouriertransformation

$$\mathcal{F}\{f(x, y)\} = F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-2\pi i(ux+vy)} dx dy$$

$$F(u, v) = |F(u, v)| e^{j\Phi(u, v)}$$

$$\mathcal{F}^{-1}\{F(u, v)\} = f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{2\pi i(ux+vy)} du dv$$

$$|F(u, v)| \dots \text{Fourierspektrum}$$

$$e^{i\phi} = \cos \phi + i \sin \phi$$

$$\Phi(u, v) \dots \text{Phase}$$

$$e^{-i\phi} = \cos \phi - i \sin \phi$$

$$F(u, v) = \frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) e^{-2\pi i \left( \frac{ux+vy}{N} \right)}$$

$$f(x, y) = \frac{1}{N} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} F(u, v) e^{2\pi i \left( \frac{ux+vy}{N} \right)}$$

Filterung:

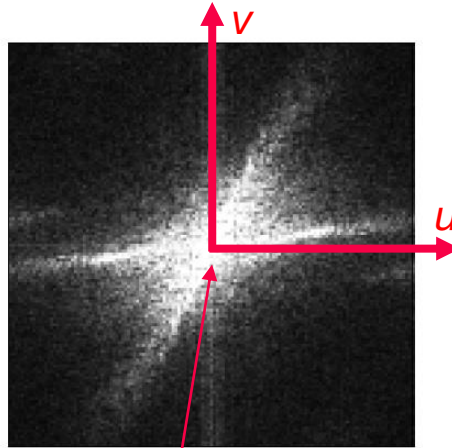
$$f(x, y) * g(x, y) = \mathcal{F}^{-1}\{F(u, v)G(u, v)\}$$

$$f * g \quad \circ \text{---} \bullet \quad F \cdot G$$

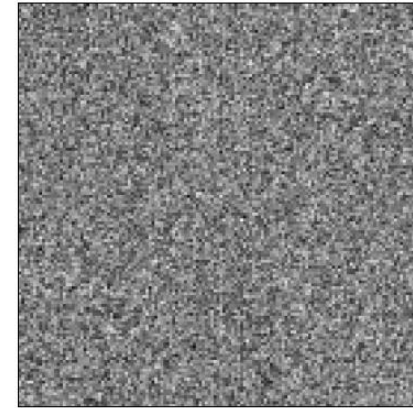
# Fouriertransformation – Beispiel (1)



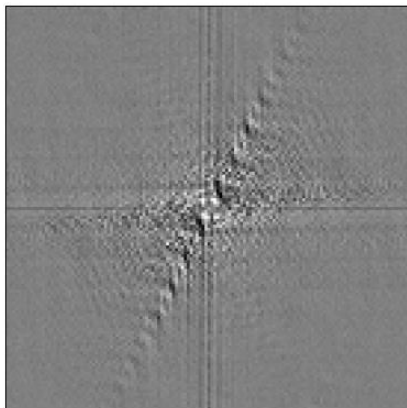
(a) Originalbild



(b) Spektrum  $|F(u,v)|$



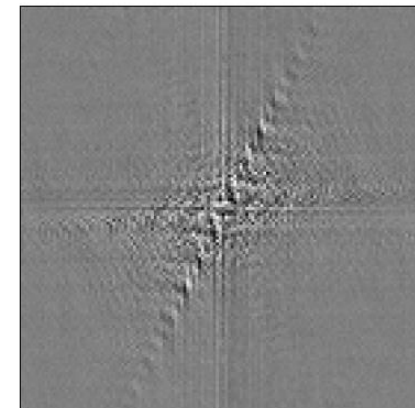
(c) Phase  $\Phi(u,v)$



(d) Realteil

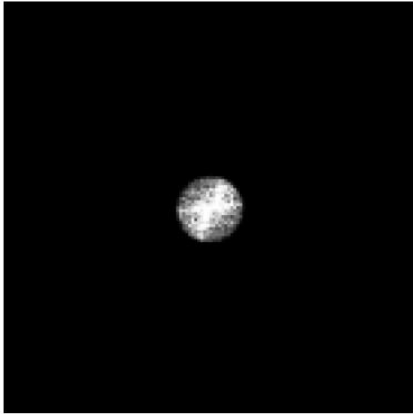
$(u,v)=(0,0)$ :  
Orsfrequenz  $\mu_0=0$

[Pinz, Bildverstehen, 1994]

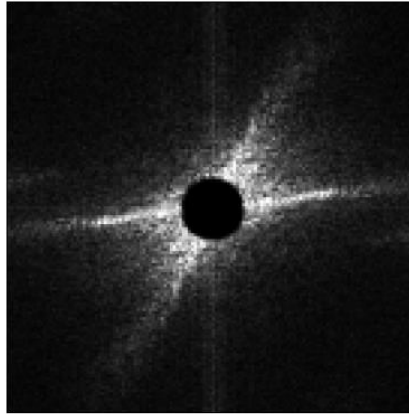


(e) Imaginärteil

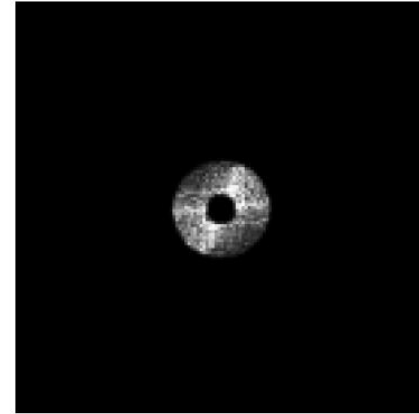
# Fouriertransformation – Beispiel (1) [Pinz, 1994]



(a) Tiefpaß



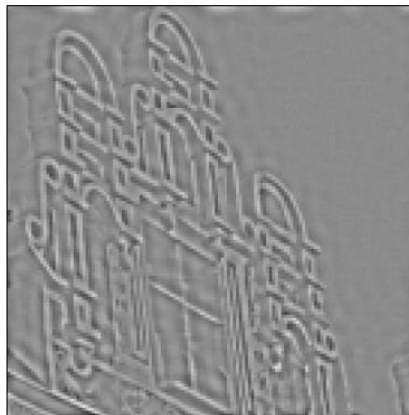
(b) Hochpaß



(c) Bandpaß



(d) Tiefpaß



(e) Hochpaß

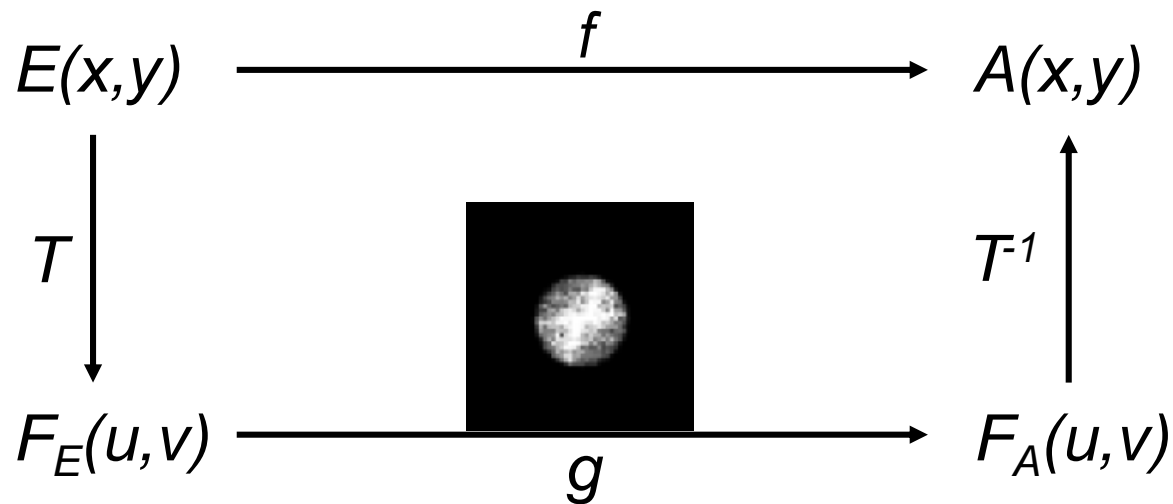


(f) Bandpaß

# Ein Resultat, viele mögliche Lösungswege



```
>> E=imread('peppers.jpg');
>> G5=fspecial('gaussian',15,5);
>> A=imfilter(E,G5);
```



# Fouriertransformation in Matlab

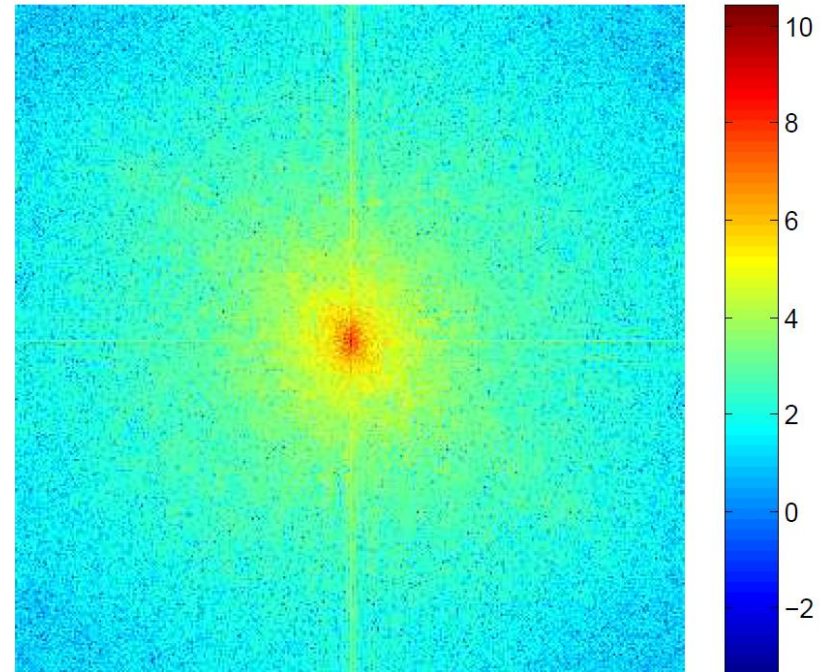
```

>> E=im2double(imread('peppers_BW.jpg'));
>> F=fft2(E);
>> F=fftshift(F);
>> figure(1), hold off, imagesc(E), axis off, colormap gray, axis image, title('intensity image');
>> figure(2), hold off, imagesc(log(abs(F))), axis off, colormap jet, axis image, colorbar, title('log fft magnitude');
    
```

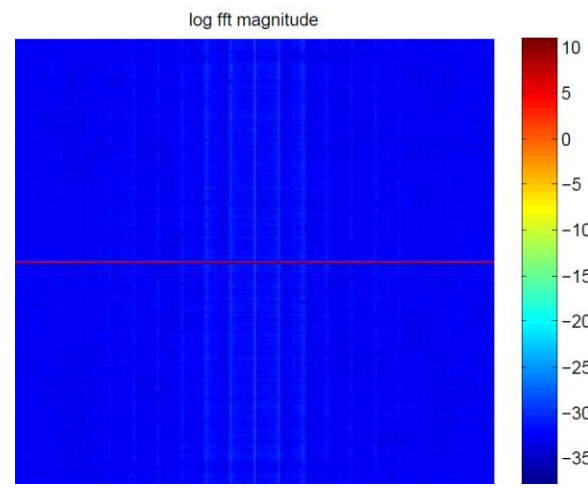
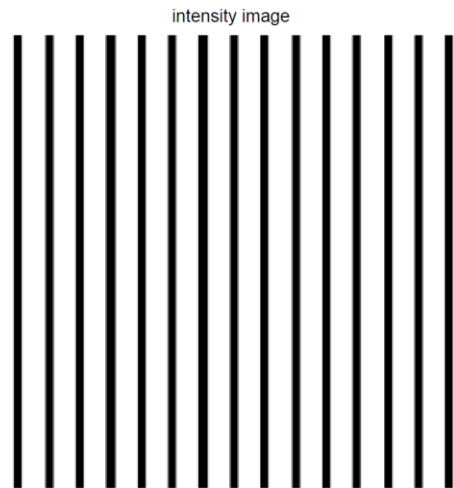
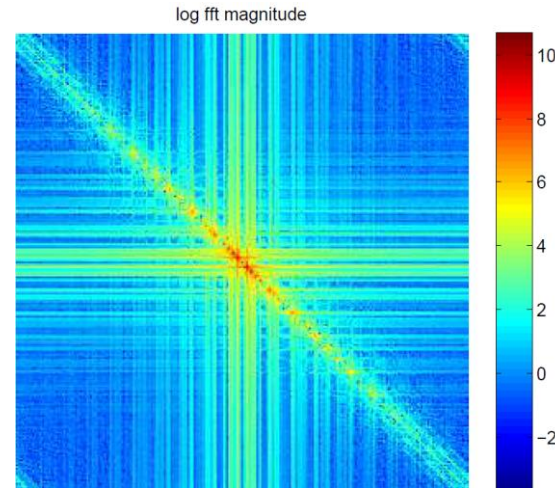
intensity image



log fft magnitude



# Fouriertransformation – Beispiel (2)



*Vgl: rezeptives Feld von komplexen Zellen im visuellen Cortex!*

# Segmentation

Unterteilen eines Bildes  $B$  in disjunkte Regionen  $R_i$

**Formale Definition** von Segmentation benötigt ein **Homogenitätskriterium  $H$** :

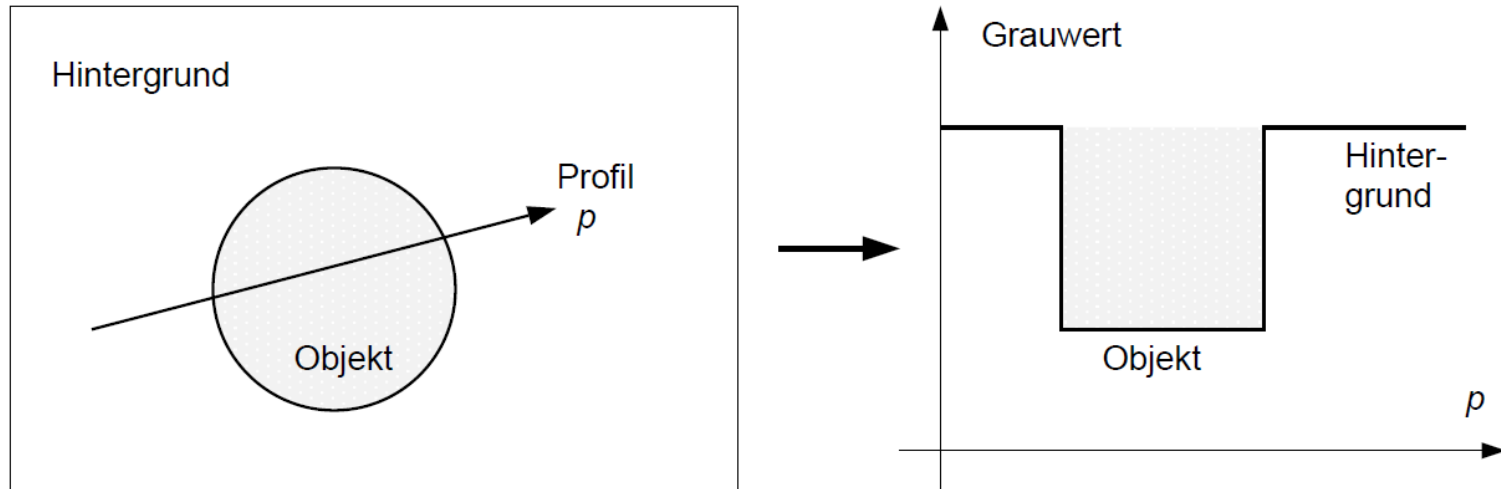
$$\bigcup_{i=1}^{N_R} R_i = B, \quad \text{wobei } \forall i : R_i \subseteq B$$

$$\forall i \neq j : R_i \cap R_j = \emptyset$$

$$H(R_i) = \text{true}$$

$$H(R_i \cup R_j) = \text{false}, \quad \text{wenn } R_i \text{ Nachbar von } R_j \text{ ist.}$$

# Flächen- / kantenbasierte Segmentation



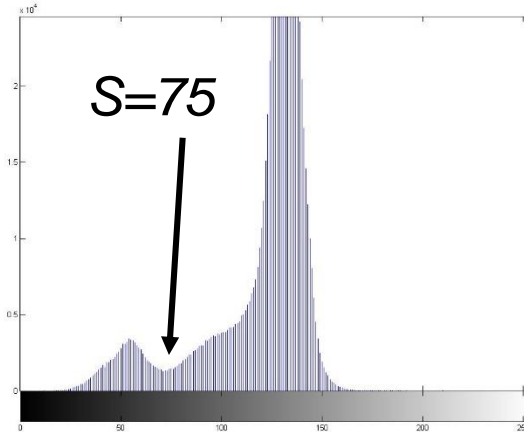
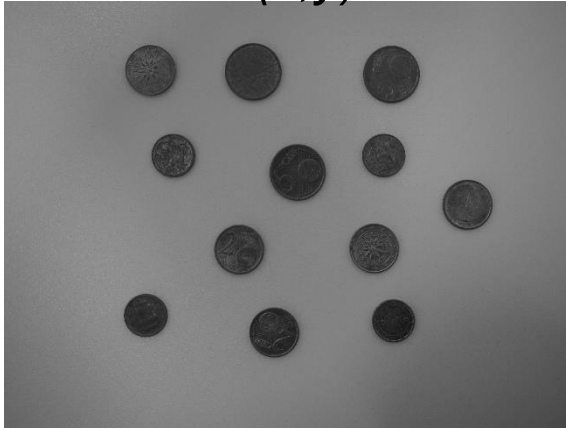
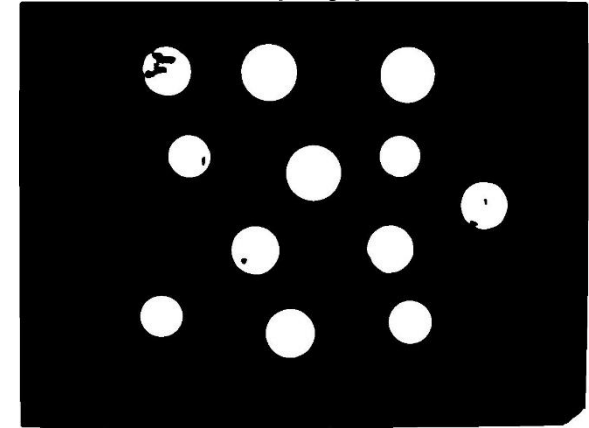
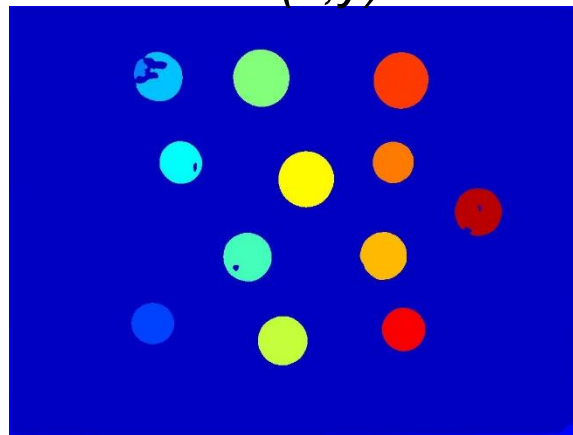
Segmentation durch:

1. Finden der Objekte als homogene *Flächen*
2. Finden der *Grenzen* zwischen Objekt und Hintergrund.

[Pinz, Bildverstehen, 1994]



# Beispiel: Regionenbasierte Segmentation mittels Schwellwert

 $E(x,y)$ 

 $A(x,y)$ 

 $A2(x,y)$ 


1. Glätten (Gauß)
2. Schwellwert
3. Region labelling
4. Segmentation

```

>> G5=fspecial('gaussian',15,5);
>> EG=imfilter(E,G5);
>> A=1-im2bw(EG, 75/255);
>> A1=bwconncomp(A);
>> A2=labelmatrix(A1);
    
```

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Digitale Bildanalyse

- Digitales Rasterbild, Kenngrößen
- Bildverarbeitungsoperationen, Segmentation
- **Salient point detection + description**

# Detektion und Beschreibung auffälliger Punkte “salient points” / “corners” / „Ecken“

## Beispiel 1:

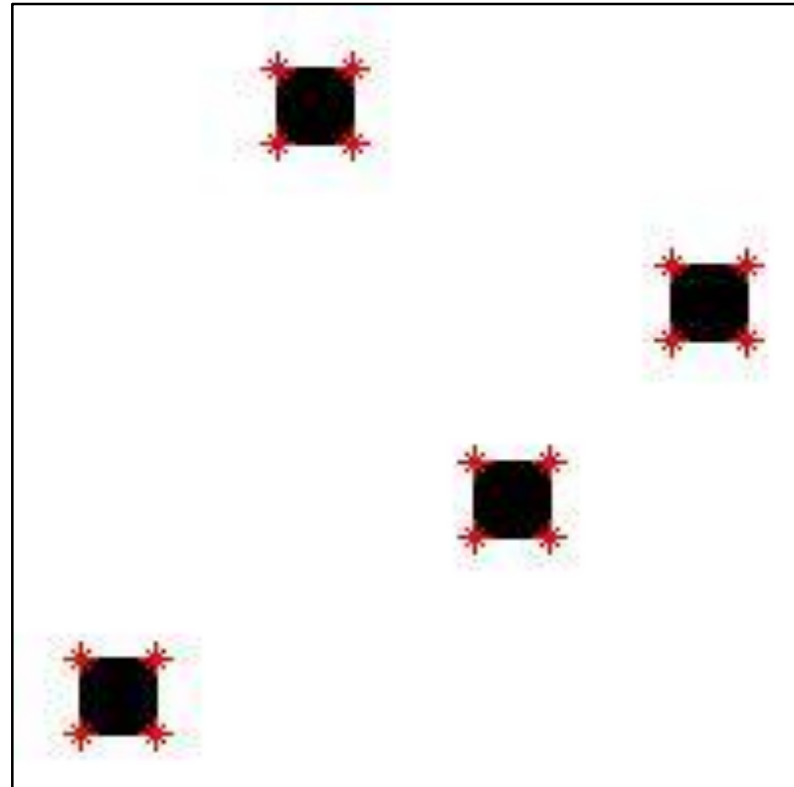
```
>> E=imread('peppers_val.jpg');  
>> C=corner(E, 'Harris');  
>> imshow(E);  
>> hold on;  
>> plot(C(:,1), C(:,2),'r*');
```



# Detektion und Beschreibung auffälliger Punkte “salient points” / “corners” / „Ecken“

## Beispiel 2:

```
>> E=imread('squares.jpg');
>> C=corner(E, 'Harris');
>> imshow(E);
>> hold on;
>> plot(C(:,1), C(:,2),'r*');
```

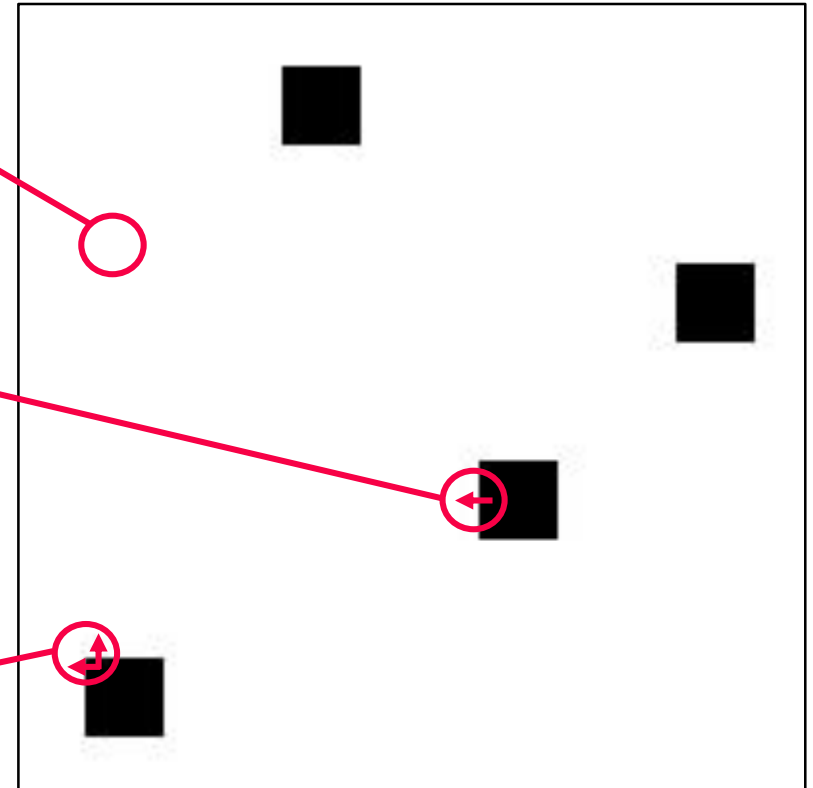


# Was definiert eine „Ecke“?

Homogene Bildregion: **Kein Gradient**

Kante: Gradient in **1 Richtung**

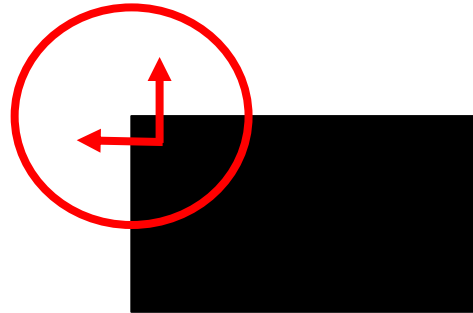
Ecke: Mindestens **2 Gradienten-Richtungen** im „rezeptiven Feld“



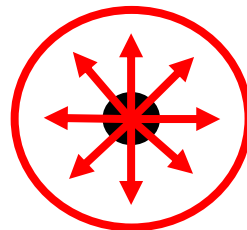
Gradient  $\leftrightarrow$  erste Ableitung

# Ecke $\leftrightarrow$ “salient point”

Ecke: Gradienten in 2 Richtungen



Auch das ist ein “salient point”: Gradienten in *alle* Richtungen



*Vgl: “On-Off” und “Off-On” Zellen !*

# Point Correspondences - Example



Structure and motion from  
“natural” landmarks [Schweighofer]

→ Stereo reconstruction of Harris corners



# Salient points (corners) based on 1<sup>st</sup> derivatives

- Autocorrelation of 2D image signal [Moravec]
  - Approximation by sum of squared differences (SSD)
  - Window  $W$
  - Differences between grayvalues in  $W$  and a window shifted by  $(\Delta x, \Delta y)$

$$\begin{aligned}
 f(x, y) &= \sum_{x_w = -\delta}^{\delta} \sum_{y_w = -\delta}^{\delta} [I(x_w, y_w) - I(x_w + \Delta x, y_w + \Delta y)]^2 \\
 &= \sum_{(x_w, y_w) \in W} [I(x_w, y_w) - I(x_w + \Delta x, y_w + \Delta y)]^2
 \end{aligned}$$

- Four different shift directions  $f_i(x, y)$ :  $f_{\text{Moravec}} = \sum_{i=1}^4 f_i$
- A corner is detected, when  $f_{\text{Moravec}} > th$



# Salient points (corners) based on 1<sup>st</sup> derivatives

- Autocorrelation (*second moment*) matrix:

- Avoids various shift directions

- Approximate  $I(x_w + \Delta x, y_w + \Delta y)$  by Taylor expansion:

$$I(x_w + \Delta x, y_w + \Delta y) \approx I(x_w, y_w) + \left( I_x(x_w, y_w) \quad I_y(x_w, y_w) \right) \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

- Rewrite  $f(x, y)$ :

Gradient in x-Richtung

$$f(x, y) = \sum_{(x_w, y_w) \in W} \left[ \left( I_x(x_w, y_w) \quad I_y(x_w, y_w) \right) \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \right]^2$$

$$= \sum_{(x_w, y_w) \in W} \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} \begin{pmatrix} I_x(x_w, y_w) \\ I_y(x_w, y_w) \end{pmatrix} \left( I_x(x_w, y_w) \quad I_y(x_w, y_w) \right) \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

$$= \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} \mathbf{M} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

“second moment matrix **M**”

# Salient points (corners) based on 1<sup>st</sup> derivatives

- Autocorrelation (*second moment*) matrix:

$$\mathbf{M} = \mu = \begin{pmatrix} \sum_{(x_w, y_w) \in W} I_x^2(x_w, y_w) & \sum_{(x_w, y_w) \in W} I_x(x_w, y_w) I_y(x_w, y_w) \\ \sum_{(x_w, y_w) \in W} I_x(x_w, y_w) I_y(x_w, y_w) & \sum_{(x_w, y_w) \in W} I_y^2(x_w, y_w) \end{pmatrix}$$

- M can be used to derive a measure of “cornerness”
- Independent of various displacements ( $\Delta x, \Delta y$ )
- Corner: significant gradients in >1 directions  $\rightarrow$  rank  $\mathbf{M} = 2$
- Edge: significant gradient in 1 direction  $\rightarrow$  rank  $\mathbf{M} = 1$
- Homogeneous region  $\rightarrow$  rank  $\mathbf{M} = 0$
- Several variants of this corner detector:
  - KLT corners, Förstner corners

# Salient points (corners) based on 1<sup>st</sup> derivatives

- Harris corners

- Most popular variant of a detector based on  $\mathbf{M}$
- Local derivatives with “derivation scale”  $\sigma_D$
- Convolution with a Gaussian with “integration scale”  $\sigma_I$
- $\mathbf{M}_{\text{Harris}}$  for each point  $\mathbf{x}$  in the image

$$\mathbf{M}_{\text{Harris}}(\mathbf{x}, \sigma_I, \sigma_D) = \mu(\mathbf{x}, \sigma_I, \sigma_D) = \sigma_D^2 G(\sigma_I) * \begin{pmatrix} I_x^2(\mathbf{x}, \sigma_D) & I_x(\mathbf{x}, \sigma_D)I_y(\mathbf{x}, \sigma_D) \\ I_x(\mathbf{x}, \sigma_D)I_y(\mathbf{x}, \sigma_D) & I_y^2(\mathbf{x}, \sigma_D) \end{pmatrix}$$

- Corneriness  $c_{\text{Harris}}$  does not require to compute eigenvalues

$$c_{\text{Harris}} = \det \mathbf{M} - \alpha \text{trace } \mathbf{M}$$

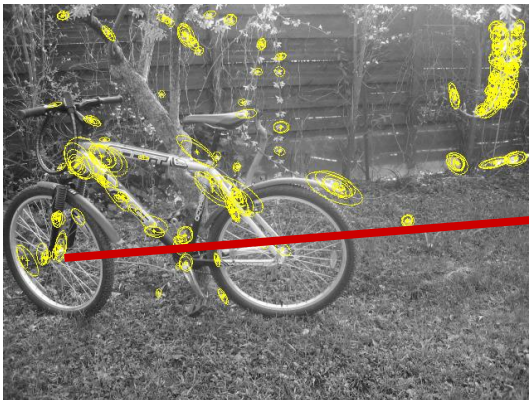
- Corner detection:  $c_{\text{Harris}} > t_{\text{Harris}}$

$$\sigma_I = 2, \quad \sigma_D = 0.7$$

$$\alpha = 0.05, \quad t_{\text{Harris}} = 25000$$

# Descriptors (1)

- Representation of salient regions
- “descriptive” features  $\rightarrow$  feature vector
- There are *many* possibilities !
- Matching, calibration, specific object recognition:
  - Search for high descriptive power
  - Tolerate viewpoint variations



$$\mathbf{f}_n = (f_{n,1}, \dots, f_{n,j})^T$$

feature vector extracted  
from patch  $P_n$

## Descriptors (2)

- Grayvalues
  - Raw pixel values of a patch  $P$
  - “local appearance-based description”

- General moments of order  $p+q$ :

$$m_{pq} = \sum_{x \in P} \sum_{y \in P} x^p y^q I(x, y)$$

- Moment invariants:

- Central moments  $\mu_{pq}$ : invariant to *translation*

$$\bar{x} = \frac{m_{10}}{m_{00}}, \bar{y} = \frac{m_{01}}{m_{00}} : \mu_{pq} = \sum_{x \in P} \sum_{y \in P} (x - \bar{x})^p (y - \bar{y})^q I(x, y)$$

# Descriptors (3)

additional  
(optional)  
material

- Moment invariants:

- Normalized central moments

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^\gamma}, \quad \text{with } \gamma = 1 + \frac{p+q}{2}$$

- Translation, rotation, scale invariant moments  $\Phi_1 \dots \Phi_7$  [Hu]

$$\Phi_1 = \eta_{20} + \eta_{02}$$

$$\Phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

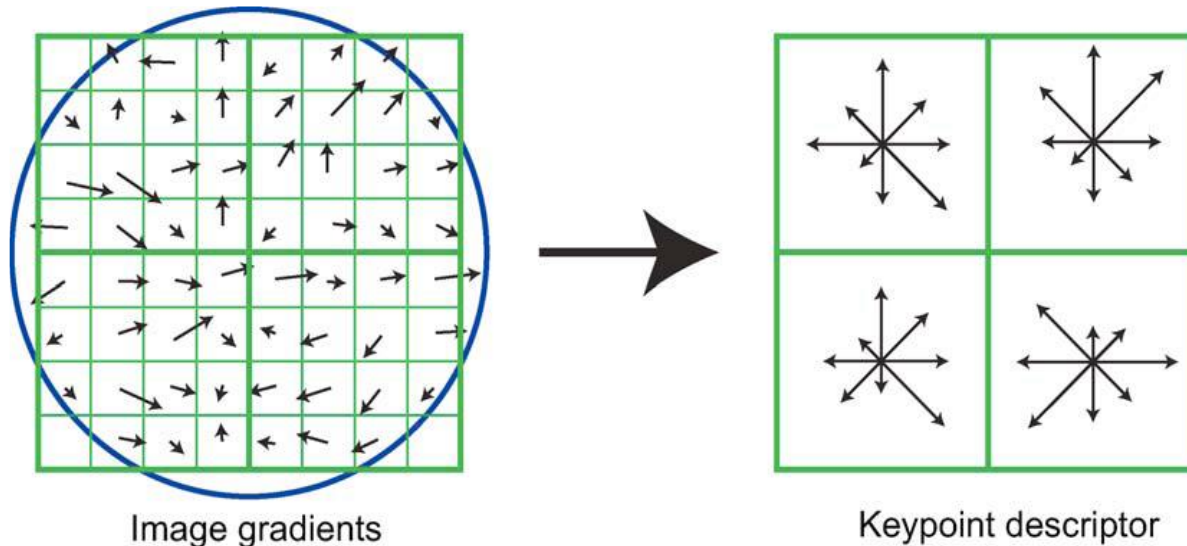
- Geometric/photometric, color invariants [vanGool et al.]

- Filters

- “local jets” [Koenderink+VanDoorn]
- Gabor banks, steerable filters, discrete cosine transform DCT

# Descriptors (4)

- SIFT descriptors [Lowe]
  - Scale invariant feature transform
  - Calculated for local patch  $P$ : 8x8 or 16x16 pixels
  - Subdivision into 4x4 sample regions
  - Weighted histogram of 8 gradient directions:  $0^\circ, 45^\circ, \dots$
  - SIFT vector dimension: 128 for a 16x16 patch



[Lowe]

# Wo braucht man Punktdetektoren und -deskriptoren?

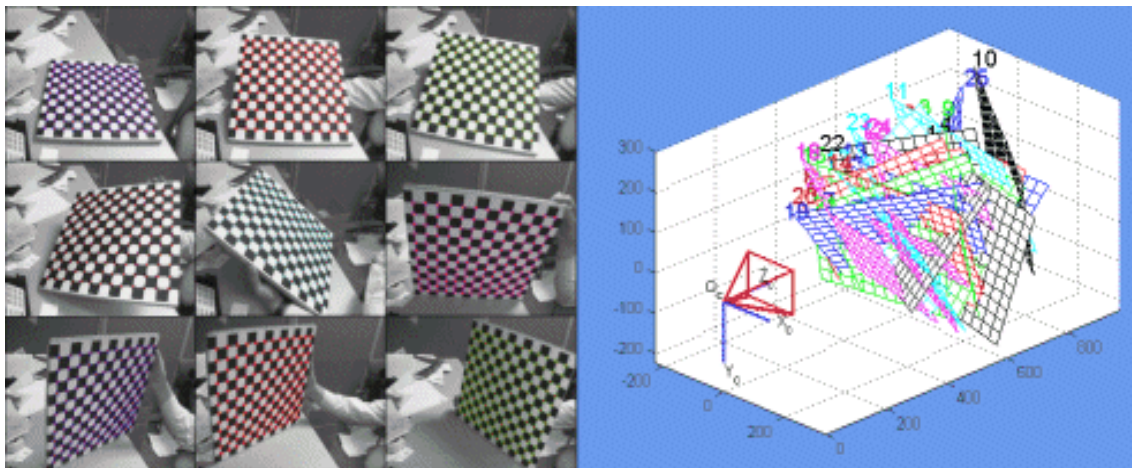
Immer wenn es nötig ist, *Punktkorrespondenzen* herzustellen!

Kamerakalibrierung

3D Tiefe aus Stereokorrespondenz

...

Immer, wenn aus Bildern *gemessen* werden soll!





# Messen optischer Größen, Messen aus Bildern

## Übersicht

Optische Strahlung, Sensorik

Geometrie, Photogrammetrie

- Kamerakalibrierung
- Stereo

Menschliche Wahrnehmung

- Neurophysiologie
- Kognitive Psychologie



Digitale Bildanalyse

- Digitales Rasterbild, Kenngrößen
- Bildverarbeitungsoperationen, Segmentation
- Salient point detection + description