Institute of
Applied Mechanics
Institut für Baumechanik

TU Graz
Graz University of Technology

Bachelorarbeit im Studiengang Bauingenieurwissenschaften
am Institut für Baumechanik

Technische Universität Graz

Symbolic calculation of beam structures

Digitalisation of teaching Strength of Materials at University

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Graz, 30. Mai 2022

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Michael Helmut Gfrerer

Statutory Declaration

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[1] Beschluss der Curricula-Kommission für Bachelor-, Master- und Diplomstudien vom 10.11.2008; Genehmigung des Senates am 1.12.2008

Abstract

This project is part of a digitalisation offensive at Graz Technical University (TU Graz). It will enhance the digital learning in Engineering Mechanics at TU Graz in multiple departments. The innovative part being that it is possible to create individual scalable assignments for every student.

To make this possible a python script was written in order to automatically create and solve homework assignments. For the calculation of beam structures often numerical solutions are used, however in the course Strength of Materials the mechanical systems should be solved in a symbolic way.

Therefore, using the Euler-Bernoulli beam theory in the governing equations a system of linear equations has been set up and later on solved. The most challenging part was to implement beams with varying flexural and axial rigidity as well as stiff elements. Furthermore, joints and different types of bearings can be used at the nodes.

Zusammenfassung

Dieses Bachelorprojekt ist Teil einer Digitalisierungsoffensive an der Technischen Universität Graz. Es stärkt die digitale Lehre im Bereich Mechanik an der TU Graz an verschiedenen Instituten. Innovativ hierbei ist es, dass skalierbare individuelle Übungsaufgaben für alle Student:innen erstellt werden können.

Es wurde hierfür ein Python Skript verfasst, um das Erstellen und Lösen von Hausübungsbeispielen für die Übung Baumechanik 2 an der TU Graz zu automatisieren. Die Lehrveranstaltungen zur Festigkeitslehre verwenden für die Berechnung von Stabtragwerken keine numerische Methode. Daraus ergibt sich die spezielle Anforderung, dass Stabtragwerke symbolisch gelöst werden müssen.

Es wurde ein System von linearen Gleichungen aufgestellt und später gelöst. Dabei wurde die Euler-Bernoulli Balkentheorie für die bestimmenden Gleichungen angewendet. Die größte Herausforderung war es, Stäbe mit unterschiedlichen Biege- und Dehnsteifigkeiten sowie starren Stäbe zu implementieren. Außerdem ist es möglich unterschiedliche Auflager sowie Momentengelenke an den Knoten einzufügen.

Danksagung

I want to thank Ass. Prof. Michael Helmut Gfrerer for the continuing support to complete this bachelor thesis.

CONTENTS

1. INTRODUCTION

Teaching theoretical principles of Strength of Materials is an essential part of university education for civil engineers. However, to create individual supervision for each student is often not possible due to limited resources in teaching staff. Thus, digitalisation is one way to efficiently enhance the existing capabilities.

For calculating the deformation of a beam structure often numerical solutions are used. But in the course *Baumechanik 2* the systems are solved in a symbolic way [1]. The following method was necessary to automatically create examples and solutions so that each student can have a different assignment.

The goal was to implement beams, joints and bearings so that varying beam structures can be designed. Furthermore, it should be possible to use an individual stiffness for each beam as well as using an infinite stiffness and therefore rigid beams.

The governing equations are based on Euler-Bernoulli beam theory. These equations are adapted so that they can be used for setting up a system of linear equations. We do not look at each beam in a system but we set up the linear equations for each node. At each node governing equations are combined with supporting conditions of the bearings and joints [2].

2. GOVERNING EQUATIONS FOR A SINGLE BEAM

In this section the governing equations for a single beam are introduced and are used for assembling a system of beams in Chapter 3. The following governing equations are based on the well-known Euler-Bernoulli beam theory, see for instance [3] or [4]. Along the beam axis constant geometrical and material parameters are assumed. In the following, E is the Youngs Modul, A the cross section area and I the area moment of inertia.

The axial deformation $u(x)$ is governed by the equilibrium equation

$$N'(x) = -n(x), \tag{2.1}$$

and the constitutive equation

$$N(x) = EAu'(x). \tag{2.2}$$

Here, $n(x)$ is the external and $N(x)$ is the internal axial force and EA the axial rigidity. Furthermore, $()'$ denotes the derivative with respects to x, where x describes the position within the beam $x \in [0, \ell]$, where ℓ is the length of the beam. In the following, c_o - c_5 are the integration constants.

For the case $EA < \infty$, we use the approach

$$u(x) = c_0 x + c_1 - \frac{1}{EA} \iint n(x)\, dx, \tag{2.3}$$

which results in the following derived equations

$$u'(x) = c_0 - \frac{1}{EA} \iint n(x)\, dx, \tag{2.4a}$$

$$N(x) = EAc_0 - \int n(x)\, dx. \tag{2.4b}$$

Whereas, for the case $EA \to \infty$ we use

$$u(x) = c_1, \tag{2.5a}$$

$$N(x) = c_0 - \int n(x)\, dx. \tag{2.5b}$$

The transversal deformation $w(x)$ is governed by the equilibrium equation

$$M''(x) = Q'(x) = -q(x), \tag{2.6}$$

and the constitutive equation

$$M(x) = -EIw''(x), \tag{2.7}$$

where $M(x)$ is the internal bending moment, $Q(x)$ the internal shear force and $q(t)$ the external transversal loading force, EI describing the flexural rigidity and $w'(x)$ is related to the rotation of the beam.

For $EI < \infty$ we use the approach

$$w(x) = c_2x^3 + c_3x^2 + c_4x + c_5 + \frac{1}{EI}\iiiint q(x)\ dx, \tag{2.8}$$

which results in the following derived equations

$$w'(x) = 3c_2x^2 + 2c_3x + c_4 + \frac{1}{EI}\iiint q(x)\ dx, \tag{2.9a}$$

$$w''(x) = 6c_2x + 2c_3 + \frac{1}{EI}\iint q(x)\ dx, \tag{2.9b}$$

$$M(x) = -EI(6c_2x + 2c_3) - \iint q(x)\ dx, \tag{2.9c}$$

$$Q(x) = -EI6c_2 - \int q(x)\ dx. \tag{2.9d}$$

If $EI \to \infty$ we use the approach

$$w(x) = c_4x + c_5, \tag{2.10a}$$

$$M(x) = c_2x + c_3 - \iint q(x)\ dx, \tag{2.10b}$$

which results in the following derived equations

$$w'(x) = c_4, \tag{2.11a}$$

$$Q(x) = c_2 - \int q(x)\ dx. \tag{2.11b}$$

3. SOLUTION OF A SYSTEM OF BEAMS

3.1. Mechanical system of beams

In this section a system of beams is introduced. The systems can contain multiple nodes, where an arbitrary number of beams can be attached. Joints and bearings can be located at nodes. On beams different types of distributed external loads can be applied. An example is illustrated in Figure 3.1.

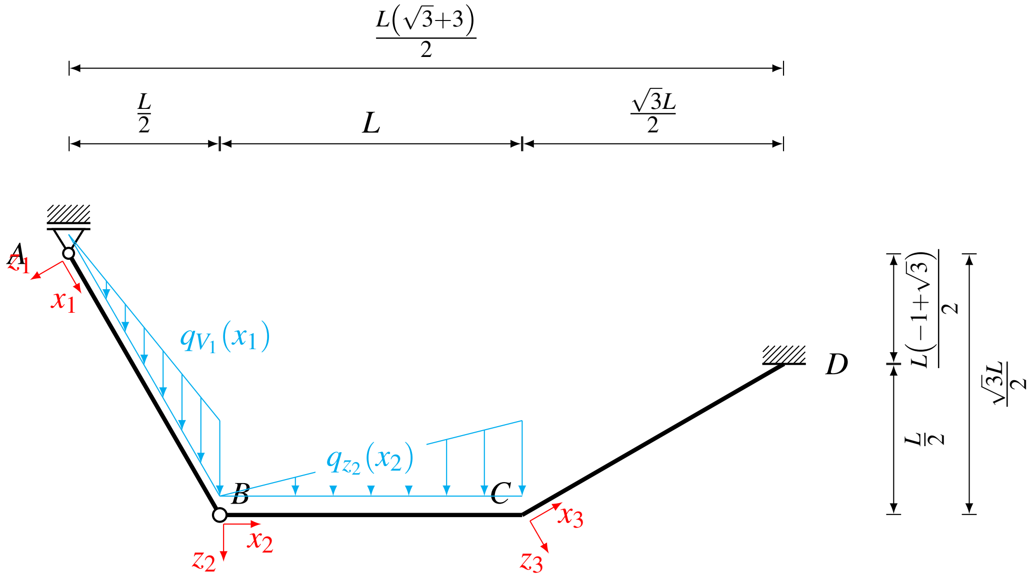


Figure 3.1.: Example for a system of beams

The vectors $\vec{n}_i$ and $\vec{t}_i$ are introduced to describe the direction of beam i

$$\vec{t}_i = \begin{pmatrix} t_x \\ t_y \end{pmatrix} = \frac{\vec{x}_B - \vec{x}_A}{|\vec{x}_B - \vec{x}_A|}, \tag{3.1a}$$

$$\vec{n}_i = \begin{pmatrix} -t_y \\ t_x \end{pmatrix}. \tag{3.1b}$$

$\vec{n}_i$ is the normal vector and $\vec{t}_i$ is the parallel vector to the axis of the beam as seen in Figure 3.2.

$\vec{n}_i$ and $\vec{t}_i$ are later on used to separate the components of certain equations in x- and y-direction.

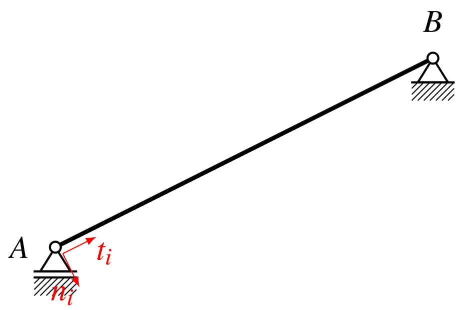


Figure 3.2.: Depiction of $\vec{n}_i$ and $\vec{t}_i$

3.2. System of equations

In order to solve a system of beams a system of linear equations is set up. The system of linear equations reads

$$\mathbf{Ac} = \mathbf{r}, \tag{3.2}$$

where $\mathbf{A}$ is the system matrix, $\mathbf{c}$ the solution vector and $\mathbf{r}$ is right hand side due to distributed loads. The number of equations is depended on the number of elements, e.g. it holds

$$number\,equations = 6 * number\,elements. \tag{3.3}$$

3.3. Setting up the equations

The linear equations are set up for each node separately. The governing equations are combined with the supporting conditions of the bearings and joints.

The system matrix and the right hand side are built in several steps. The first step is to consider the translational movement.
The basic idea is, that with the exception of the *Querkraftgelenk* the translational movement of all attached elements at one node must be equal. Also to be considered is if the node movement is restricted and therefore a fixed displacement is needed.

3.3.1. Displacement compatibility

The first set of equations we describe considers the displacement compatibility. This means that the displacement at one node has to be equal for all attached beams. We formulate this condition for node k, where we assume that n_k beams are attached. Therefore, for two beams i and $i+1$ the equation reads

$$w_{i|k}\vec{n}_i + u_{i|k}\vec{t}_i = w_{i+1|k}\vec{n}_{i+1} + u_{i+1|k}\vec{t}_{i+1}, \tag{3.4}$$

where we used the notation that for a quantity $y_i(x)$, $y_{i|k}$ denotes the evaluation at node k, i.e. $y_i(x_k)$. In total (3.4) is formulated for $i = 1, \ldots, n_k - 1$.

In view of the implementation we reformulate (2.8) to

$$w_i = \hat{w}_i C_i + \frac{1}{EI_i} \iiiint q_i(x)\, dx, \tag{3.5}$$

where we introduced the row vector $\hat{w}_i$,

$$\hat{w}_i = \begin{cases} \left[0, 0, x^3, x^2, x, 1\right] & \text{for} \quad EI < \infty \\ \left[0, 0, 0, 0, x, 1\right] & \text{for} \quad EI \to \infty \end{cases}, \tag{3.6}$$

and the column vector C_i with the integration constants related to element i,

$$C_i = \begin{bmatrix} C_i \\ C_{i+1} \\ C_{i+2} \\ C_{i+3} \\ C_{i+4} \\ C_{i+5} \end{bmatrix}. \tag{3.7}$$

The displacement $u(x)$ in (2.3) is reformulated to

$$u_i = \hat{u}_i \vec{C}_i - \frac{1}{EA_i} \iint n_i(x)\, dx, \tag{3.8}$$

where we introduced the row vector $\hat{u}_i$

$$\hat{u}_i = \begin{cases} \left[x, 1, 0, 0, 0, 0\right] & \text{for} \quad EA < \infty \\ \left[0, 1, 0, 0, 0, 0\right] & \text{for} \quad EA \to \infty \end{cases}. \tag{3.9}$$

Rearrangement of (3.4) yields

$$w_{i+1|k}\vec{n}_{i+1} + u_{i+1|k}\vec{t}_{i+1} - w_{i|k}\vec{n}_i - u_{i|k}\vec{t}_i = 0 \tag{3.10}$$

and collecting terms with integration constants on the left side and known terms on the right side yields

$$(\hat{w}_{i+i|k} C_{i+1})\vec{n}_{i+1} + (\hat{u}_{i+1|k} C_{i+1})\vec{t}_{i+1} - (\hat{w}_{i|k} iCi)\vec{n}_i - (\hat{u}_{i|k} C_i)\vec{t}_i = \vec{r}_i - \vec{r}_{i+1}, \tag{3.11}$$

where we introduced

$$\vec{r}_i = \vec{n}_i \frac{1}{EI_i} \iiiint q_i(x)\ dx - \vec{t}_i \frac{1}{EA_i} \iint n_i(x)\ dx. \tag{3.12}$$

Note that in the case of $EI \to \infty$ we have

$$\vec{r}_i = -\vec{t}_i \frac{1}{EA_i} \iint n_i(x)\ dx, \tag{3.13}$$

whereas in the case of $EA \to \infty$ it is

$$\vec{r}_i = \vec{n}_i \frac{1}{EI_i} \iiiint q_i(x)\ dx. \tag{3.14}$$

Furthermore, if $EI \to \infty$ and $EA \to \infty$ we have simultaneously

$$\vec{r}_i = 0. \tag{3.15}$$

3.3.2. Fixed displacement

The displacement of a node can already be a given condition, e. g. when there are pinned or clamped support types. In this case we have the condition

$$w_{i|k}\vec{n}_i + u_{i|k}\vec{t}_i = 0. \tag{3.16}$$

Here only one attached element is considered. The condition is applied to other elements of the node through displacement coupling.

In case of a roller bearing the condition reads

$$(w_{i|k}\vec{n}_i + u_{i|k}\vec{t}_i) \cdot \vec{N} = 0, \tag{3.17}$$

where $\vec{N}$ is the direction in which the bearing prevents movement see Figure 3.3.

3.3.3. Rotation compatibility

Rotation compatibility is used when all attached beams must have the same rotation. This is the case when there is no joint and not a bearing of the type clamped or parallel guide. For the rotation compatibility only w' is considered and the condition reads

$$w'_{i|k} = w'_{i+1|k}. \tag{3.18}$$

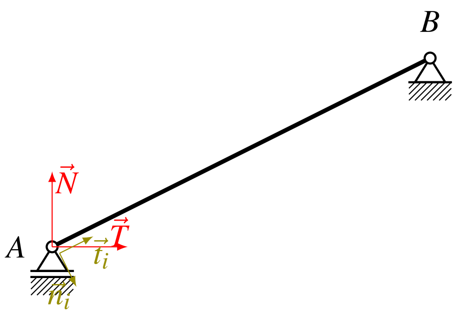


Figure 3.3.: Roller bearing with $\vec{N}$ and $\vec{T}$

In view of the implementation we reformulate (2.9b) to

$$w_i' = \hat{w}_i' C_i + \frac{1}{EI_i} \iiint q_i(x)\, dx, \tag{3.19}$$

where $\hat{w}_i'$ is depended on the stiffness

$$\hat{w}_i' = \begin{cases} \left[0, 0, 3x^2, 2x, 1, 0\right] & \text{for} \quad EI < \infty \\ \left[0, 0, 0, 0, 1, 0\right] & \text{for} \quad EI \to \infty \end{cases}, \tag{3.20}$$

and C_i used as in (3.7).

Rearrangement of (3.18) and collecting known terms on the right side yields

$$\hat{w}_i' C_i - \hat{w}_{i+1}' C_{i+1} = \vec{r}_{i+1} - \vec{r}_i, \tag{3.21}$$

where $\vec{r}_i$ is depended on flexural rigidity

$$\vec{r}_i = \frac{1}{EI_i} \iiint q_i(x)\, dx. \tag{3.22}$$

In case of $EI \to \infty$ this leads to

$$\vec{r}_i = 0. \tag{3.23}$$

3.3.4. Fixed rotation

The rotations of beams attached to one node can be a given condition, e. g. when there are bearings of the type pinned or clamped. Therefore, the equations reads

$$w_{i|k}' = 0. \tag{3.24}$$

3.3.5. Internal bending moments

For internal moments two different scenarios have to be considered. The first one is a joint. In this case the internal moment of each beam at the given node has to be zero [5],

$$M_{i|k} = 0. \tag{3.25}$$

The equation (3.25) can be reformulated to

$$\hat{M}_i C_i - \iint q_i(x)\, dx = 0, \tag{3.26}$$

where we introduced the row vector $\hat{M}_i$

$$\hat{M}_i = \begin{cases} \left[0, 0, -6EI_i x, -2EI_i, 0, 0\right] & \text{for} \quad EI < \infty \\ \left[0, 0, x, 1, 0, 0\right] & \text{for} \quad EI \to \infty \end{cases}, \tag{3.27}$$

and C_i used as in (3.7).

The known term is than arranged on the right side what leads to

$$\hat{M}_i C_i = -\vec{r}_i, \tag{3.28}$$

where $\vec{r}_i$ is in this case independent from flexural rigidity and can always be formulated as

$$\vec{r}_i = -\iint q_i(x)\, dx. \tag{3.29}$$

If there is no node as well as no bearing of the type clamped or parallel guide the sum of all internal moments at the node has to be zero,

$$\sum_{i \in \beta_k} M_{i|k} = 0, \tag{3.30}$$

where β_k is the index set of the beams attached to the node k [6].

3.3.6. Internal Forces

The sum of all internal forces at one node has to be zero when there is no bearing,

$$\sum_{i \in \beta_k} Q_{i|k} \vec{n}_i + N_{i|k} \vec{t}_i = 0. \tag{3.31}$$

The components of the equation can also be written like

$$Q_i = \hat{Q}_i C_i - \int q_i(x)\, dx, \tag{3.32}$$

where we introduced the row vector $\hat{Q}_i$,

$$\hat{Q}_i = \begin{cases} \left[0, 0, -6EI_i, 0, 0, 0\right] & \text{for} \quad EI < \infty \\ \left[0, 0, 1, 0, 0, 0\right] & \text{for} \quad EI \to \infty \end{cases}, \tag{3.33}$$

and C_i used as in (3.7).

The second component reads

$$N_i = \hat{N}_i C_i - \int n_i(x)\, dx, \tag{3.34}$$

where we introduced the row vector $\hat{N}_i$,

$$\hat{N}_i = \begin{cases} \left[EA_i, 0, 0, 0, 0, 0\right] & \text{for} \quad EA < \infty \\ \left[1, 0, 0, 0, 0, 0\right] & \text{for} \quad EA \to \infty \end{cases}. \tag{3.35}$$

Now we can formulate (3.31) to

$$\sum_{i \in \beta_k} (\hat{Q}_i C_i)\vec{n}_i + (\hat{N}_i C_i)\vec{t}_i = -\vec{r}_i, \tag{3.36}$$

where $\vec{r}_i$ is independent from flexural and axial rigidity and therefore always yields

$$\vec{r}_i = -\int q_i(x)\, dx - \int n_i(x)\, dx. \tag{3.37}$$

In case of a roller bearing, the sum of all internal forces has to be zero in the direction of $\vec{T}$ as seen in Figure 3.3, normal to the direction in which the bearing prevents movement. This leads to

$$\sum_{i \in \beta_k} (Q_{i|k}\vec{n}_i + N_{i|k}\vec{t}_i) \cdot \vec{T} = 0. \tag{3.38}$$

4. APPLICATION IN UNIVERSITY COURSES

This bachelor project is part of an initiative to enhance digital learning in Engineering Mechanics. The aim was to create scalable individual assignments.

It should be possible to:

1. Automatically create individual assignments
2. Automatically create the solution for each assignment
3. Digitally distribute the assignments
4. That students can digitally hand in their answers
5. That these answers are checked automatically
6. Digitally judge the input and give back an evaluation to the student

It was important that lots of assignments can be created, that are similar in complexity and needed work effort, but can not be solved in the same way. Therefore, the assignments have to be automatically scalable and designed with abstract rules.

The points 1 and 2 have been realized with a python-script. The points 3-6 were implemented with Moodle-STACK-questions, a feature of the online learning platform *TeachCenter* used at TU Graz.

There are two main reasons why individual scalable assignments are important for mechanical comprehension:

1) Even though the assignments are similar when it comes to the needed effort, they can not be solved in the exact same way. Therefore students need to find their own solution but it still helps to communicate with other students about how they solved their problem.

2) The students are automatically judged and get an evaluation immediately. Therefore, they have the opportunity to correct their mistakes without the fear of judgement from teaching staff.

The following assignment see Figure 4.1 is an example for what kind of systems can be created. It is one of 370 assignments which were generated for the course *Baumechanik 2* held by the Institute of Applied Mechanics. A solution file is also compiled. Different homework assignments are included in the appendix.

BAUMECHANIK 2 — TU GRAZ

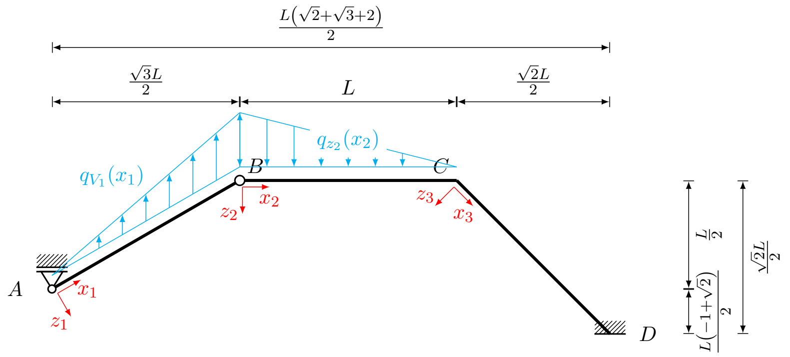
SOMMERSEMESTER 2022 — INSTITUT FÜR BAUMECHANIK

HAUSÜBUNG SYSTEMBERECHNUNG

Berechnen Sie für das dargestellte mechanische System

a) die Biegelinie $w_2(x_2)$, sowie

b) den Verschiebungsverlauf $u_1(x_1)$.



Die Belastungsfunktionen lauten:

$$q_{V_1}(x_1) = \frac{qx_1}{L}$$

$$q_{z_2}(x_2) = \frac{q\,(L - x_2)}{L}$$

Die Steifigkeiten der Einzelstäbe betragen:

$$EI_1 \to \infty$$

$$EA_1 = \frac{EI}{L^2}$$

$$EI_2 = EI$$

$$EA_2 \to \infty$$

$$EI_3 \to \infty$$

$$EA_3 \to \infty$$

Figure 4.1.: Example assignment 1

5. CONCLUSION

In the present thesis a method for the symbolic calculation of arbitrary plane systems has been developed and implemented in python. As a special feature the method can deal with flexible as well as rigid elements. In contrast to the formulation of the boundary and interface conditions in [2], the present formulation avoids bearing forces in the system matrix. Thus, the size of the system of linear equations is reduced and therefore the method is more efficient.

Furthermore, the method has been successfully used to create individual assignments for over 300 students. The assignments were automatically generated based on abstract rules and also automatically checked by using Moodle-STACK-questions.

The system has potential to be further developed. In future work the implementation of springs, single moments and loads, half joints and *Zwangseinbau* should be considered. Then all of the practise assignments used in *Baumechanik 2* can be recreated.

Considering the digitalisation aspect of this project the interface between the python script and the online learning platform has the most potential to be further optimised. For example, the feedback given back to the students can be more specific.

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A. EXAMPLE ASSIGNMENTS

BAUMECHANIK 2 — TU GRAZ

SOMMERSEMESTER 2022 — INSTITUT FÜR BAUMECHANIK

HAUSÜBUNG SYSTEMBERECHNUNG

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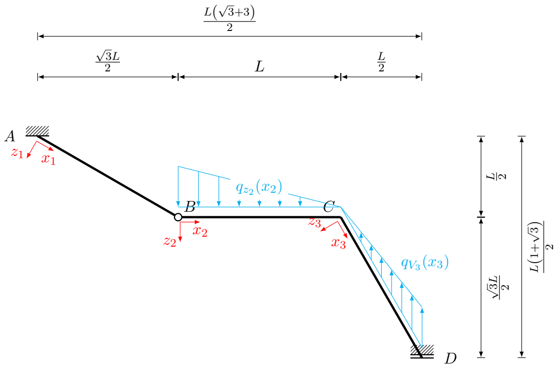


Die Belastungsfunktionen lauten:

$$q_{z_2}(x_2) = \frac{q\,(L - x_2)}{L}$$

$$q_{V_3}(x_3) = \frac{q x_3}{L}$$

Die Steifigkeiten der Einzelstäbe betragen:

$$EI_1 \to \infty$$

$$EA_1 = \frac{EI}{L^2}$$

$$EI_2 = EI$$

$$EA_2 \to \infty$$

$$EI_3 \to \infty$$

$$EA_3 \to \infty$$

Figure A.1.: Example assignment 2

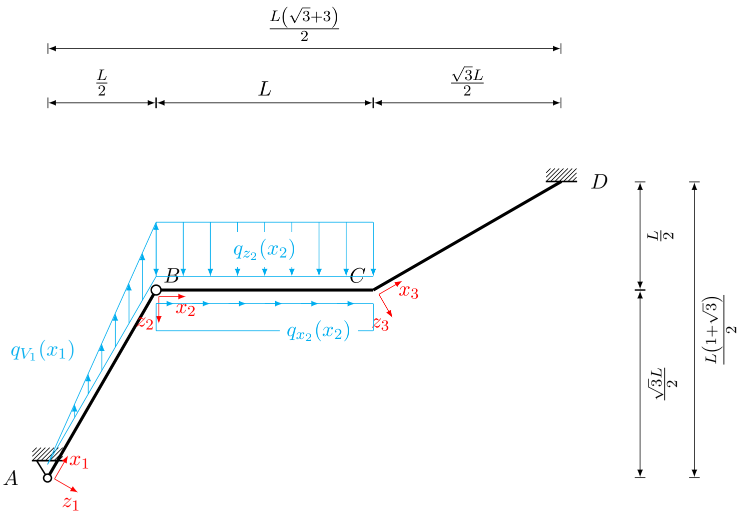
Baumechanik 2 TU Graz
Sommersemester 2022 Institut für Baumechanik

Hausübung Systemberechnung

Berechnen Sie für das dargestellte mechanische System

a) die Biegelinie $w_2(x_2)$, sowie

b) den Verschiebungsverlauf $u_1(x_1)$.



Die Belastungsfunktionen lauten:

$$q_{V_1}(x_1) = \frac{qx_1}{L}$$
$$q_{x_2}(x_2) = q$$
$$q_{z_2}(x_2) = q$$

Die Steifigkeiten der Einzelstäbe betragen:

$$EI_1 \to \infty$$
$$EA_1 = \frac{EI}{L^2}$$
$$EI_2 = EI$$
$$EA_2 \to \infty$$
$$EI_3 \to \infty$$
$$EA_3 \to \infty$$

Figure A.2.: Example assignment 3

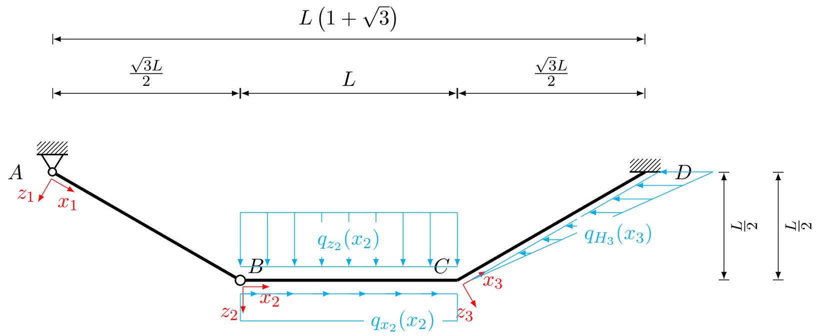
BAUMECHANIK 2 | TU GRAZ
SOMMERSEMESTER 2022 | INSTITUT FÜR BAUMECHANIK

HAUSÜBUNG SYSTEMBERECHNUNG

Berechnen Sie für das dargestellte mechanische System

a) die Biegelinie $w_2(x_2)$, sowie

b) den Verschiebungsverlauf $u_1(x_1)$.



Die Belastungsfunktionen lauten:

$$q_{x_2}(x_2) = q$$
$$q_{z_2}(x_2) = q$$
$$q_{H_3}(x_3) = \frac{qx_3}{L}$$

Die Steifigkeiten der Einzelstäbe betragen:

$$EI_1 \to \infty$$
$$EA_1 = \frac{EI}{L^2}$$
$$EI_2 = EI$$
$$EA_2 \to \infty$$
$$EI_3 \to \infty$$
$$EA_3 \to \infty$$

Figure A.3.: Example assignment 4