

# Multiphasic modelling of thrombus formation and growth

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# Outline

1. Introduction
2. Theory of Porous Media
3. Modelling
4. Numerical Example

# Introduction

## Type B Aortic Dissection (TBAD)

- Causes: High blood pressure, weakening of aortic wall, pre-existing defects in the aortic valve.

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- Acute Cases: 0.5 to 6.3 per 100,000 people annually.

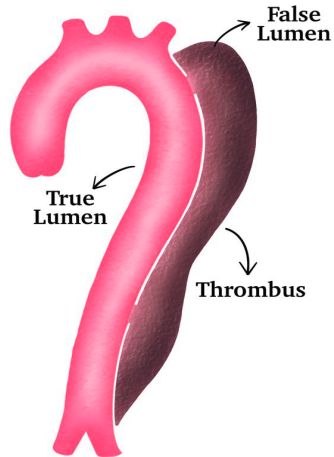
# Introduction

## Type B Aortic Dissection (TBAD)

- Causes: High blood pressure, weakening of aortic wall, pre-existing defects in the aortic valve.
- Acute Cases: 0.5 to 6.3 per 100,000 people annually.
- High Mortality Rate: Greater than 50% for patients with acute TBAD.

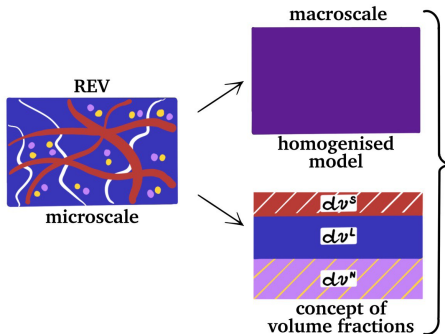
# Introduction

## Type B Aortic Dissection (TBAD)



# Theory of Porous Media

## Representative Element Volume (REV) of Microstructure



$$\begin{aligned}
 n &= \frac{dv}{dv} \\
 &= \frac{dm}{dv} \\
 R &= \frac{dm}{dv} \\
 &= n^R
 \end{aligned}$$

where :

$n$  : volume fraction

$m$  : partial density

$R$  : realistic density

# Theory of Porous Media

## Field Equations

$$\text{mass balance : } (\rho)^{\circ} + \text{div} \mathbf{x}^{\circ} = \dot{\rho}$$

$$\text{momentum balance : } \mathbf{x}^{\circ\circ} = \text{div} \boldsymbol{\sigma} + \mathbf{b} + (\dot{\rho} \mathbf{x}^{\circ})$$

$$\text{moment of momentum balance : } \mathbf{m} = (\boldsymbol{\sigma})^T$$

$$\rho^S + \rho^L + \rho^N = 0$$

$$\dot{\rho}^S + \dot{\rho}^L + \dot{\rho}^N = 0$$



# Assumptions

Materially Incompressible :

$$R = \text{constant}$$

Quasi static condition :

$$\mathbf{x}^{00} = \mathbf{0}$$

Isothermal condition :

$$= \text{constant}$$

Fully saturated condition :

$$n^S + n^L + n^N = 1$$

Velocity :

$$\mathbf{x}_L^0 = \mathbf{x}_N^0 = \mathbf{x}_F^0$$

Pressure :

$$p^{LR} = p^{NR} = p$$

Isotropic Neo-Hookean Stress :

$$\frac{n^S}{n_{0S}^S} \mathbf{B} + \log J_S \mathbf{I}$$

$$\Lambda^L = 0 \quad \Lambda^S = \Lambda^N$$

T. Ricken, A. Schwarz, and J. Bluhm. A Triphasic Theory for Growth in Biological Tissue-Basics and Applications. *Materialwissenschaft und Werkstofftechnik*, volume 37, issue 6, pages 446-456.

# Mass Supply

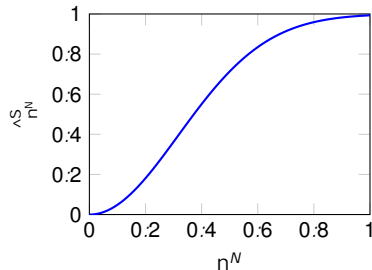
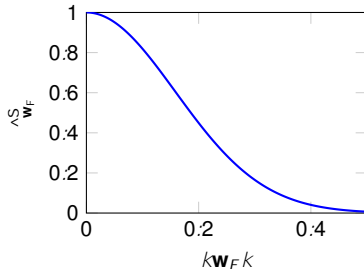
- The mass exchange is between solid and nutrient phase.

$$\hat{\Lambda}^S = \hat{\Lambda}^S(\mathbf{w}_F; n^N)$$

$$\hat{\Lambda}_{\mathbf{w}_F}^S = \exp[-k \mathbf{w}_F k^2 - 1]$$

$$\hat{\Lambda}^S = C \hat{\Lambda}_{\mathbf{w}_F}^S \hat{\Lambda}_{n^N}^S$$

$$\hat{\Lambda}_{n^N}^S = \exp[-(n^N)^2 - 2] + 1$$



8  
 Darcy's Law

- Describes the flow of fluid through a porous medium.

$$\mathbf{n}^F \mathbf{w}_F = \frac{n^F}{n_{0S}^F} \frac{K^S}{\mu^F} (\text{grad } p - \mathbf{g})$$

where :

$\mathbf{w}_F$  : seepage velocity

$K^S$  : intrinsic permeability

$\mu^F$  : dynamic fluid viscosity

# Weak Formulation I

Unknowns :  $\bar{f}_U^S; n^S; n^N; pg$

# Weak Formulation I

Unknowns :  $f u_S; n^S; n^N; p; g$

Volume Balance of Mixture :

$$\int_Z \text{div } \mathbf{x}_S^\theta p \, dv + \int_Z n^F \mathbf{w}_F \text{grad } p \, dv + \int_Z q p \, da = 0$$

$$+ \int_Z \wedge^S \left( \frac{1}{NR} - \frac{1}{SR} \right) p \, dv$$

Momentum Balance of Mixture :

$$\int_Z \left( \frac{S}{E} p \mathbf{I} \right) : \text{grad } \mathbf{u}_S \, dv + \int_Z \left( \frac{S}{E} + \frac{F}{E} \right) \mathbf{g} \cdot \mathbf{u}_S \, dv$$

$$+ \int_Z \wedge^S \mathbf{w}_F \cdot \mathbf{u}_S \, dv + \int_Z \mathbf{t} \cdot \mathbf{u}_S \, da = 0$$

where:

$$q = n^F \mathbf{w}_F \cdot \mathbf{n}$$

$$\mathbf{t} = \left( \frac{S}{E} n_S p \mathbf{I} \right) \cdot \mathbf{n}$$

# Weak Formulation II

Volume Balance of Solid :

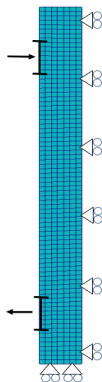
$$\int_Z (n^S)^0_S n^S dv + \int_Z n^S \operatorname{div} \mathbf{x}_S^0 n^S dv - \int_Z \frac{\wedge S}{SR} n^S dv = 0$$

Volume Balance of Nutrients :

$$\int_Z (n^N)^0_S + n^N \operatorname{div} \mathbf{x}_S^0 - \frac{\wedge N}{NR} n^N dv - \int_Z n^N \mathbf{w}_F : \operatorname{grad} n^N dv + \int_Z n^N \mathbf{w}_F : \mathbf{n} da = 0$$

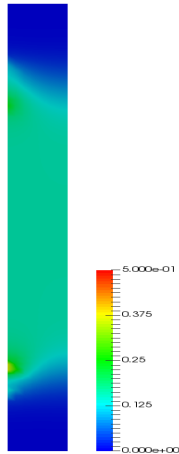
# Numerical Example

- Non-Linear FE formulation
- Implemented in PANDAS



# Numerical Example

## Norm of Seepage Velocity





# Numerical Example

## Change in Solid Volume Fraction (Thrombosis)

$t = 0$  hours

# Numerical Example

## Change in Solid Volume Fraction (Thrombosis)

$t = 0$  hours

$t = 84$  hours

# Numerical Example

## Change in Solid Volume Fraction (Thrombosis)

$t = 0$  hours

$t = 84$  hours

$t = 168$  hours

# Conclusion

- A tri-phasic model has been developed for growth of the thrombus.
- Growth is patient specific. It can be adapted to different scenarios using the constants in growth term.

$$\begin{aligned} \hat{w}_F^S &= \exp[ k w_F k^2 - 1 ] \\ \hat{n}^S &= \exp[ (n^N)^2 - 2 ] + 1 \\ \hat{S} &= C \hat{w}_F^S \hat{n}^S \end{aligned}$$

- Open Question : Realistic Material and Medical Data?

## Acknowledgement

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**Mechanics, Modeling and Simulation of Aortic Dissection**

*[www.biomechaorta.tugraz.at](http://www.biomechaorta.tugraz.at)*



A joint project of the

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