TU Graz

Multiphasic modelling of thrombus formation and growth

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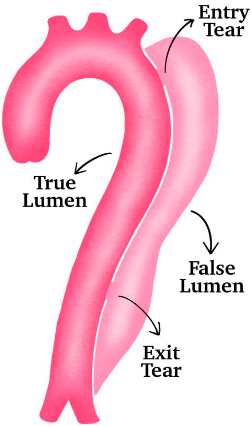
AORTIC DISSECTION
MECHANICS - MODELING - SIMULATION

Outline

Introduction

Type B Aortic Dissection (TBAD)



- Causes: High blood pressure, weakening of aortic wall, pre-existing defects in the aortic valve.

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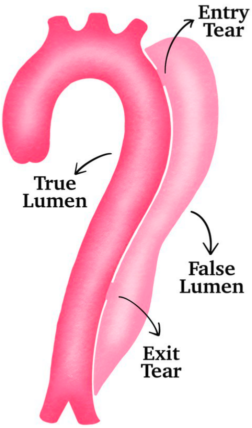

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- Acute Cases: 0.5 to 6.3 per 100,000 people annually.

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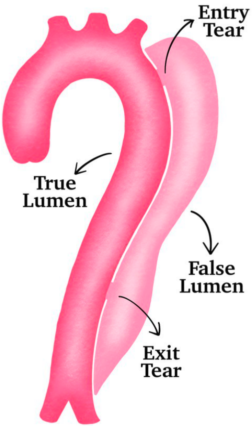

- Causes: High blood pressure, weakening of aortic wall, pre-existing defects in the aortic valve.
- Acute Cases: 0.5 to 6.3 per 100,000 people annually.
- High Mortality Rate: Greater than 50% for patients with acute TBAD.

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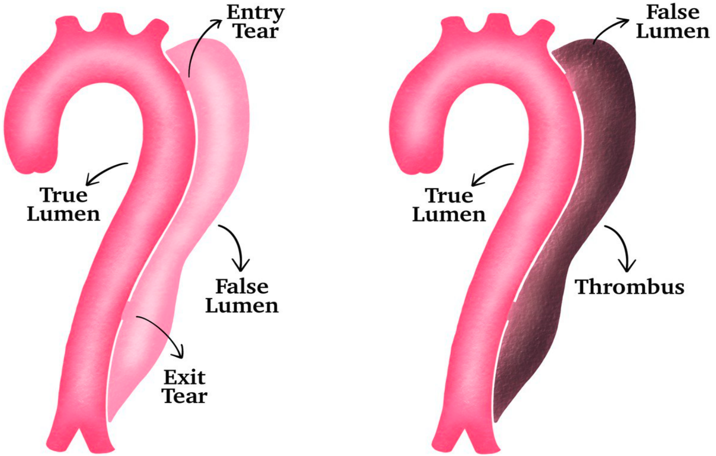
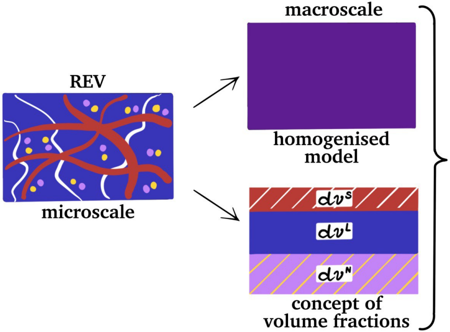


Theory of Porous Media

Representative Element Volume (REV) of Microstructure



$$n^\alpha = \frac{dv^\alpha}{dv}$$

$$\rho^\alpha = \frac{dm^\alpha}{dv}$$

$$\rho^{\alpha R} = \frac{dm^\alpha}{dv^\alpha}$$

$$\rho^\alpha = n^\alpha \rho^{\alpha R}$$

where :

n^α : volume fraction

ρ^α : partial density

$\rho^{\alpha R}$: realistic density

Theory of Porous Media

Field Equations

mass balance : $(\rho^\alpha)'_\alpha + \rho^\alpha \operatorname{div} \mathbf{x}'_\alpha = \hat{\rho}^\alpha$

momentum balance : $\rho^\alpha \mathbf{x}''_\alpha = \operatorname{div} \boldsymbol{\sigma}^\alpha + \rho^\alpha \mathbf{b}^\alpha + (\hat{\mathbf{p}}^\alpha - \hat{\rho}^\alpha \mathbf{x}'_\alpha)$

moment of momentum balance : $\boldsymbol{\sigma}^\alpha = (\boldsymbol{\sigma}^\alpha)^T$

$$\boxed{\hat{\rho}^S + \hat{\rho}^L + \hat{\rho}^N = 0}$$

$$\boxed{\hat{\mathbf{p}}^S + \hat{\mathbf{p}}^L + \hat{\mathbf{p}}^N = 0}$$

Assumptions

Materially Incompressible : $\rho^{\alpha R} = \text{constant}$

Quasi static condition : $\mathbf{x}_\alpha^{\prime\prime} \equiv 0$

Isothermal condition : $\theta^\alpha \equiv \theta = \mathit{constant}$

Fully saturated condition : $\mathrm{n}^S + \mathrm{n}^L + \mathrm{n}^N = 1$

Velocity : $\mathbf{x}_L^\prime = \mathbf{x}_N^\prime = \mathbf{x}_F^\prime$

Pressure : $p^{LR} = p^{NR} = p$

Isotropic Neo-Hookean Stress : $\left(\frac{\mathrm{n}^S}{\mathrm{n}_{0S}^S}\right)^{n+1} \left[\mu \mathbf{B} + \left(\lambda^S \mathit{log} J_S - \mu\right) \mathbf{I}\right]$

$$\boxed{\hat{\rho}^L = 0 \longrightarrow \hat{\rho}^S = -\hat{\rho}^N}$$

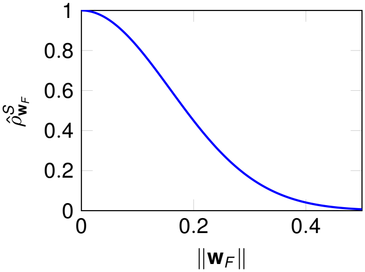
T. Ricken, A. Schwarz, and J. Bluhm. A Triphasic Theory for Growth in Biological Tissue-Basics and Applications. *Materialwissenschaft und Werkstofftechnik, volume 37, issue 6*, pages 446-456.

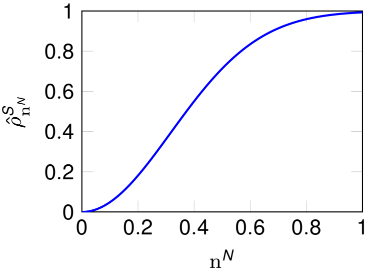
Mass Supply

- The mass exchange is between solid and nutrient phase.

$$\hat{\rho}^S = \hat{\rho}^S(\mathbf{w}_F, \mathrm{n}^N)$$
$$\hat{\rho}^S_{\mathbf{w}_F} = exp[-\|\mathbf{w}_F\|^2/\beta_1]$$

$$\hat{\rho}^S = \mathrm{C}\, \hat{\rho}^S_{\mathbf{w}_F}\, \hat{\rho}^S_{\mathrm{n}^N}$$
$$\hat{\rho}^S_{\mathrm{n}^N} = -exp[-(\mathrm{n}^N)^2\, \beta_2] + 1$$





Darcy’s Law

- Describes the flow of fluid through a porous medium.

$$\mathrm{n}^F \mathbf{w}_F = -\left(\frac{\mathrm{n}^F}{\mathrm{n}^F_{0S}}\right)^m \frac{K^S}{\mu^{FR}} (grad\, p - \rho^{FR} \mathbf{g})$$

where :

$\mathbf{w}_F$: seepage velocity

K^S : intrinsic permeability

μ^{FR} : dynamic fluid viscosity

Weak Formulation I

Unknowns : $\{\mathrm{u}_S, \mathrm{n}^S, \mathrm{n}^N, p\}$

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Volume Balance of Mixture :

$$\int_\Omega div\, \mathbf{x}_S' \,\delta p \,dv - \int_\Omega \mathrm{n}^F \mathbf{w}_F \cdot grad\, \delta p\, dv + \int_\Omega \hat{\rho}^S \left(\frac{1}{\rho^{NR}} - \frac{1}{\rho^{SR}}\right) \delta p\, dv + \int_{\Gamma_q} \bar{q}\, \delta p\, da = 0$$

Momentum Balance of Mixture :

$$\int_\Omega (\boldsymbol{\sigma}_E^S - p\,\mathbf{I}) : grad\, \delta \mathbf{u}_S\, dv - \int_\Omega (\rho^S + \rho^F)\,\mathbf{g} \cdot \delta \mathbf{u}_S\, dv - \int_\Omega \hat{\rho}^S \mathbf{w}_F \cdot \delta \mathbf{u}_S\, dv - \int_{\Gamma_t} \bar{\mathbf{t}} \cdot \delta \mathbf{u}_S\, da = 0$$

where:

$$\bar{q} = \mathrm{n}^F \mathbf{w}_F \cdot \mathbf{n}$$

$$\bar{\mathbf{t}} = (\boldsymbol{\sigma}_E^S - n_S p\,\mathbf{I}) \cdot \mathbf{n}$$

Weak Formulation II

Volume Balance of Solid :

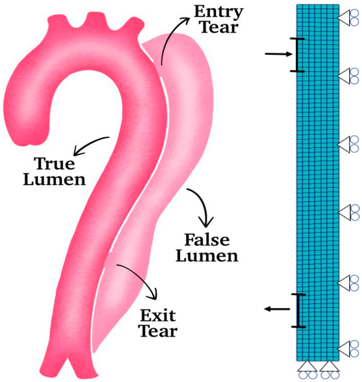
$$\int_\Omega (\mathrm{n}^S)'_S \, \delta \mathrm{n}^S \, dv + \int_\Omega \mathrm{n}^S \, div \, \mathbf{x}'_S \, \delta \mathrm{n}^S \, dv$$
$$- \int_\Omega \frac{\hat{\rho}^S}{\rho^{SR}} \, \delta \mathrm{n}^S \, dv = 0$$

Volume Balance of Nutrients :

$$\int_\Omega \left((\mathrm{n}^N)'_S + \mathrm{n}^N \, div \, \mathbf{x}'_S - \frac{\hat{\rho}^N}{\rho^{NR}} \right) \delta \mathrm{n}^N \, dv$$
$$- \int_\Omega \mathrm{n}^N \mathbf{w}_F \cdot grad \, \delta \mathrm{n}^N \, dv + \int_{\Gamma_v} \mathrm{n}^N \mathbf{w}_F \, . \, \mathbf{n} \, da = 0$$

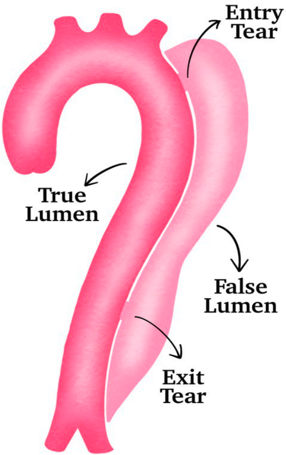
Numerical Example

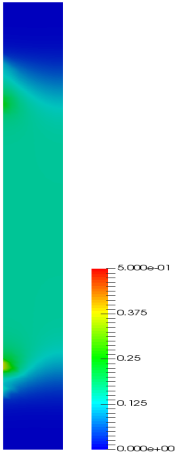
- Non-Linear FE formulation
- Implemented in PANDAS



Numerical Example

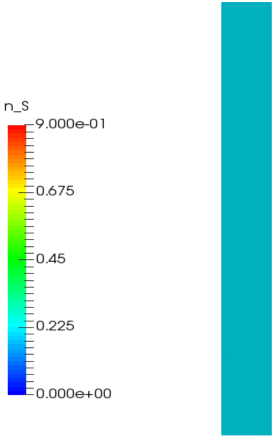
Norm of Seepage Velocity





Numerical Example

Change in Solid Volume Fraction (Thrombosis)



t = 0 hours

Numerical Example

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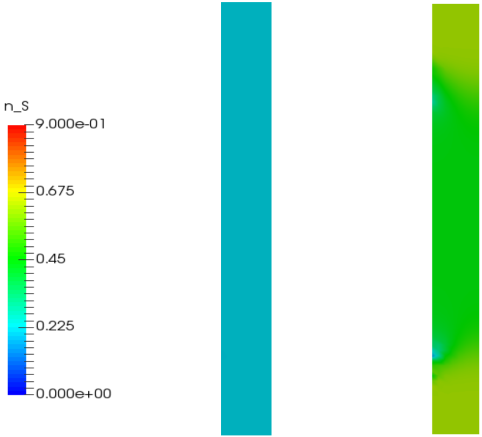
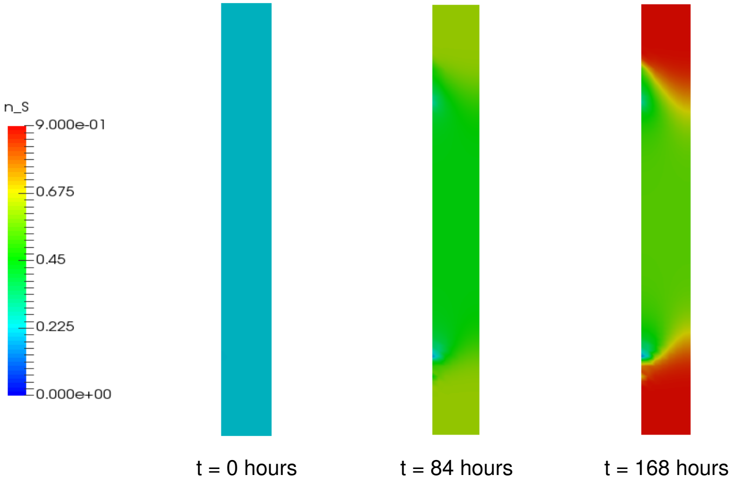


t = 0 hours

t = 84 hours

Numerical Example

Change in Solid Volume Fraction (Thrombosis)



Conclusion

- A tri-phasic model has been developed for growth of the thrombus.
- Growth is patient specific. It can be adapted to different scenarios using the constants in growth term.

$$
\begin{aligned}
\hat{\rho}^S_{\mathbf{w}_F} &= exp[-\|\mathbf{w}_F\|^2/\beta_1] \\
\hat{\rho}^S_{\mathrm{n}^N} &= -exp[-(\mathrm{n}^N)^2\ \beta_2] + 1 \\
\hat{\rho}^S &= \mathrm{C}\ \hat{\rho}^S_{\mathbf{w}_F}\ \hat{\rho}^S_{\mathrm{n}^N}
\end{aligned}
$$

- Open Question : Realistic Material and Medical Data?

Acknowledgement

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Mechanics, Modeling and Simulation of Aortic Dissection

www.biomechaorta.tugraz.at

A joint project of the