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Advanced Computation Method for Value-Based Distribution Systems Reliability Evaluation

A. S. Nazmul HUDA[1], Rastko ZIVANOVIC[2]

[1]The University of Adelaide, School of Electrical & Electronic Engineering, SA 5005, Australia, asnazmul.huda@adelaide.edu.au (Corresponding Author).

[2]University of Applied Sciences Upper Austria, 4600 Wels, Austria, Tel.: +43 5 0804 43475, rastko.zivanovic@fh-wels.at.

Abstract: Value-based distribution system reliability evaluation determines the worth of reliability in terms of customer expected cost of interruption (ECOST). Accelerating the accurate estimation of ECOST using advanced computation methods could be an important feature in the distribution systems management tool. In fact, currently practice simulation technique based on Monte Carlo (MC) method can provide accurate result. However, one basic challenge related to the MC method is the high computational burden while a large number of approximations is required to achieve a high accuracy. In this paper, a new approach for the estimation of this reliability index is presented. This approach is based on the novel Multilevel Monte Carlo method that can reduce huge computational cost needed for estimating the index accurately. To illustrate the method, we study reliability evaluation of distribution systems of Roy Billinton Test System. The impact on interruption costs assessment associated with the network reinforcement, load types, time-varying load and cost models are investigated. We conclude that significant savings in computational cost are possible for these studies using the proposed method compared to the standard MC method.

Keywords: Interruption cost, Monte Carlo method, Multilevel Monte Carlo method, Speedup, Distribution systems.

1 Introduction

It has been reported that approximately 80% of all customer interruptions occur due to failures in the distribution systems [1]. The ultimate goal of distribution systems planning is to decrease the impact of the failures for providing reliable and high-quality electricity supply to customers at a reasonable rate. System customer expected interruption cost (ECOST) evaluation shows the monetary worth of the interruptions which can assist in the decision of reliability planning. However, a prominent problem in distribution systems planning studies is the accurate and fast estimation of the ECOST.

Estimation of ECOST using average load and cost models can provide a result with a large error. ECOST varies unevenly with the random nature of the failures and associated time-varying load and cost models. Knowledge of the probability distributions associated with the index can be used to more accurately estimate the ECOST rather than using only average value. Conventionally used Monte Carlo (MC) method [2] can estimate ECOST by taking account of its random nature. However, the limitation of the MC method is that large computational burden arises for obtaining a high accuracy level. Therefore, the objective of this research is to speed up the computation of ECOST estimation. A novel Multilevel Monte

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Carlo (MLMC) method [3] has been applied for this purpose. We verify the performance of the proposed method by comparing to MC method in terms of computational efficiency and accuracy. Using the proposed method, the following studies are carried out and presented in the paper.

- Developing a general simulation framework based on MLMC method for ECOST evaluation.
- Calculating the impacts of time-varying load and cost models on the estimation.
- Analyzing the effects of network reinforcement, complexity and load types on the estimation process.

2 Value-based reliability evaluation

2.1 Generation of operating history

For estimating the operating history of any component, the stochastic model of component Time-to-Failure (TTF) is first developed. Consider λ_i and r_i are the failure rate and repair time of component i, respectively. Also, consider the stochastic differential equation (SDE) of TTF is driven by the Brownian motion [4]. If $S_{\lambda i(t)}$ is the TTF of an event i at a time t, then SDE of TTF with defined drift μ, volatility σ and initial TTF can be modeled using the Brownian motion W on the whole time interval $[0, T]$ [5] as follows:

$$dS_{\lambda i(t)} = \mu\left[S_{\lambda i(t)}, t\right]dt + \sigma\left[S_{\lambda i(t)}, t\right]dW, \tag{1}$$

In this paper, the SDE is solved by Milstein discretisation scheme [6]. The discretisation scheme with n time-steps, step size $h = T/n$ and Brownian increments ΔW_m could be written as:

$$S_{\lambda i(m+1)} = S_{\lambda i(m)} + \mu\left[S_{\lambda i(m)}, t_m\right]h + \sigma\left[S_{\lambda i(m)}, t_m\right]\Delta W_m + \frac{1}{2}\sigma^2\left[S_{\lambda i(m)}, t_m\right](\Delta W_m^2 - h), \tag{2}$$

where ΔW_m are the independent normally distributed random variables. $\Delta W_m = W_{m+1} - W_m$ $[m = 0, \dots\dots, n-1]$ and $t_m = kh$ $[k = 0, \dots\dots, n]$. Using the above Eqn. (2), the operating history of component i, T_{ui} is generated as:

$$T_{ui} = -S_{\lambda i(m+1)}\, ln(U) \tag{3}$$

where U is a uniformly distributed random variable between [0,1].

2.2 Modeling of load

Usually load level of a specific customer type fluctuates due to the discrepancy between the hourly consumption levels. In addition, seasonal inconsistency in the weather contributes prominently to loading level diversity [7]. Evaluation of interruption cost based on average load level without considering time-varying diversity does not reflect the time-varying nature of system ECOST. Thus accurate approximation of ECOST needs the consideration of modeling of loads throughout a 24-hour period depending on seasons.

For modeling time-varying load $L_p(t)$ of a load point P at an hour t, annual peak load (L_{peak}), weekly peak load as a percentage of annual peak (W_p), daily peak load as a percentage of

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weekly peak (D_p) and hourly peak load as a percentage of daily peak (H_p) are formulated as [8]:

$$L_p(t) = L_{peak} \times W_p \times D_p \times H_p(t), \text{ MW} \quad (4)$$

Consider, the interruption of load point P starts and ends at ts and te hours, respectively. Then the average time-varying load level of this load point is evaluated as follows:

$$L_p = L_{peak} \times W_p \times D_p \times \frac{\sum_{t=ts}^{te} H_P(t)}{te-ts+1}, \text{ MW} \quad (5)$$

2.3 Modeling of per unit interruption cost

For modeling customer per unit interruption cost based on average cost model (C_{avg}), only sector customer damage function (SCDF) [9] is analyzed to found the cost (C_p) related to a load point P interruption for a duration r_p, i.e. $C_p = C_{avg}(r_p)$.

On the other hand, for modeling interruption cost $C_p(t)$ at an hour t based on time-varying cost model, the multiplication of C_{avg} from SCDF and weight factor $W_p(t)$ is used, i.e. $C_p(t) = C_{avg} \times W_p(t)$. Then average time-varying cost level of a load point P for above failure period can be formulated as:

$$C_p = C_{avg} \times \frac{\sum_{t=ts}^{te} W_p}{te-ts+1}, \text{ (\$/kW)} \quad (6)$$

2.4 Modeling of system ECOST

For a component failure i, the value of average outage rate B_i could be calculated using the following expression:

$$B_i = \frac{M}{\sum_{n=1}^{N} T_{ui}}, \text{ (occ./yr)} \quad (7)$$

where M is the number of times component i fails during whole simulation period and N is the desired number of simulated periods.

For load point P, average outage rate F_p is evaluated as follows by accumulating the outage rate of all the failure events connected to this load point.

$$F_p = \sum_{i=1}^{n_i} B_i, \text{ (occ./yr)} \quad (8)$$

where n_i denotes the number of outage events interrupting the service of the load point P. Using Eqns. (5), (6) and (8), overall distribution system ECOST can be evaluated as follows.

$$ECOST = \sum_{p=1}^{n_p} F_p L_p \, C_p. \text{ (k\$/yr)} \quad (9)$$

where n_p is the total number of supply points in the system.

3 Multilevel Monte Carlo Approach

In the MC method, the estimation of ECOST is carried out on a single fine grid level $l = L$ [10]. l is a nonnegative integer. Therefore, the relation between level and step size of Eqn. 2 can be written as $h = 2^{-L}T$. Since all the samples are run on the finest level, both the output accuracy and computation time of the ECOST is high.

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On the other hand, the basic difference between MC and MLMC methods is that same output value is estimated in the MLMC method using multiple levels as $l = 0, 1, \dots\dots, L$. The whole simulation is carried out starting from the coarsest level $l = 0$ to the finest level $l = L$. On each level, expectations are calculated using different time-step size and number of samples in such a way which could reduce the overall variance for a target accuracy level. Initially, most of the samples are run on the coarsest level $l = 0$. On the next level $l = 1$, a correction value is added that initiates to decrease the bias. Based on this correction value, the expected difference from one level to the next fine level is added until the finest grid accuracy is achieved. By this way, the less accurate estimate on the coarsest level is sequentially corrected by the estimations on the subsequent fine levels. Due to the smaller value of the difference, very few samples will be needed on finer levels to precisely estimate the expected value. Let X denotes the system ECOST. X_A is the approximation of $E[X]$ and X_l is the approximation on level l. If $E[X_L]$ is the expected value on the finest level i.e. $E[X_L] = E[X_A]$, then the MLMC expectation can then be expressed as:

$$E[X_A] = E[X_0] + \sum_{l=1}^{L} E[X_l - X_{l-1}] = \sum_{l=0}^{L} E[Z_l], \tag{10}$$

The estimator for this expectation can be written as:

$$\hat{Z}_{ML} = \hat{Z}_0 + \sum_{l=1}^{L} \hat{Z}_l = \sum_{l=0}^{L} \hat{Z}_l. \tag{11}$$

where $\hat{Z}_0$ is an unbiased estimator for $E[X_0]$ using samples N_0 and $\hat{Z}_l$ is the estimator for $E[X_l - X_{l-1}]$ using N_l samples for $l \geq 1$.

$$\hat{Z}_0 = \frac{1}{N_0} \sum_{i=1}^{N_0} X_0^{(i)}, \tag{12}$$

$$\hat{Z}_l = \frac{1}{N_l} \sum_{i=1}^{N_l} \left(X_l^{(i)} - X_{l-1}^{(i)} \right), \tag{13}$$

$E[X_l - X_{l-1}]$ is approximated as $E[X_l^f - X_{l-1}^c]$. $E[X_l^f]$ and $E[X_{l-1}^c]$ are estimated on the fine and coarse levels using different time-step sizes h_f and h_c, respectively [6]. where

$$h_f = 2^{-l} T, \tag{14}$$

$$h_c = 2^{-(l-1)} T. \tag{15}$$

The statistical properties of X_l are unchanged whether the estimation of $E[X_l - X_{l-1}]$ from level $(l-1)$ to l or $E[X_{l+1} - X_l]$ from l to $(l+1)$. This means that both $E[X_l - X_{l-1}]$ and $E[X_{l+1} - X_l]$ have the same expected value, i.e., $E\left[X_l^f\right] = E[X_l^c]$. The convergence criteria of the MLMC method is the target rms error which could be written as follows:

$$\varepsilon^2 = \sum_{l=0}^{L} N_l^{-1} V_l + [E(X_A - X)]^2. \tag{16}$$

To obtain an overall mean square error $MSE \leq \varepsilon^2$, both the variance and weak error of MLMC estimator could be reduced below $\varepsilon^2/2$. The variance could be reduced by setting optimal number of samples N_l on each level as follows [3].

$$N_l = 2\varepsilon^{-2} \sqrt{V_l / C_l} \left(\sum_{l=0}^{L} \sqrt{V_l C_l} \right). \tag{17}$$

where the cost to compute one sample on the level l is $C_l = c_3 2^{\gamma l}$ for a constant c_3 and some $\gamma > 0$. γ is the rate of computation cost increase with the level l.

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For weak convergence, the test tries to confirm that $[E(X_A - X)]^2 \leq \varepsilon^2/2$. Consider, the convergence rate of $E(X_A - X)$ with l for constant c_1 is measured by a positive value α, i.e., $|E[X_l - X_{l-1}]| \leq c_1 2^{-\alpha l}$ [3].

The remaining error is

$$E(X_A - X) = E[X_L - X_{L-1}]/(2^\alpha - 1) \quad (18)$$

and the target convergence criterion is therefore

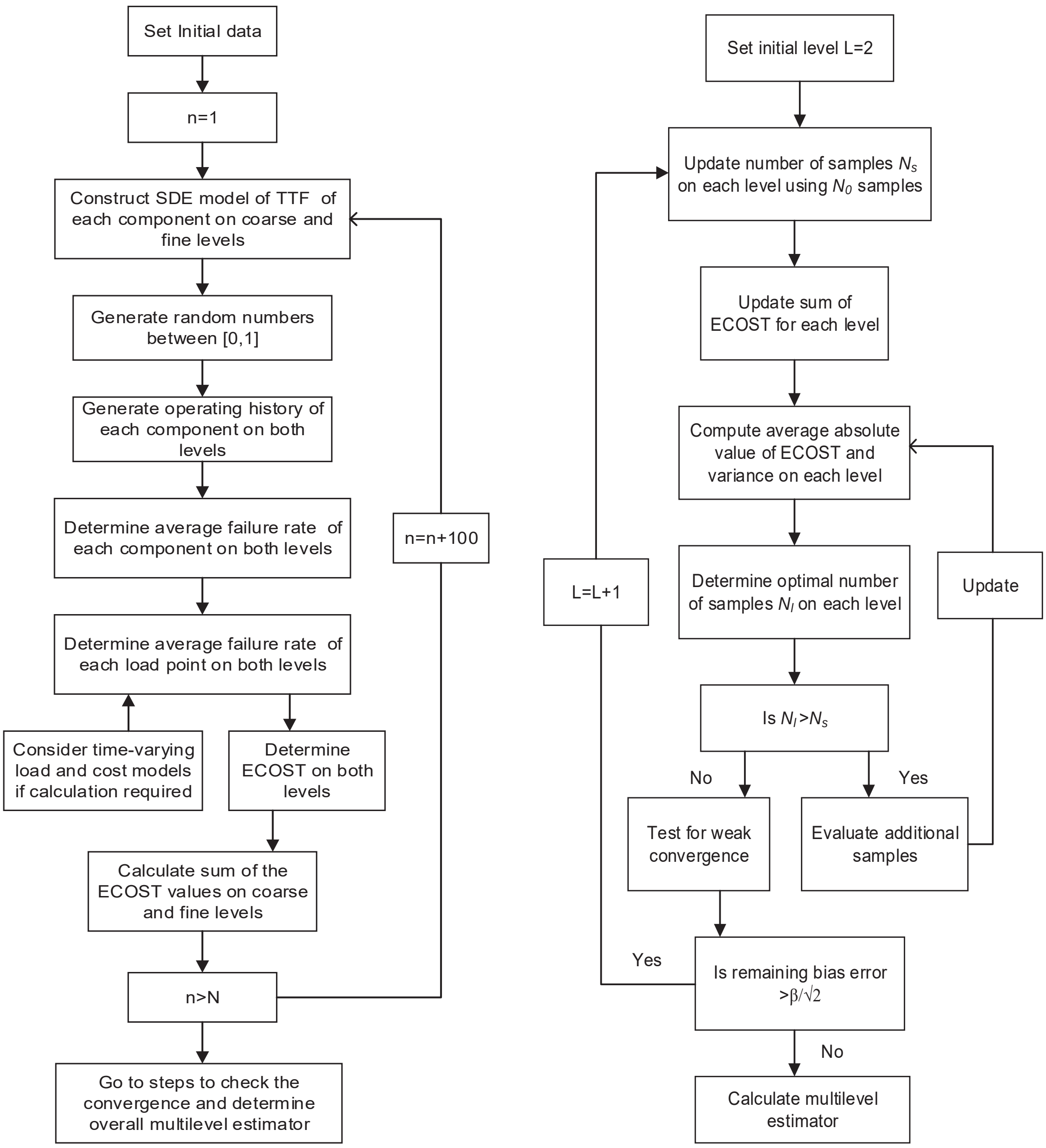
$$E[X_L - X_{L-1}]/(2^\alpha - 1) < \varepsilon/\sqrt{2}. \quad (19)$$

β is assumed as the convergence rate of variance with l for a constant c_2 i.e., $V_l \leq c_2 2^{-\beta l}$.

4 Methodology

In methodology, there are two phases. In the 1st phase, the stochastic model of ECOST is established on both coarse and fine levels. Initially, failure rate, repair/switching time of each component of the distribution system are defined. Additionally, the values of sample size for convergence test, initial sample size on each level, drift, volatility and target accuracy level are defined. The model of a component is represented by up-down states. The operating history of each component is generated according to the exponential probability distribution using Eqn. (3). After this using Eqn. (7), the value of each component average failure rate is calculated. The value of each load point average failure rate is calculated by accumulating the individual component value connected to the relevant load point by following Eqn. (8). System ECOST is then computed using Eqn. (9). Using Eqn. (5), time-varying load model of each load point during failure period is established based on peak load, hourly, daily and weekly load diversity factors. Similarly, time-varying cost model of each load point is established based on SCDF and cost weight factors by following Eqn. (6). A flowchart of the ECOST estimation on coarse and fine levels is shown in Figure 1(a).

In the 2nd phase, overall MLMC estimator is calculated through satisfying the target convergence criteria of the simulation. Initially, the finest grid level of simulation is set at $L = 2$. The number of samples N_s on each level is then determined using initial sample size. The sum of ECOST values on coarse and fine levels is simultaneously updated. Then, absolute average value of the index and variance are calculated on each level. The optimal sample size N_l on each level is determined based on Eqn. (17). Next, the optimal sample size on each level is compared to the already computed N_s on this level. If the N_l is larger than N_s, then the additional samples on each level are evaluated and the values of mean, variance on each level are also updated. The purpose of determining the optimal N_l is to make the variance term of Eqn. (16) smaller than $\varepsilon^2/2$. The test for the weak convergence is performed using Eqn. (19) which ensures the remaining bias error < $\varepsilon/\sqrt{2}$. If the bias error is remained greater than $\varepsilon/\sqrt{2}$, then the finest level is reset as $L = L + 1$. The entire process is repeated again until the target accuracy level is achieved. Finally, the combined multilevel estimator for ECOST is computed using Eqn. (11). A flowchart for convergence test is presented in Figure 1(b).



(a) (b)

Figure 1. Flowchart (a) ECOST estimation on coarse and fine levels (b) Convergence test

5 Case studies and simulation results

5.1 Test system

A typical urban distribution system connected to Bus 2 (B2) of the Roy Billinton test system (RBTS) [11] is proposed to conduct the case studies. Figure 2 presents the RBTS B2 system which consists of four main feeder sections (FI, F2, F3 and F4) and 22 load points. Detailed information of feeders, customers and loads are found in [11]. For both MC and MLMC methods, a target accuracy level of 3% is specified. In all studies, the computations are performed in a MATLAB platform using an Intel Core i7-4790 3.60 GHz processor.

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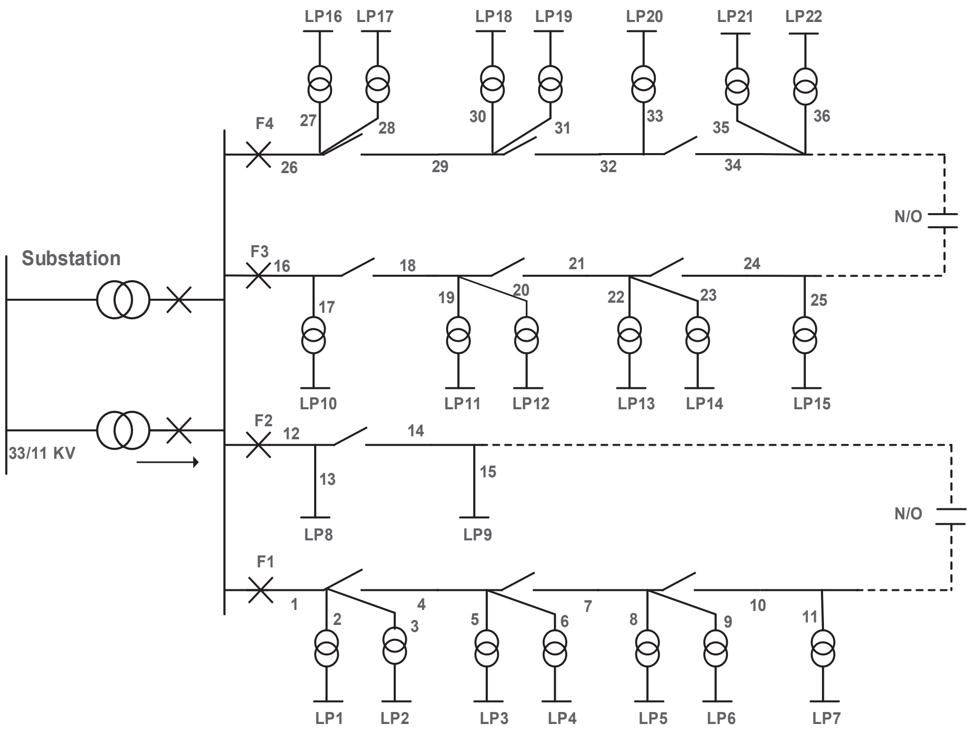

Figure 2. Distribution system for RBTS Bus 2

5.2 Test results

The variations of ECOST values and their estimation time demonstrate the effect of network reinforcement on cost value and estimation time, as shown in Figure 3 and Table 2, respectively. Six case studies are carried out as Table 1, where availability of protective devices and switches are considered in various combinations. Figure 1 shows that the maximum and minimum ECOST values are found in case B and E, respectively. In case E, the availability of switches, fuses, alternative supply is considered with the restoration of low-voltage transformer action by replacement. On the other hand, in case B, all these protective equipment are unavailable with transformer action restoration by time-consuming repairing. In fact, the more investment in the protective equipment reduces the interruption effect and as a result the value of ECOST is also reduced.

Table 2 shows the computational performance of the MC and MLMC methods for all case studies. By comparing with the MC based estimation, the proposed method can estimate ECOST with an acceptable accuracy and the proposed method is considerably more efficient than standard MC. The maximum and minimum simulation times are required for case B and E, respectively. In these cases, the percentage of time-saving using proposed method are 95.66% and 96.52%, respectively. The high value of output increases the computation time for both methods which indicates the reinforcement effect in computation time.The MLMC method improves the calculation efficiency of the MC simulation by reducing the number of iterations on the finest level. For example, the proposed method requires 2262 iterations on the finest level and the MC method needs about 15000 iterations for convergence in case A-ECOST estimation. Due to a large number of samples, MC method provides ECOST with noticeably high accuracy compared to the MLMC method.

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Table 1. Case studies for network reinforcement effect analysis

Case	Disconnecting Switches	Fuses	Alternative Supply	Transformer Action Restoration
A	Yes	Yes	Yes	Repairing
B	No	No	No	Repairing
C	No	Yes	No	Repairing
D	Yes	No	Yes	Repairing
E	Yes	Yes	Yes	Replacement
F	Yes	No	No	Repairing

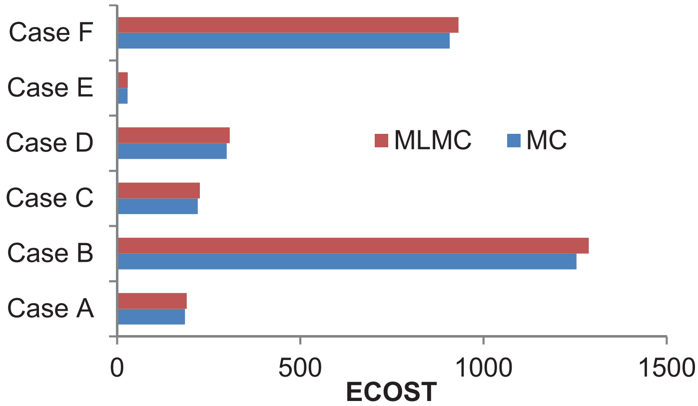

Figure 3. ECOST (k$/yr) variation due to network reinforcement

Table 2. Effect of network reinforcement on cost estimation time

Method	Case A	Case B	Case C	Case D	Case E	Case F
MC (s)	35.23	1110.13	49.67	80.19	0.92	317.25
MLMC (s)	1.14	48.19	1.71	2.76	0.032	25.15

Figure 4 shows the effect of time-varying load and cost models in ECOST estimation while considering the starting of load point interruption at different times. Three different starting time depending on peak and off-peak hours are considered here. It can be clearly seen from Figure that ECOST values vary with the failure starting time and there is a significant difference when compared to average load and cost model based ECOST. At $ts = 8$ am, due to the peak hours for almost all load types during this period, maximum ECOST is achieved.

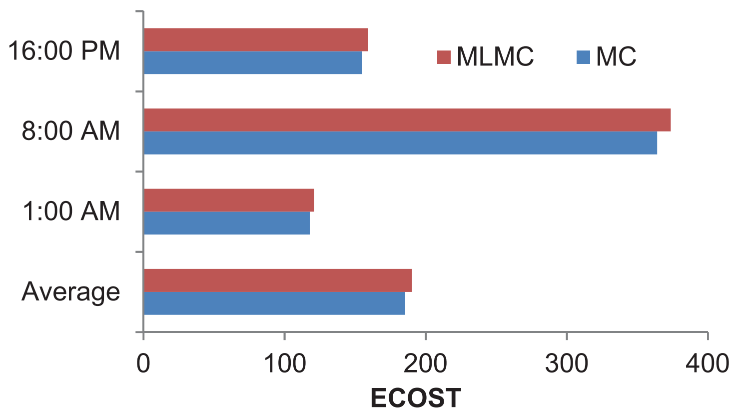

Figure 4. ECOST (k$/yr) variation due to time-varying load and cost models

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Table 3 shows the impacts of starting time on time-varying ECOST computation time using MLMC and MC methods. The results obtained for the test system show that ECOST computation cost will be varied for different starting times. It is possible to reduce the computation time of a system by using the MLMC method. Overall, we can save more than 90% of estimation time compared to MC based estimation.

Table 3. Effect of time-varying load and cost models on computation time

Method	Average	1:00 AM	8:00 AM	16:00 PM
MC (s)	35.23	15.87	134.37	25.43
MLMC (s)	1.14	0.51	5.04	0.87

Figure 5 presents the effects of network size and load types in ECOST variation. The RBTS distribution systems connected to five load buses (Bus2 to Bus6) is considered for this purpose [12]. It is seen that the maximum and minimum system ECOST values are found in B3 and B6 systems, respectively. From Eqn. (9), we can find that the amount of system ECOST depends on the failure rate, load level and interruption cost of the connected load points. In the B3 system, there are five types of loads such as residential, large users, small industrial, commercial and office building users and the total amount of average load for all 44 load points is 52.63 MW. On the other hand, in the B6 system, there are four types of loads such as residential, small industrial, commercial and agricultural and total amount of average load for all 40 load points is 10.7155 MW. In most of the cases, average load level per load point in the B3 system is higher than the B6 system. Due to having load points with high interruption cost and duration in B3, it gives the large value of ECOST than the B6 system.

As displayed in Table 4, the maximum and minimum computation times are required for the distribution systems connected to B3 and B6, respectively. In all cases, the percentage of computation speedup is above 90%. For example, the proposed and MC method need 1.85 and 49.33 seconds, respectively for RBTS B5 system.

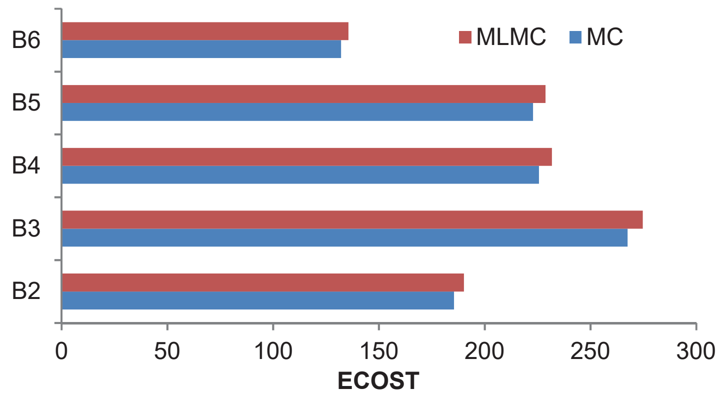

Figure 5. ECOST (k$/yr) variation due to network size and load types

Table 4. Effect of network size and load types on computation time

Method	B2	B3	B4	B5	B6
MC (s)	35.23	69.27	49.33	47.40	27.06
MLMC (s)	1.14	3.65	1.85	1.58	0.9

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6 Conclusion

Accurate and fast evaluation of reliability worth value of the distribution system is very important for its cost-effective planning, design, operation and power market structure. The paper has presented a new method for customer interruption cost estimation based on the Multilevel Monte Carlo method. The method can accelerate the estimation process by providing all benefits of traditionally used Monte Carlo method. The effects of different aspects are taken into consideration for accurate estimation such as network reinforcement, time-varying cost and load models rather than considering only average load models and customer damage function based cost models. The proposed method can be easily modified for cost estimation by considering the momentary interruption and the effects of time-varying weather dependent failure rates.

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