

# Efficient heat pump operation: a comparative analysis of model-predictive and rule-based heat pump control

Bernd RIEDERER, Daniel MUSCHICK, Astrid LEITNER, Mathias SCHWENDT, Valentin KAISERMAYER, Markus GÖLLES

BEST – Bioenergy and Sustainable Technologies GmbH, Inffeldgasse 21b, 8018 Graz, +43 5 02378-9201, [office@best-research.eu](mailto:office@best-research.eu), [www.best-research.eu](http://www.best-research.eu)

**Abstract:** With the energy system transitioning towards decentralized and renewable-based energy systems advanced digital solutions and technologies are required. Especially when it comes to heat pumps, which are directly coupling the electric and the thermal energy sector, and are therefore increasingly deployed, their optimal operation becomes crucial. Therefore, this study evaluates the performance of an optimization-based, model-predictive energy management system (EMS) for residential heat pumps and compares it with a traditional rule-based control strategy. To this end simulation studies are performed to investigate the annual performance of heat pumps in different residential system configurations, using heat and electricity demand data from a representative single-family home. The research demonstrates that for systems with a low degree of flexibility and shift potentials as well as moderately sized heat pumps, rule-based approaches can already provide efficient operating strategies. However, for more flexible systems model-predictive EMS – when implemented within a supervisory controller – achieves superior efficiency and cost savings by fully exploiting system flexibility. By maintaining identical operating conditions for both approaches, the analysis isolates the impact of control logic, highlighting the potential of digitalized optimization for decentralized, sector-coupled energy systems, which is up to 48% under optimal conditions within the most flexible scenarios.

**Keywords:** Energy efficiency, heat pump, control strategies, energy management, comparative analysis, single-family home

## 1 Introduction & Motivation

The transition towards decentralized and renewable-based energy systems requires advanced digital solutions to exploit flexibility regarding the electricity sector in buildings and residential environments. Particularly in the heating sector – responsible for around 60% (2023) of final energy demand in the EU [1] – air-source heat pumps coupled with photovoltaic (PV) systems, and thermal storage play a crucial role. Their integration offers significant potential for increasing energy efficiency [2] and reducing greenhouse gas emissions, yet also introduces operational challenges due to volatile PV generation, fluctuating thermal demand, varying coefficients of performance (COP) and dynamic electricity tariffs.

Heat pumps sit directly at the interface between electrical and thermal energy and are therefore highly sensitive to control strategies. Traditional rule-based approaches often fail to make full use of available flexibility or to coordinate PV generation, tariffs and heat demand effectively. Optimization-based model-predictive energy management systems (EMS) can inherently address these challenges and are therefore commonly studied within simulation studies.

These studies focus on the impact of the building mass as an additional thermal storage and other flexibility measures, while often being done without a comparison to a real-world applicable reference control strategy.

This work discusses a comparative analysis of such control strategies compared to rule-based ones that are currently implemented in real-world heat pump systems. The investigation uses several scenarios to study the impact of flexibility measures and storage sizes on the operational behaviour and net costs. Simulation studies are used to investigate the annual performance of a commonly sized 8 kW<sub>th</sub> and an oversized 12 kW<sub>th</sub> heat pump in different residential system configurations. Key performance indicators related to costs, energy efficiency, run time behaviour, and grid interaction are analysed. In the upcoming section 2 the different control strategies are explained in more detail. Section 3 deals with the study design and further details about the data preparation and the simulation scenarios. The results of the simulation studies are shown in section 4. Finally, the results and implications are discussed alongside remaining open questions in section 5.

## 2 Controller strategies

We compare operating strategies for the operation of a heat pump, for systems with either two individual buffer storages, where one is being used for space heating demand and another one for domestic hot water (DHW) demand, or a single buffer configuration with a freshwater module. In the following, the model-predictive optimization-based and the rule-based control strategy are explained in detail.

### 2.1 Model-predictive optimization-based control strategy

The supervisory MPC controller is based on a modular, model-predictive, optimization-based control framework that automatically constructs and solves a mixed-integer linear programme (MILP) for each considered system setup. The framework models typical components of residential energy systems, such as heat pumps, thermal storage, photovoltaic systems, and household electricity consumption, using linear models. Using these models, an optimization problem for determining the optimal operating schedule as well as the ideal buffer management is devised. This system-wide optimization problem thus provides an ideal control trajectory over a prediction horizon. The technical details on the models and the framework in general can be found in [3], [4].

For the controller to become fully predictive it also needs to integrate multiple forecast signals for quantities that cannot directly be derived from the models of the individual technologies. For this purpose, the following quantities are obtained via external services:

- electricity price (variable or fixed tariffs),
- ambient temperatures,
- solar irradiation.

Within the framework these signals are then further integrated into the optimization problem. The electricity prices can directly be used within the cost function of the optimization problem for all electricity import, as well as exports through the point of common coupling to the electricity grid. Ambient temperatures have a twofold usage: on the one hand directly influencing the model of the heat pump, and on the other hand as an input variable for internal

forecasts of heat and DHW demand. For the heat pump model, we use a variable COP that depends on both the feed temperature and the ambient temperature, which allows the optimization to take environmental conditions into account. For estimating the hot water demand within the building, we use a multi-linear regression (MLR) model [5], [6] that takes the time of the day, ambient temperature as well as historic demand data from up to 30 days before the time of actual operation scheduling into account. Thus, it becomes a self-learning and adapting model of the hot water demand. Finally, the solar irradiation is used for obtaining the power output of the PV cells for the prediction horizon. The obtained profile is also directly included within the optimization problem, allowing the control schedule to incorporate the additional power input.

Additionally, for specific occasions it is necessary to enforce a specific behaviour within the optimization problem, for instance in the case of modelling some additional low-level control behaviour, or external requirements such as shutdowns for specific day times due to e.g. noise constraints on the operation of the heat pump. These constraints can be explicitly added to the optimization problem

When it comes to deploying the framework as an operational controller within systems it is usually operated by re-solving the new optimization problems using updated measurements and forecasts on a regular basis, e.g. every 15 minutes.

## 2.2 Rule-based control strategy

The rule-based control (RBC) completely relies on setting threshold values for temperatures to be available at temperature sensors placed close to certain outlets within the thermal storages. Depending on the measured temperatures at the outlets and on additional day time and room temperature constraints, the heat pump is requested and started with a specific setpoint. Additionally, the availability of surplus power from e.g. PV generation is considered as an additional feature for the control strategy.

Within the current approach there are different operating strategies depending on whether there is a demand from the heating circuit, from domestic hot water or from surplus power. The details of the operating strategies are under disclosure, but the general principles will be discussed here shortly for the configuration with two buffers, as the single buffer strategy will be directly derived from this afterwards.

A heating demand triggers a heat pump startup if the temperature sensor close to the consumer outlet of the buffer used solely for heating falls under a certain temperature, the ambient temperature is below a certain temperature and the room temperature also falls below a variable setpoint. The setpoint for the room temperature depends on the time of day and is separated into comfort times and down times with different setpoint values respectively. After the heat pump has been started the compressor speed is varied to keep a constant temperature difference between the condenser inlet and the outlet. Once the buffer is filled, which is determined by another temperature threshold for a sensor close to the consumer inlet ports, the heat pump is shut down.

The trigger for the domestic hot water demand and the respective buffer is in principle the same as the above, however differing in detail concerning the temperatures, the constant temperature increase produced by the heat pump, as well as not being dependent on the room

or ambient temperature. To avoid a heat pump operation during especially cold times, operation is not allowed during night times.

For the optional surplus power control strategy, the startup of the heat pump is triggered when there is enough surplus power available. Whether this is the case is obtained from the current amount of required and produced electrical power and a nominal electrical power using an averaged heat pump COP. Apart from the startup the operation is the same as for the domestic hot water demand and is also used to feed into the domestic hot water storage. The shutdown in this case however is triggered at a higher threshold temperature for the consumer inlet port temperature of the buffer.

As a final side remark, the heat pump is always operated such that it can feed into a single storage at a time, and a switch needs to be done once a demand changes. The highest priority is always on domestic hot water demand, which would trigger a change of operating point. However, when switching from one buffer to another the heat pump is powered down and needs some down time before being started again for filling the other buffer storage. Additionally, the algorithm above describes the more complicated configuration setup with two individual buffer storages. For the single buffer storage configurations, the general rules stay the same, without the need for switching between filling modes, and the threshold temperatures being used for the respective consumer in- and outlets of the single storage, instead of the individual storages.

### **3 Methodology and Studied Scenarios**

The study is carried out by performing annual simulations of various configurations, based on real heating and domestic hot water demand data from a single-family house in Austria. The details on the configurations are disclosed in the upcoming section. This is then followed by a description of the data preprocessing and reconciliation procedure, which is done to obtain a gap-less time series for the whole simulation period. Finally, the simulation procedure itself is described.

#### **3.1 Study design and scenarios**

To investigate the benefits and strengths of the two different control strategies the following quantities have been varied:

- The maximum thermal output of the heat pump
- The size and existence of a PV system
- The configuration of the thermal buffer setup (single-buffer or two-buffer system)
- The storage capacities of the thermal buffers
- The electricity costs due to either a variable or a fixed tariff

Each of these quantities modifies the flexibility within the energy system and therefore provides a possibility for efficiency and cost improvements through varying operational strategies. From these parameters a total of 16 scenarios have been chosen to investigate the individual influences on the resulting operational behaviours for the different control strategies. The 8 different scenarios with a PV cell included in the simulations are listed in

Table 1. These scenarios are also simulated again without a PV system. The scenarios without solar power systems will be denoted with an additional asterisk (\*) next to the scenario name. The layouts of the different scenarios are shown in Figure 1.

Table 1: Different configurations for the individual simulation studies. 'HP' indicates the maximal thermal output of the heat pump, 'PV' the sizing of the PV cell, 'Buffer' and 'DHW' the sizing of the respective storages for heat- and domestic hot water demand, and 'Electricity tariff' whether a fixed or a variable tariff is used.

Scenario	HP	PV	Buffer	DHW	Electricity tariff
A-1	13 kW <sub>th</sub>	15 kW <sub>p</sub>	1500 l	-	variable
A-2	13 kW <sub>th</sub>	15 kW <sub>p</sub>	500 l	-	variable
B-1	8 kW <sub>th</sub>	10 kW <sub>p</sub>	300 l	300 l	variable
B-2	8 kW <sub>th</sub>	10 kW <sub>p</sub>	500 l	300 l	variable
B-3	8 kW <sub>th</sub>	10 kW <sub>p</sub>	750 l	300 l	variable
C-1	8 kW <sub>th</sub>	10 kW <sub>p</sub>	300 l	300 l	fixed
C-2	8 kW <sub>th</sub>	10 kW <sub>p</sub>	500 l	300 l	fixed
C-3	8 kW <sub>th</sub>	10 kW <sub>p</sub>	750 l	300 l	fixed

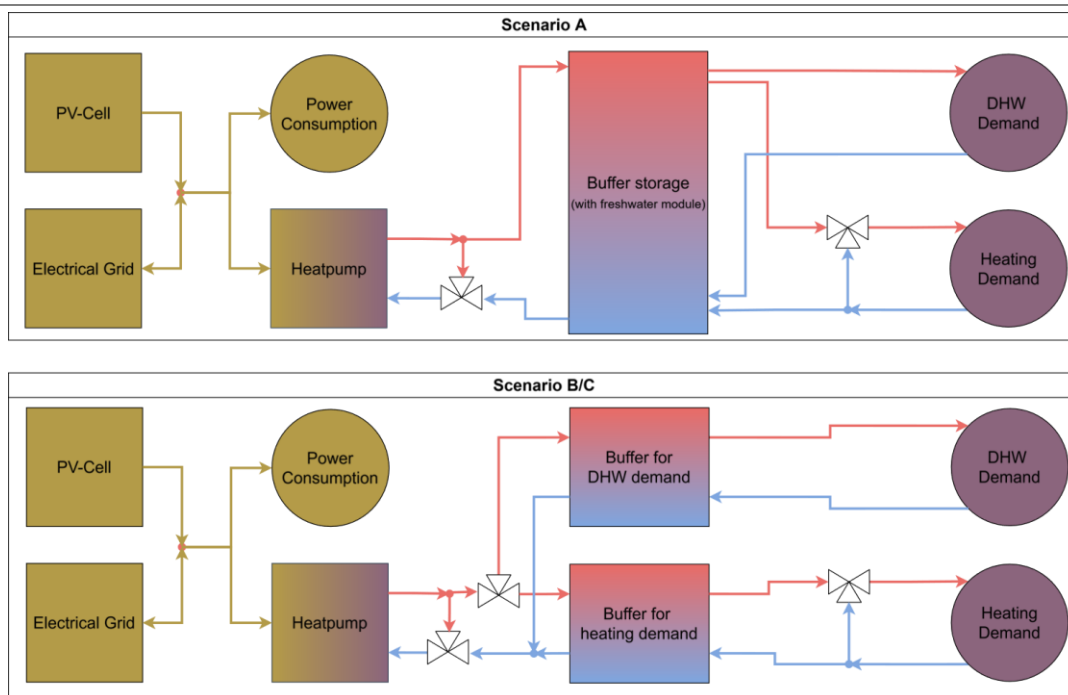


Figure 1: Setup of the different configurations for the studied scenarios.

For the different electricity tariff scenarios only the electricity costs have been used, neglecting any additional taxes or fees. The variable tariff uses EPEX SPOT prices from 2023 for import costs and export revenue. The fixed tariff uses 12 ct/kWh as import costs and 8 ct/kWh as export revenue, which is roughly the annual average of the variable tariffs which are approximately 11 ct/kWh and 8 ct/kWh respectively.

### 3.2 Data preparation

For the simulations to be carried out data from a real single-family home is required. The input data for this study includes domestic hot water demand, space heating demand, power consumption, global horizontal irradiation for the PV simulation, and measurements of room

temperature, and ambient temperature, as they are needed for the RBC strategies. The data has been collected for the year 2023 within a typical Austrian single-family home.

Due to individual sensor faults that have occurred during the recording time it has not been possible to continuously collect data with a single sensor and replacements were necessary. In this case the data collected by the individual sensors has been combined into one time series. Additionally, some datapoints simply have temporarily not been stored or measured properly, resulting in missing data for individual timestamps. In this case data from the previous day has been taken over as a naïve reconciliation method. However, the overall amount of data that required pre-processing stayed below 0.3% for the whole year.

The solar yield for our reference setup is also not directly available, since the real single-family home does not contain a PV system at the time of interest. Instead, the solar yield has been calculated from the GHI data available for the location of the home. For this purpose, it has been estimated that the PV cell has a southern orientation and an ideal inclination of  $37^\circ$ , with an overall efficiency of 15%. To obtain the time series for the solar the GHI data is used to scale DNI data calculated using the clear sky irradiance and then converted together using commonly known conversion models similar to those in *pvl*ib [7].

### 3.3 Simulation details

To assess the performance of the different control strategies on a comparable basis, the models and implementations of the optimization-based control framework, which is implemented using the Julia programming language [8], are used as a simulation backbone. In this framework, a multi-energy system representation is generated and both applications, i.e. the supervisory MPC and the RBC strategies, can be implemented and tested.

For the simulation using the supervisory MPC control strategies the input data for the demand has been directly passed to the MPC controller. Therefore, the controller can directly integrate prediction data into the simulations, thus using perfect foresight. To reduce the simulation time an optimization problem is being formed for each individual day with a prediction horizon of 48 hours. The first 32 hours are discretized into 15-minute time intervals and the remaining 16 hours into 4-hour intervals. By solving the optimization problem, the operation schedule for a single day with a resolution of 15 minutes is obtained. Then the simulation proceeds to the next day and the same procedure is performed again. The simulations for the supervisory MPC control strategy have been conducted using the *Gurobi* optimizer [9].

The rule-based strategies are done in the same moving horizon fashion but using a higher update frequency of 15 minutes per iteration instead of a full day. This is necessary because the controller has no knowledge about the future and can only react to new information. Each simulation step consists of using the input data as well as the current state of operation from the previous step, and of deriving the operating point for the next iteration according to the rules outlined in section 2.2. These are implemented by constraints on the same model also used for optimization that enforce the rule logic and fully determine the operating trajectory. While relying on optimization is not needed for rule-based control, it guarantees that identical physical models, boundary conditions, and numerical handling are used for all simulations. The rule-based simulations have been conducted using the *HiGHS* optimizer [10].

## 4 Key Results: Heat Pump Performance and Energy Efficiency

In the following we will present the results from the annual simulations for the individual scenarios. In the upcoming section, selected operation schedules from the single-buffer scenarios A and A\* will be shown to illustrate the changes in operational behaviour with and without PV, alongside the net cost reductions achieved. This is followed by the results for the obtained net costs of the two-buffer scenarios, as well as import costs and export revenues, as a function of the thermal buffer size, and a short discussion thereof.

### 4.1 Operation schedules and cost savings for large heat pumps (Scenarios A)

For the scenarios with a larger heat pump of 13 kW<sub>th</sub>, which would be considered oversized for a typical single family home, the operation schedules obtained from the simulation studies are shown in Figure 2 for 2 weeks at the end of March and beginning of April 2023.

When comparing the operation schedules for the heat pump using the RBC with the schedule from the MPC control in Figure 2 it can immediately be seen that for the RBC the heat pump is operated in a repetitive pattern, while the MPC control makes heavy usage of the modulation capability of the heat pump. By using this operational flexibility, the MPC control is able to operate the heat pump in times of high yield from the PV cells, as well as in times with a better performance of the heat pump. This is again verified by the plots on the side for the energy and heat pump and COP: It can be seen that the imported energy (blue bar/most left) is reduced in the MPC case, while the export (orange bar/second) is still in a similar range. Due to the higher average COP, the same heat demand can still be met, while in total consuming less electrical energy. Inspection of other periods of the year show the same pattern throughout all scenarios A. Also, it should be mentioned that we do not consider the surplus power rules for the rule-based strategy, as outlined in section 2.2, and thus the operational schedules stay unaffected for the scenarios without a PV cell. For the MPC simulations, however, the operational schedule changes slightly, having to optimize only for COP and cost-efficiency in these cases.

When comparing the annual net costs from the simulation study as shown in Table 2, it can be seen that in all scenarios the costs for the MPC control strategy are reduced compared to the rule-based strategy and are reaching up to a relative reduction of electricity costs by 48%. While the impact of the buffer size is almost negligible, a significant reduction can be achieved by using a PV cell.

*Table 2: Obtained annual net costs (rounded to €) for the scenarios A and A\*, for the different control strategies, alongside the relative change in costs for comparing the MPC strategy with the rule-based one. Negative costs indicate net benefit due to power export rewards.*

Scenario	RBC	MPC	Relative change
A-1	-384 €	-538 €	-40%
A-2	-334 €	-496 €	-48%
A-1*	1083 €	913 €	-16%
A-2*	1111 €	959 €	-14%

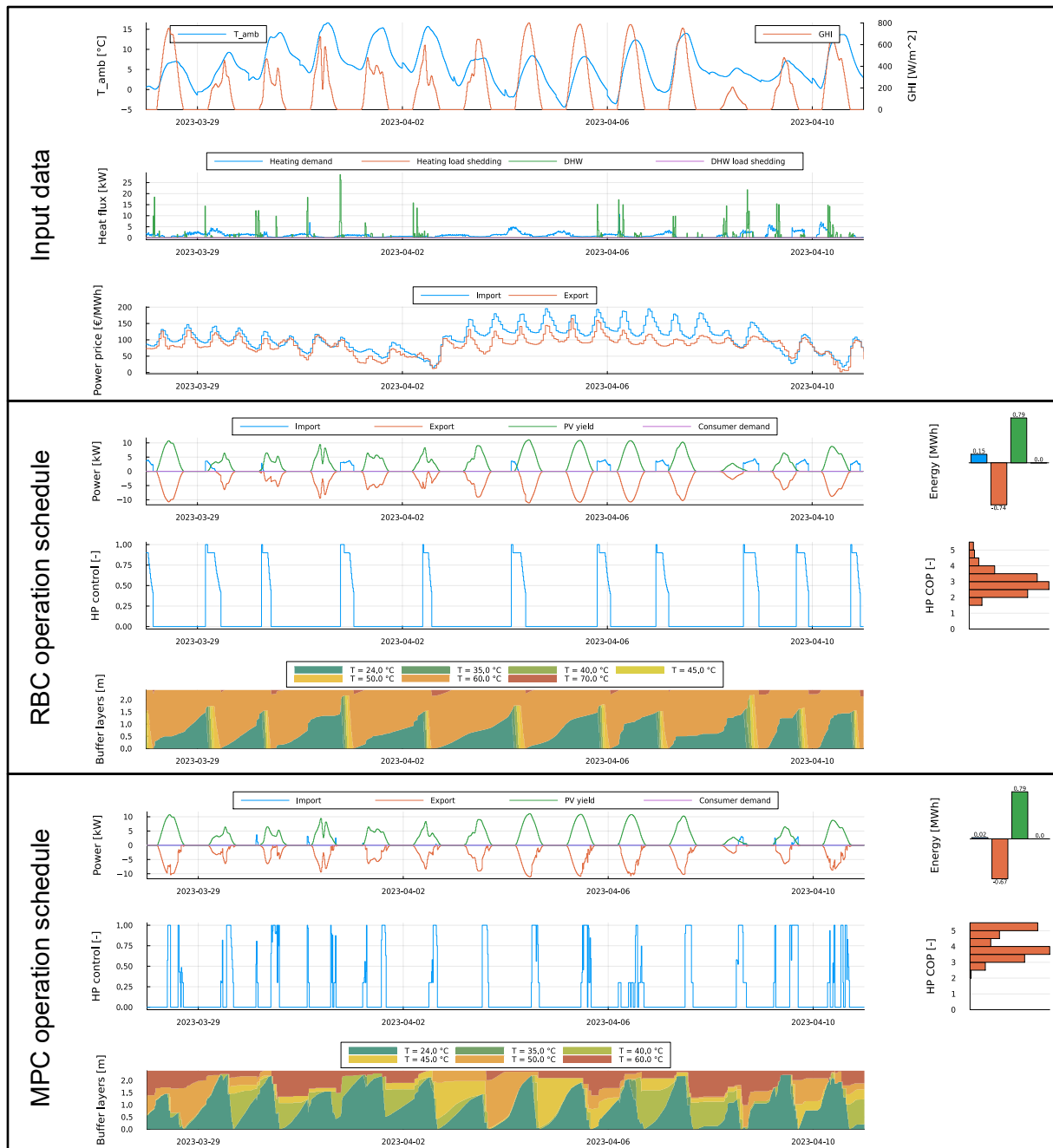


Figure 2: Comparison of operation schedules for the different control strategies, for 2 weeks at the end of the heating season. In the top panel the input data of the simulation is shown. In the middle and the bottom panel, the power time series, the operation schedule of the heat pump and the buffer fill-states for the two operating strategies (rule-based control and MPC) are shown. Additionally, the total power import, export and self-production is shown on the right-hand side, as well as a histogram of the heat pump COPs. Power demand has been set to zero for the sake of data privacy.

#### 4.2 Cost efficiency vs. flexibility (Scenarios B and C)

Following the detailed investigation of the operation schedules for scenario A in the previous section, we are now going to show the results of scenarios B and C, which focus on studying the cost efficiency as a function of the buffer storage size in the two buffer configurations.

We start with the scenarios without a PV cell. The results for the annual net costs obtained in the simulations for scenarios B\* and C\* are shown in Figure 3. For both control strategies, the scenario C with fixed import and export tariffs are more expensive than the variable tariffs when

using data from the year 2023. In the case of fixed tariffs, it turns out that the MPC strategy results in higher annual operating costs, which is due to the small buffer storages. In these scenarios the two-buffer setup requires the optimization problem to switch between filling the individual buffers and is therefore forced to operate much more like the rule-based approach in general. The worse performance can then be traced back to the average COP within the simulation: the average COP is lower for all MPC strategy simulations compared to the rule-based ones. While this indicates that a rule-based approach may be sufficient or better performing, the trends of the costs as a function of buffer size clearly indicate that the MPC strategy becomes more efficient with increasing storage capacity, while the rule-based one stays unchanged overall.

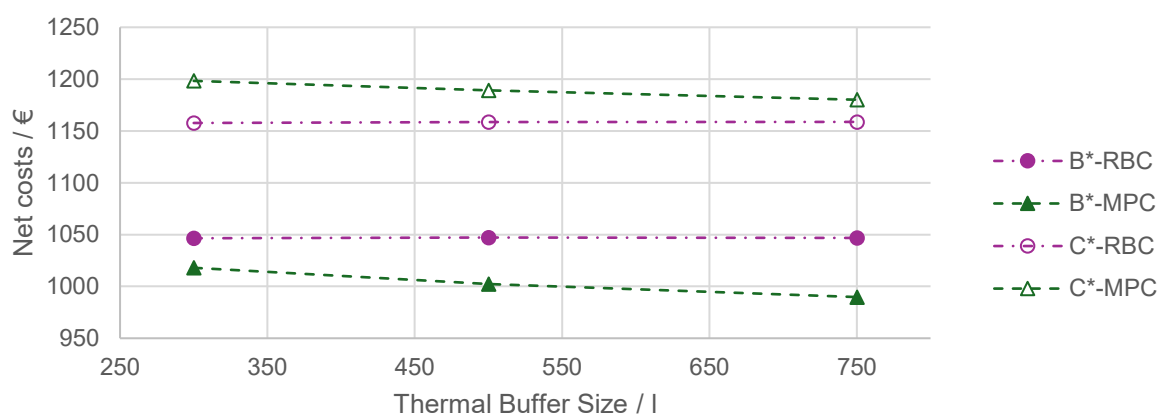


Figure 3: Annual net costs for the rule-based control strategy (violet circles) and the MPC strategy (green triangles) as a function of thermal buffer size for the scenarios B\* and C\* without a PV cell. Lines are added to guide the eye.

For the scenarios with a PV cell included the situation, however, is drastically different. In Figure 4 the results for annual power import are shown and in Figure 5 the results for the annual import costs, the export revenue as well as the net costs are shown for the RBC and the MPC strategy. In this case the additional rule for considering surplus power from the PV cell within the rule-based strategy is also simulated individually.

From the annual import costs in Figure 5a we see that the rule-based strategies show a slight upwards trend for increasing buffer sizes. This is consistent with the fact that larger buffer storages require more energy until the temperature thresholds at the sensors are reached and therefore result in a longer heat pump operation on average, which in turn will result in a more averaged performance of the heat pump overall. This is further supported by the behaviour of the overall power import shown in Figure 4a. The surplus power usage strategy reduces the import power and consequently the costs compared to the usual strategy as expected.

The MPC strategy however manages to reduce the import power and therefore also significantly reduces the import costs compared to either of the rule-based strategies due to its predictive nature. While in the case of the smallest buffer the fixed price scenario (scenario C, green unfilled triangle) the costs are within the same range as the rule-based approaches for small buffer sizes this is drastically different for all other scenarios. When looking at the annual export revenue in Figure 5b however, the MPC strategy is continuously losing on revenue. This is due to a higher degree of self-sufficiency compared to the rule-based approaches which can be seen in Figure 4b. The rule-based approaches are again almost unaffected by the buffer size, but the revenue is larger for the fixed price scenarios (unfilled

symbols), contrary to the MPC strategy, which achieves higher revenues for the variable tariff. When combining the results for import and export into the annual net costs, which are shown in Figure 5c we see that while all rule-based strategies tend to have increasing costs for larger buffer sizes, the MPC strategies significantly reduce the costs for larger buffer sizes. While this trend is not expected to be continued for arbitrary buffer sizes, it is a good indication that with increasing flexibility MPC control strategies become superior compared to rule-based approaches.

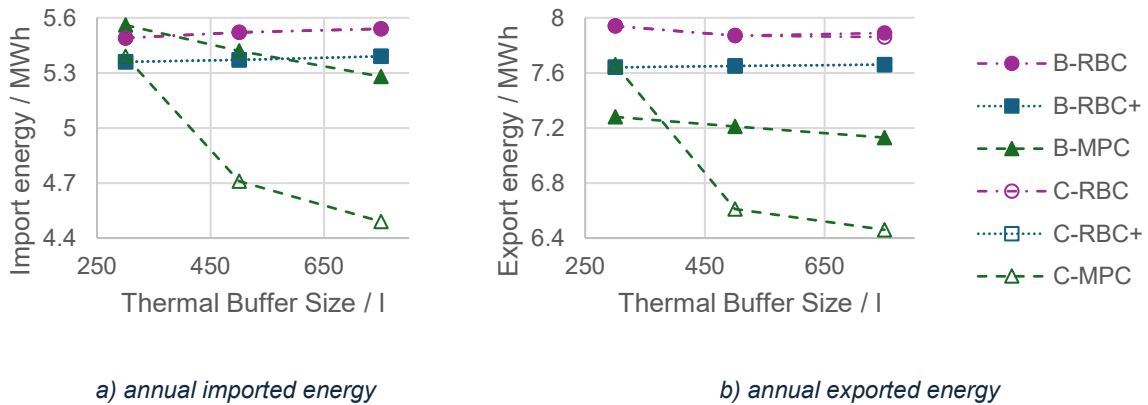


Figure 4: Annual imported (a) and exported energy (b) for the usual RBC strategy (violet circles), the RBC with surplus utilization (RBC+/blue squares), and the MPC strategy (green triangles) as a function of thermal buffer size for the scenarios B and C with a PV cell. Lines are added as a guide to the eye.

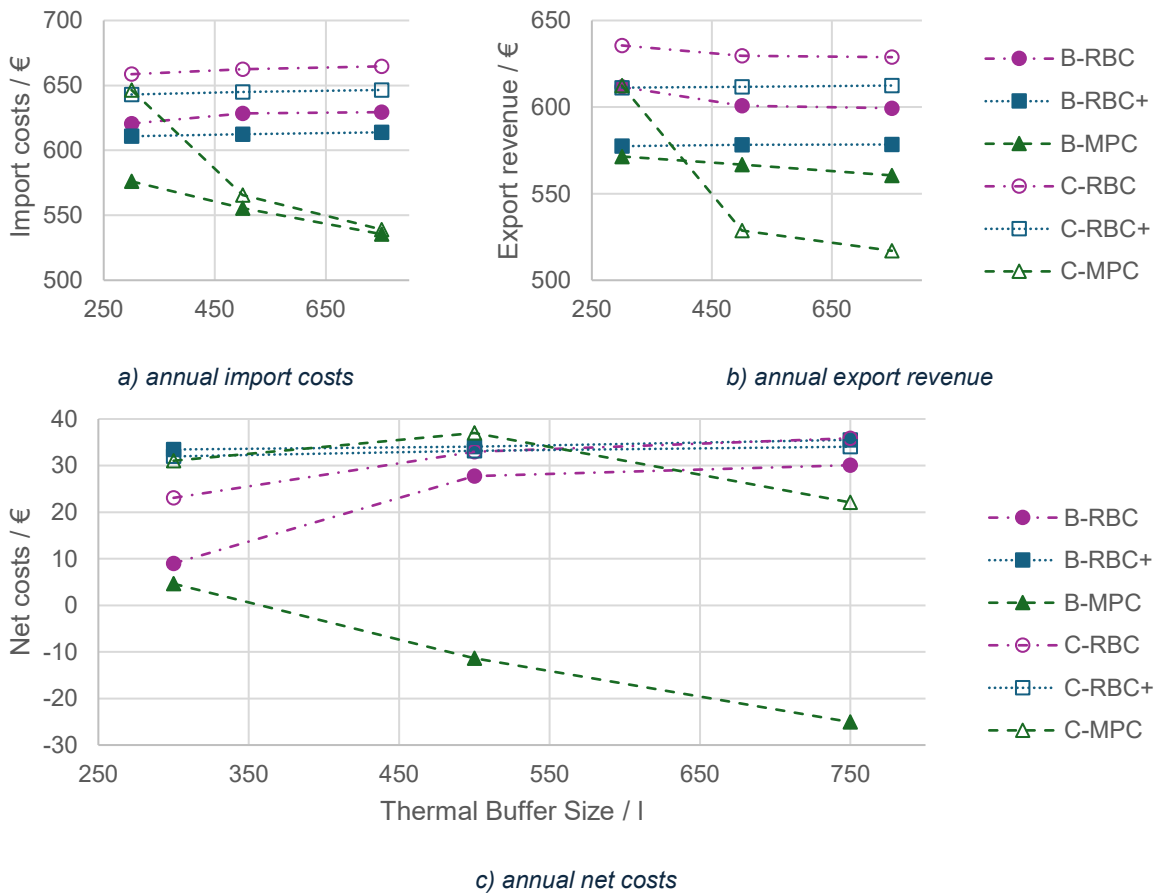


Figure 5: Annual import costs (a), export revenue (b) and net costs (c) for the usual RBC strategy (violet circles), the RBC with surplus utilization (RBC+/blue squares), and the MPC strategy (green triangles) as a function of thermal buffer size for the scenarios B and C with a PV cell. Lines are added as a guide to the eye.

## 5 Conclusion & Outlook

This study demonstrates the substantial efficiency and cost-saving potential of optimization-based, model-predictive energy management for residential heat pump systems in case of available flexibility. The consistent execution of both strategies using the same supervisory controller framework – either as a fully predictive EMS or with rule-based behaviour implemented through additional constraints – ensures that observed differences are attributable solely to the control logic itself, showcasing the maximal potential of such systems. From the study results it can be concluded that for commonly sized systems rule-based strategies can already lead to efficient operation of the systems. Additional rules for the usage of surplus power from PV have shown to not reduce the net operating costs but increases the self-sufficiency of the system. However, it needs to be mentioned that the time resolution of simulation is much lower than it would be used in a real system and thus might lower the gains for this rule-based control strategy due to slow reaction times to surplus availability.

On the other hand, the results showed that a supervisory control strategy substantially benefits from systems with more flexibility either provided by higher output powers of the heat pump, larger buffer storages, availability of PV systems or from variable electricity tariffs. In each case, the net operating costs for the energy system decreased with an increase in flexibility. Especially, in the cases of very high flexibility, as for instance the configuration with a single 500l buffer, a 15 kW<sub>p</sub> PV cell and a 13 kW<sub>th</sub> heat pump (scenario A-1) the net costs, without additional fees and taxes, for the annual operation have been reduced by up to 48%.

For this reason, systems with a high degree of flexibility should generally consider the usage of supervisory, predictive model-based control strategies for their systems. As systems are expected to grow more complex in the following years, with the integration of additional storages such as batteries, and the possibility of considering the building mass as additional thermal storage, supervisory MPC frameworks provide a suitable and economically more beneficial control approach.

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